2.2METHOD OF JOINTS

In this method, after determining the reactions at the supports, the equilibrium of every joint is considered.Thismeansthesumofalltheverticalforcesaswellasthehorizontal forces acting on a joint is equated to zero. The joint should be selected in such a way thatatanytimethereareonlytwomembers,inwhichtheforcesareunknown.Theforce in the member will be compressive if the member pushes the joint to which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.

Example2.2.1 Atrussof8mspanconsistingofsevenmemberseachof4m lengthsupportedatitsendsandloadedasshowninFig.5.2.Determinethe forces in the members by method of joints

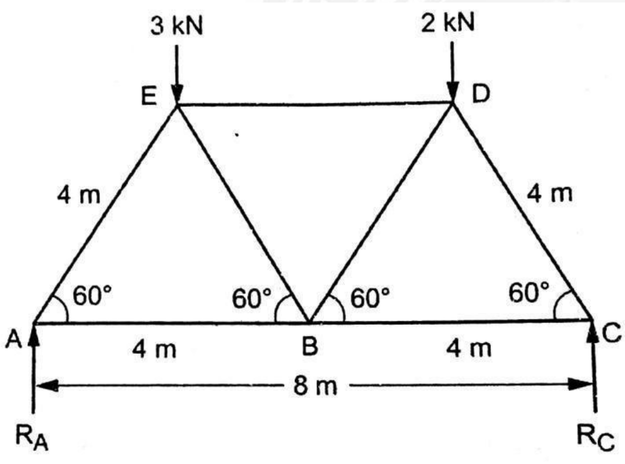

Fig.2.2.a

Solution:

Determine the reactions at A and C

Taking moment about A

$R_C \times 8 = R_D \times 6 \times R_E \times 2$ Or

$R_C \times 8 = 2 \times 6 \times 3 \times 2$ Or

$\mathbf{R_C = 2.25\ kN}$

We know that,

Up ward vertical reaction = Download vertical reaction

RA+ RC = 3 + 2

Or $R_A + 2.25 = 5$

Or $\mathbf{R_A = 2.75 kN}$

Consider the joint A.

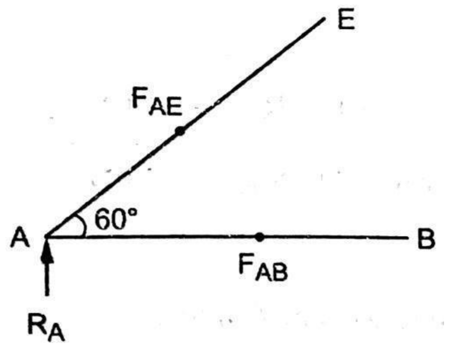

Fig.2.2.b

Assume the forces (F_{AE} and F_{AB})acting on joint A are tensile forces(acting away from joint A). If we get negative value the force in that member is compressive.

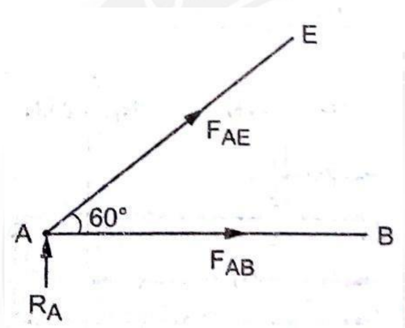

Fig.2.2.c

At joint A:

Resolving the force(F_{AE})vertically, we know that the sum of vertical forces=0.

$R_A + F_{AE} \sin 60^o = 0$

Or $2.75 = -F_{AE} \sin 60^o$

Or $\mathbf{F_{AE} = -3.17Kn}$

(Compression)

Resolving the force(F_{AE})horizontally,

Sum of horizontal forces = 0

$F_{AB} + F_{AE} \cos 60^o = 0$

Or $F_{AB} = -F_{AE} \cos 60^o$

Or $\mathbf{F_{AB} = -1.58kN}$(Tension)

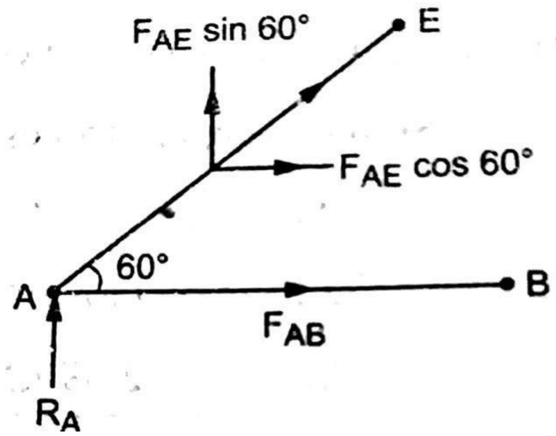

Fig.2.2.d

Consider the joint C.

