

3.1 Differential Equation of simple harmonic motion(S.H.M):

A particle executing S.H.M is called a harmonic oscillator. Consider a particle of mass ‘m’ executes S.H.M along a straight line (as in fig). Let y be the displacement of particle from mean position at any time t.



B

The restoring force F is proportional to displacement y and oppositely directed i.e:

$$F \propto -y$$

$$F = -ky \text{ -----(1)}$$

k- Constant of proportionality (spring factor)

If $a = \frac{d^2y}{dt^2}$ is acceleration at any instant t

Then by Newton’s second law of motion

$$F = \text{mass X acceleration}$$

$$= ma \text{ -----(2)}$$

From the equations (1) and (2)

$$m \frac{d^2y}{dt^2} = -ky$$

$$m \frac{d^2y}{dt^2} + ky = 0$$

$$\frac{d^2y}{dt^2} + \frac{k}{m} y = 0$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \text{ -----(3)}$$

Where $\omega^2 = \frac{k}{m}$ is a constant

ω is known as angular frequency.

The eqn(3) represents the differential equation of S.H.M.

The general solution of differential equation for S.H.M is given by

$$Y = A \sin(\omega t + \varphi)$$

Where A - Amplitude of the S.H.M

φ - is the initial phase.

