Forward and Reverse kinematics transformation for RR robot with 2 DOF with 2D

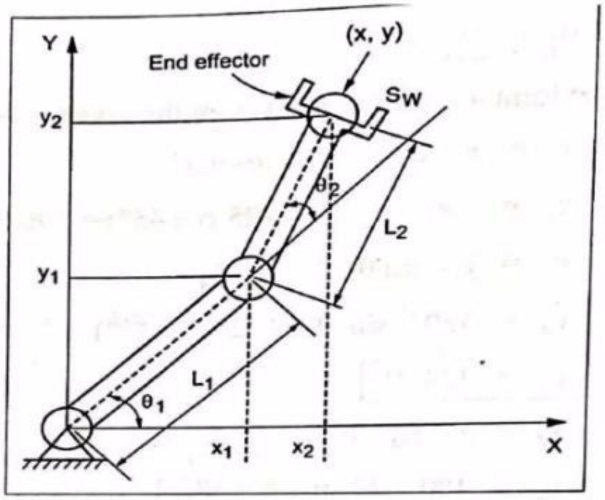

Fig. 6.13. Forward and Reverse transmission for a Robot with two joints

In the forward kinematics, if we are given the joint angles, we have to find out the position of the end effector.

- A manipulator with two degrees of freedom all rotational, in which the two joints represents a simple wrist.
- The robot has a RR configuration.
- The arm and body (R) provides position of the end of the arm and wrist 'R' provides orientation.
- The robot is still limited to the x, y plane.
- We have defined the origin of the axis system at the centre of joint 1.

Forward Kinematics/Direct Kinematics

The position of the end arm in world space by defining vector for link 1 ar^d another for link 2.

From Figure 6.13, $r_1 = L_1 \cos\theta_1,\ L_2 \sin\theta_2$ ... (6.23)

$r_2 = L_2 \cos(\theta_1 + \theta_2),\ L_2 \sin(\theta_1 + \theta_2)$ ... (6.24)

Let L_1 be the length of the arm 1.

L_2 be the length of the arm 2.

Link L_1 makes an angle θ_1 with horizontal.

Link L_2 makes an angle θ_2 with link L_1.

— The end point of the robot is at $S_W = (x, y)$

— For the forward kinematics, we can compute x and y coordinates.

$$\boxed{\begin{aligned} x &= L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) \\ y &= L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) \end{aligned}}$$

Reverse / Backward / Inverse kinematics:

✓ In reverse kinematics, we have given world coordinates (x and y) and we war^t to calculate the joint values θ_1 and θ_2.

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) \quad \text{... (6.25)}$$

$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) \quad \text{... (6.26)}$$

Squaring and adding equations (6.25) and (6.26), we get

$$\begin{aligned} x^2 + y^2 = {} & L_1^2 \cos^2\theta_1 + L_2^2 \cos^2(\theta_1 + \theta_2) + 2 L_1 L_2 \cos\theta_1 \cdot \cos(\theta_1 + \theta_2) \\ & + L_1^2 \sin^2\theta_1 + L_2^2 \sin^2(\theta_1 + \theta_2) + 2 L_1 L_2 \sin\theta_1 \cdot \sin(\theta_1 + \theta_2) \end{aligned}$$

$[\because (A + B)^2 = A^2 + B^2 + 2AB]$

$$\begin{aligned} x^2 + y^2 = {} & L_1^2 \{(\cos^2\theta + \sin^2\theta)\} + L_2^2 \{\cos^2(\theta_1 + \theta_2) + \sin^2(\theta_1 + \theta_2)\} \\ & + 2 L_1 L_2 [\cos\theta_1 \cdot \cos(\theta_1 + \theta_2) + \sin\theta_1 \cdot \sin(\theta_1 + \theta_2)] \end{aligned}$$

$$x^2 + y^2 = L_1^2 \; [1] + L_2^2 \; [1] + 2 L_1 L_2 [\cos\theta_1 \cdot \cos(\theta_1 + \theta_2) +$$

$$\sin\theta_1 \cdot \sin(\theta_1 + \theta$$

$$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2(\theta_1 + \theta_2) + \cos^2(\theta_1 + \theta_2) =$$

$$x^2 + y^2 = L_1^2 + L_2^2 + 2 L_1 L_2 [\cos\theta_1 \cos(\theta_1 + \theta_2) + \sin\theta_1 \cdot \sin(\theta_1 + \theta_2)]$$

$$x^2 + y^2 = L_1^2 + L_2^2 + 2 L_1 L_2 [\cos(\theta_1 - (\theta_1 + \theta_2))]$$

$$[\because \cos A \cos B + \sin A \cdot \sin B = \cos(A - B)]$$

$$x^2 + y^2 = L_1^2 + L_2^2 + 2 L_1 L_2 [\cos\theta_1 - \theta_1 + \theta_2]$$

$$= L_1^2 + L_2^2 + 2 L_1 L_2 [\cos\theta_2]$$

$$\cos\theta_2 = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2 L_1 L_2}$$

$$\boxed{\theta_2 = \cos^{-1}\left[\frac{x^2 + y^2 - L_1^2 - L_2^2}{2 L_1 L_2}\right]}$$