Assume the forces (F_{DC} and F_{BC}) acting on joint C are tensile forces(acting away from joint C). If we get negative value the force in that member is compressive.

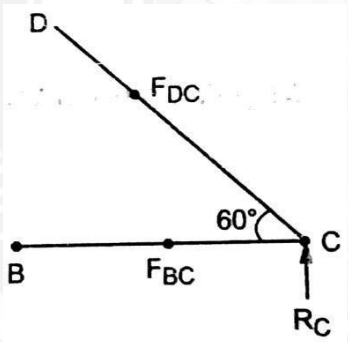

Fig.5.2.e

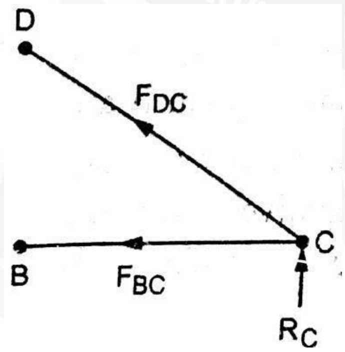

Fig.2.2.f

At joint C:

Resolving the force(F_{DC})vertically, Sum of horizontal forces = 0

Or $R_C+F_{DC}\sin 60^o=0$

Or $2.25 = -F_{DC}\sin 60^o$

Or $\mathbf{F_{DC} = -2.59kN}$

(Compression)

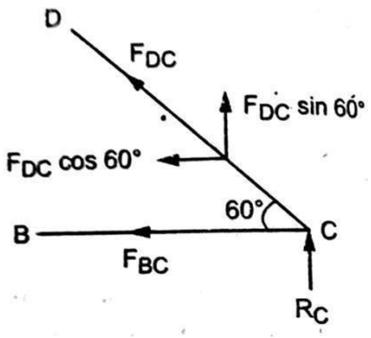

Fig.2.2.h

Resolving the force horizontally,

Sum of horizontal forces=0

$-F_{BC}-F_{DC}\cos60^o=0$

(Force acting towards left side is –ve)

Or $F_{BC} = -F_{DC}\cos60^o$

Or $F_{BC}= -2.59 \times \cos 60^o$Or

F_{BC}= 1.295kN (Tension)

Consider the joint B.

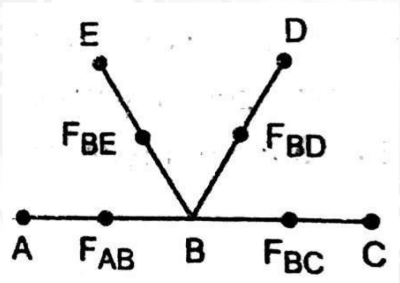

Fig.2.2.i

Assume the forces(F_{AB},F_{BC},F_{BD} and F_{BC})acting on joint B are tensile forces (acting away from joint B). If we get negative value the force in that member is compressive.

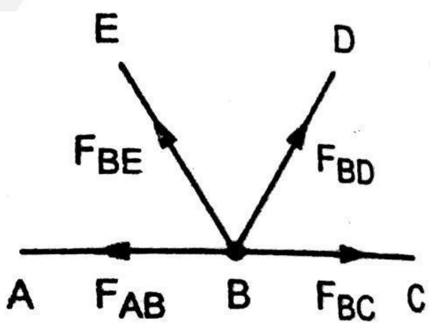

Fig.2.2.j

At joint B:

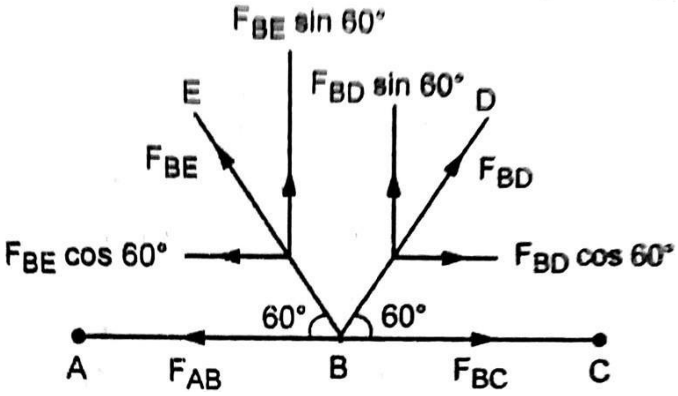

Fig.2.2.k

Resolving the force (F_{BE}&F_{BD}) vertically,

Sum of horizontal forces=0

$F_{BE}\sin 60^o + F_{BD}\sin 60^o = 0$

Or $F_{BE}\sin 60^o = -F_{BD}\sin 60^o$ Or

$\mathbf{F_{BE} = -F_{BD}}$

Resolving the force (F_{BE}&F_{BD}) horizontally,

Sum of horizontal forces = 0

$\Sigma H = 0$

$-F_{AB} - F_{BE}\cos 60^o + F_{BC} + F_{BD}\cos 60^o = 0$

(Force acting towards right side is –ve, force towards left sideis –ve)

Or $-1.58 - F_{BE}\cos 60^o + 1.29 + F_{BD}\cos 60^o = 0$

Or $-0.29 + 2F_{BD}\cos 60^o = 0$

(Since, $F_{BE} = -F_{BD}$)

Or **FBD=0.29 kN** (Tension)

Or $\mathbf{F_{BE} = -0.29}$ **kN**(Compression)

Consider the joint D.

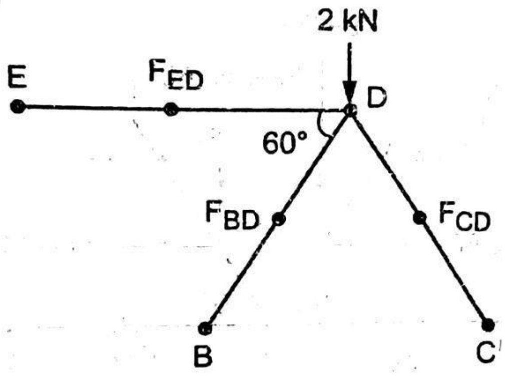

Fig.2.2.l

Assume the forces (F_{ED}, F_{BD}, and F_{CD}) acting on joint D are tensile forces (acting away from joint D). If we get negative value, the force on that member is compressive.

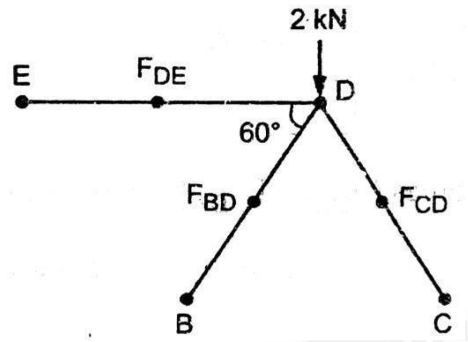

Fig.2.2.m

At joint D:

Resolving the force(F_{BD}&F_{CD})horizontally, sum of horizontal lforces=0

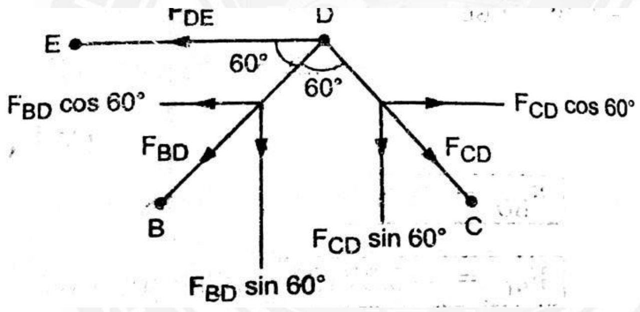

Fig.2.2.n

$-F_{DE} - F_{BD}\cos 60^o + F_{CD} + F_{BD}\cos 60^o = 0$

(Force towards right side +ve, force towards left side –ve)

Or $- F_{DE} - 0.29 \times \cos 60^o - 2.59 \times \cos 60^o = 0$

Or **F_{DE}=-1.44 kN**(Compression)

Result:

SI. No.	Member	Force (kN)	Nature of force.
1	AE	-3.17	compression
2	AB	1.58	Tension
3	CD	-2.59	compression
4	BC	1.29	Tension
5	BD	0.29	Tension
6	BE	-0.29	compression

7	**DE**	**-1.44**	**compression**

Example 2.2.2 DeterminetheforcesinthetrussshowninFig2.3.Itcarriesa horizontal load of 16 kN and vertical load of 24 kN.

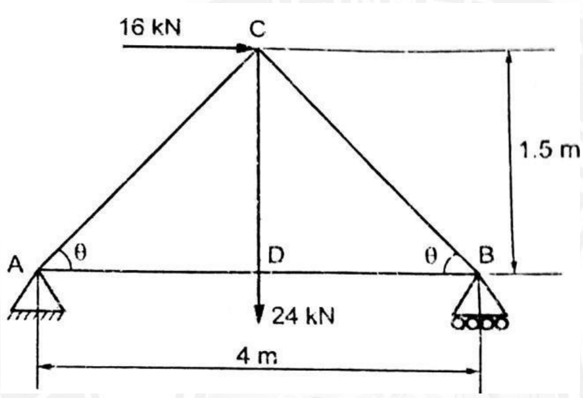

Fig.2.3.a

Solution: The truss is supported on rollers at B and hence there action at B should be vertical 9(R_B).

The truss in hinged at A and hence end A consist so horizontal reaction (H_A) and vertical reaction (R_A).

Determine the reaction at A and B (R_A and R_B).

Taking moment about A.

$$R_B \times 4 = 24 \times 2 + 16 \times 1.5$$

R_B=18kN

We know that,

Upward vertical load=Download vertical load R_A

$+ R_B = 24$

$R_A + 18 = 24$

R_A= 6kN

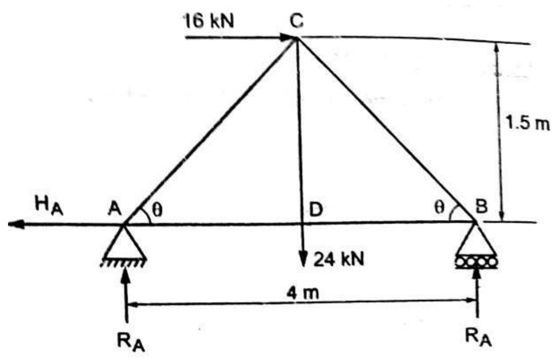

Fig.2.3.b

Right side horizontal load = Left side horizontal load

$$16 = H_A$$

HA=16kN

In the triangle BCD

$$BC^2 = CD^2 + BD^2$$
$$BC^2 = (1.5)^2 + 2^2$$

BC=2.5 m

$$\text{Sin}\,\theta = \frac{DC}{BC} = \frac{1.5}{2.5}$$
$$\text{Sin}\theta = 0.6$$
$$\boldsymbol{\theta = 36.8^0}$$

Consider the joint A.

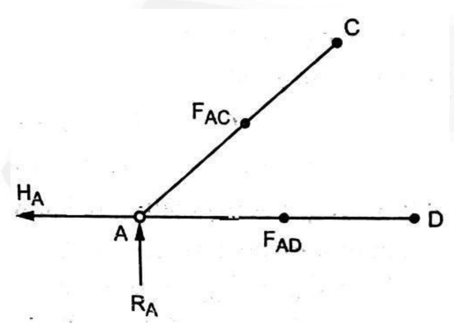

Fig.2.3.c

Assume the forces F_{AC} and F_{AD} acting on joint A are tensile forces(acting away from joint A). If we get negative value force on that member is compressive.

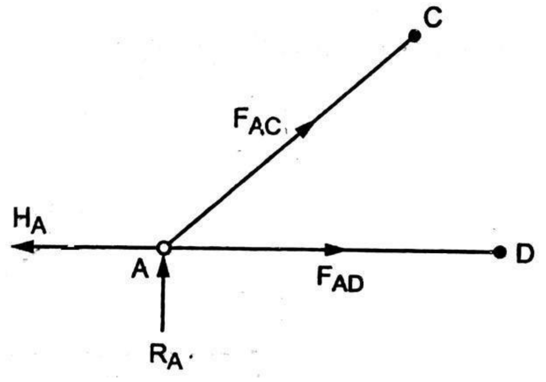

Fig.2.3.d

Atjoint A:

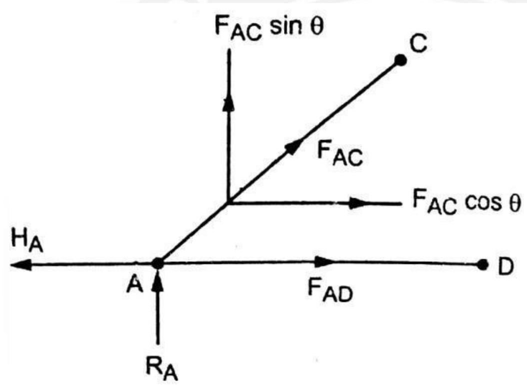

Fig.2.3.e

Resolving the force(F_{AC}) vertically, we know that

Sum of vertical forces = 0

$R_A + F_{AC}Sin\theta=0$

$6+F_{AC}Sin\ 36.8^0=0$

$6=-F_{AC}Sin36.8^0$

F_{AC} = - 10KN (Compressive)

Resolving the force(F_{AC})horizontally, we know that,

Sum of horizontal forces=0

$-H_A+F_{AD}+F_{AC}Cos\theta=0$

$-16+F_{AD}+-10\times Cos36.8^0=0$

$\mathbf{F_{AD}=24kN}$(Tension)

Consider the joint B.

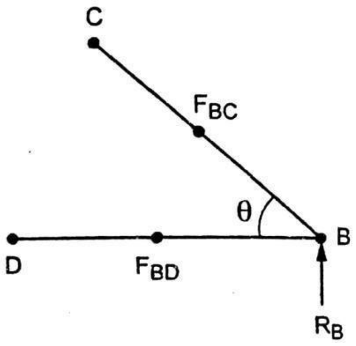

Fig.2.3.f

Assume the forces F_{BC} and F_{BD} acting on joint B are tensile forces(acting away from B). If we get negative value, force in that member is compressive.

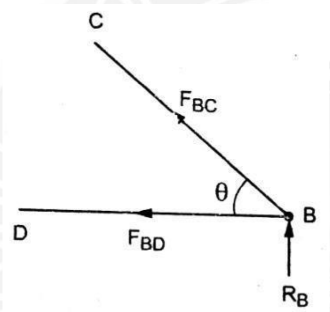

Fig.2.3.g

At joint B:

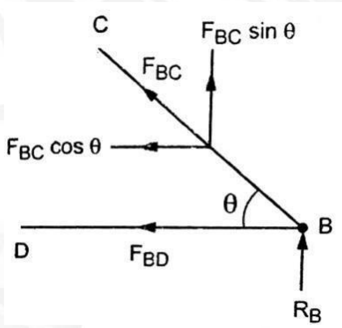

Fig.2.3.g

Resolving the force(F_{BC})vertically, we know that,

Sum of vertical forces = 0

$R_B+F_{BC}\sin\theta=0$

$18+F_{BC}\sin\theta=0$

$F_{BC} = -\frac{18}{\sin 36.8}$

F_{BC}=-30kN(Compressive)

Resolving the force (F_{BC}) horizontally,

We know that,

Sum of horizontal forces=0

$- F_{BC}Cos\theta - F_{BD} = 0$

$- F_{BC}Cos\theta = F_{BD}$

$$30 \, Cos \, \theta = F_{BD}$$

$$30 Cos \, 36.8^0 = F_{BD}$$

F_{BD}=24kN (Tension)

Result:

Sl.No.	Member	Force (kN)	Natureofforce
1	**AC**	**-10**	**Compression**
2	**AD**	**24**	**Tension**
3	**BC**	**-30**	**Compression**
4	**BD**	**24**	**Tension**

Example2.2.3 A truss is loaded as in Fig 2.4.Determine the forces in all the members of that truss.

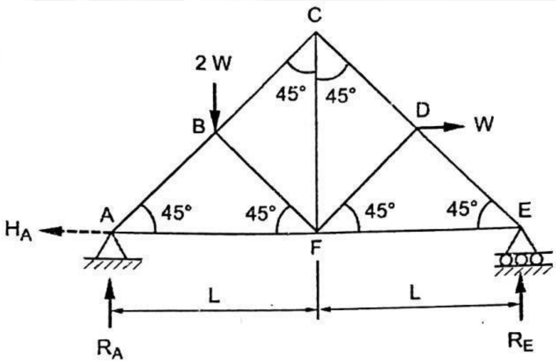

Fig2.4.a

Solution: To solve the above problem, consider W=1kN

To find the reactions at the support:

$$R_{A+}R_E = 2W = 2$$

$H_A = W = 1$

Taking moment about point A,

$$i.e., \quad \Sigma MA = 0$$

$$-(RE \times 2L) \times \left(2 \times \frac{L}{2}\right) + \left(1 \times \frac{L}{2}\right) = 0$$

$$\left(L \times \frac{L}{2}\right) = R_E \times 2L$$

3

$$\frac{}{2}L = R_E \times 2L$$

$$\mathbf{R_E = 0.75}$$

$$R_A + R_E = 2$$

$$R_A = 2 - R_E = 2 - 0.75$$

$$R_A = 1.25$$

Solving the above problem using method of joints:

At joint A:

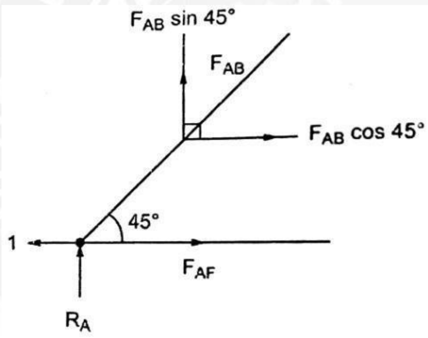

Fig2.4.b

Sum of horizontal forces $= \Sigma H = 0$

Sum of vertical forces $= \Sigma V = 0$

$\Sigma V = 0, \quad R_A + F_{AB}.\sin 45^0 = 0$

$$F_{AB}.\sin 45^0 = -R_A$$

$$F_{AB} = \frac{R_A}{\sin 45°} = \frac{1.25}{\sin 45°}$$

$\mathbf{F_{AB} = -1.77}$(Compression)

$\Sigma H = 0, \quad -1 + F_{AB}\cos 45^0 + F_{AF} = 0$

$$-1 + (-1.77 \times \cos 45^0) + F_{AF} = 0$$

$\mathbf{F_{AF} = 2.25}$ (Tension)

At joint E:

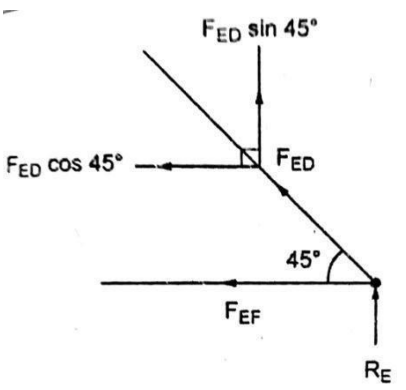

Fig2.4.c

$\Sigma V=0$

$F_{ED}.\sin 45 + R_E = 0$

$F_{ED} = \frac{E}{\sin 45°} = \frac{.}{\sin 45°}075$

F_{ED}= -1.06(Compressive)

$\Sigma H=0$

$-F_{EF} - F_{ED}\cos 45^0=0$

$-F_{EF}-(-1.06 \times \cos 45^0)=0$

$-F_{EF}+0.75 =0$

F_{EF}=0.75(Tensile)

At joint B:

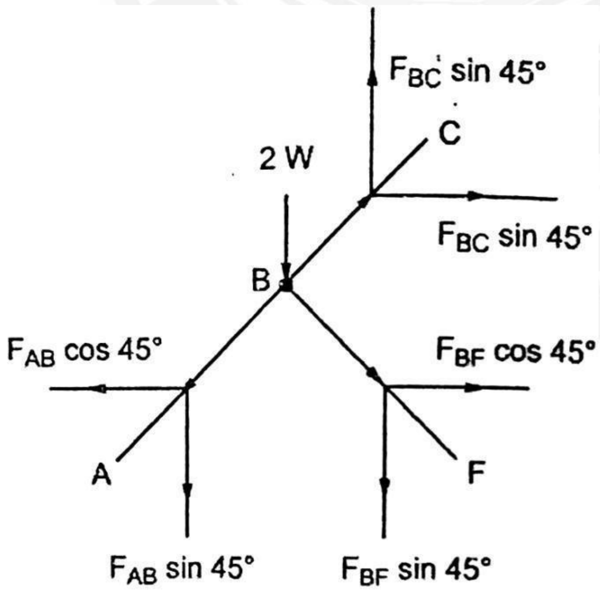

Fig2.4.d

$\Sigma H=0$

$- F_{AB}\cos45^0+F_{BC}\cos45^{0+}F_{BF}\cos45^0=0$ cos

450 $[- F_{AB}+ F_{BC}+ \quad F_{BF}] = 0$

$F_{BC}{}^{+}F_{BF}=+F_{AB}=-1.77$

$F_{BC}{}^{+}F_{BF}=-1.77$ …..(1)

$\Sigma V = 0$

$F_{BC}\sin45^0 – F_{AB}\sin45^0 – F_{BF}\sin45^0 – 2=0$

$F_{BC} \sin 45^0 – F_{BF} \sin 45^0 = 2 + F_{BA}. \sin 45^0$

$F_{BC}\sin45^0 – F_{BF}\sin45^0 = 2+(-1.77\sin45^0)$

$(F_{BC} – F_{BF})\sin45^0 = 0.75$

$F_{BC} – F_{BF} = \frac{0.75}{\sin 45°} = 1.06$

$F_{BC} – F_{BF} = 1.06$ ….. (2)

Solving equations (1)and(2),we get

F_{BC}= -0.36(Compression)

F_{BF} = -1.42(Compression)

At joint D:

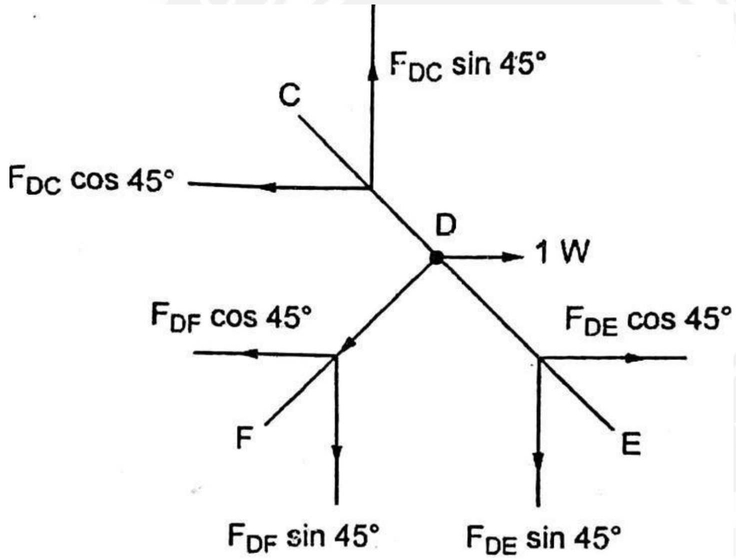

Fig2.4.d

$\Sigma H=0$

$- F_{DC}\cos45^0 – F_{DF}\cos45^{0+} F_{DE}\cos 45^0+1=0$

$- F_{DC}\cos45^0 – F_{DF}\cos45^{0-}(1.06 \times\cos45^0)+1=0$

$-F_{DC}\cos45^0 – F_{DF}\cos45^0 = - 0.25$

$-(F_{DC}+F_{DF})\cos 45^0 = - 0.25$

$F_{DC} + F_{DF} = +\frac{0.25}{\cos 45°}$

$F_{DC}+ F_{DF}=0.35$ …..(3)

$\Sigma V=0$

$+F_{DC}\sin45^0 – F_{DF}\sin45^0 – F_{DE}\sin45^0 = 0$

$F_{DC} – F_{DF} – F_{DE} = O$

$F_{DC} – F_{DF} = +F_{DE}$

$F_{DC} – F_{DF} = -1.06$ …..(4)

Solving (3) and (4),weget

$\mathbf{F_{DC} = -0.35}$(Compression)

$\mathbf{F_{DF} = 0.71}$ (Tension)

Member	Force(kN)	Nature of force
AB	**-1.77W**	**Compression**
BC	**-0.36W**	**Compression**
CD	**-0.35W**	**Compression**
DE	**-1.06W**	**Compression**
EF	**0.75W**	**Tension**
FA	**2.25W**	**Tension**
FD	**0.71W**	**Tension**
CF	**0.5W**	**Tension**
BF	**-1.42W**	**Compression**

Example2.2.4 Analyse the truss shown in Fig.2.5.usingmethodsofjoints.

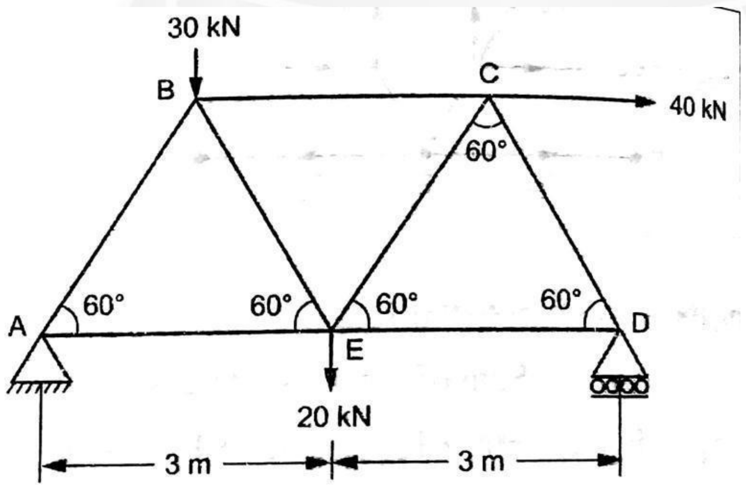

Fig 2.5.a

Solution:

The truss is supported on rollers at D and hence the reaction at D should be vertical (R_D).

The truss is hinged at A and hence end Ac consists of horizontal reaction(H_A) and vertical reaction (R_A).

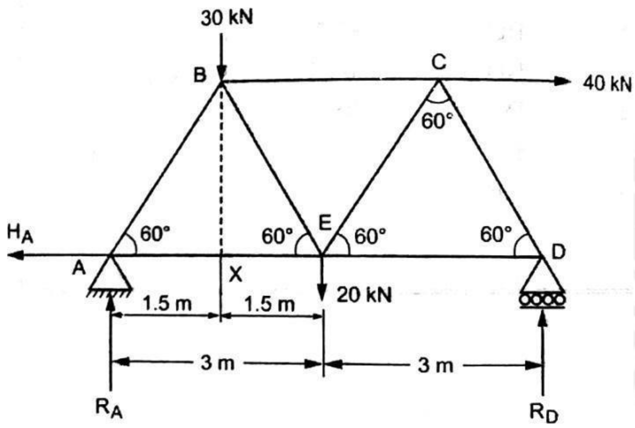

Fig2.5.b

From Δ^{le} BEX,

$\tan 60° = \frac{BX}{XE} = \frac{BX}{1.5}$

$BX = \tan 60° \times 1.5$

BX=2.6 m

Determine the reactions. Taking moment about A

$R_D \times 6 = 40 \times BX + 20 \times 3 + 30 \times 1.5$

$= 40 \times 2.6 + 20 \times 3 + 30 \times 1.5$

$\mathbf{R_D = 34.8}$ kN

We know that,

Upward vertical forces = Downward vertical forces

$R_A + R_D = 30 + 20$

$R_A = 50 - R_D$

RA= 50– 34.8

$\mathbf{R_A = 15.2\ kN}$

We know that,

Horizontal forces to wards right side=Horizontal forces towards left side

$40kN = H_{A,}$

$\therefore \mathbf{H_A = 40kN}$

Consider the joint A.

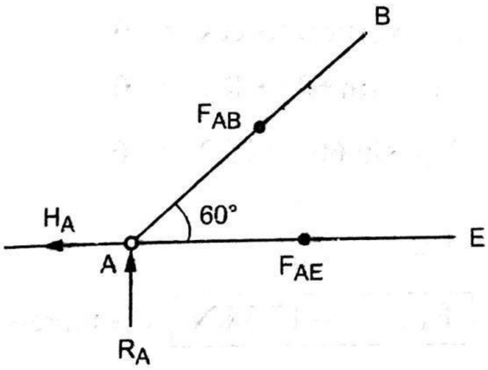

Fig2.5.c

Assume the forces F_{AB} and F_{AE} acting on joint A are tensile forces(acting away from A). If we get negative value, the force in that member is compressive.

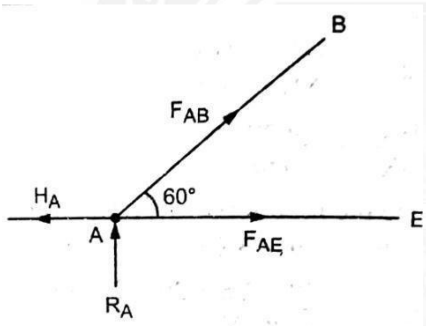

Fig2.5.d

At joint A:

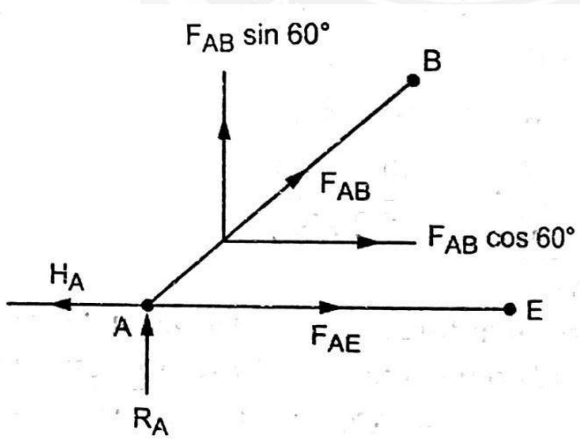

Fig2.5.e

Resolving the force vertically,

Sum of vertical forces=0

F_{AB} Sin 60° + R_A= 0

F_{AB}Sin60°+15.2=0 15 2

$F_{AB} = \frac{- \quad .}{\sin 60°}$

F_{AB}=-17.5kN(Compressive)

Resolving the force horizontally,

Sum of horizontal forces = 0

$F_{AB}\cos 60^0 - H_A + F_{AE} = 0$

$-17.5\cos 60^0 - 40 + F_{AE} = 0$

$-17.5(0.5) - 40 + F_{AE} = 0$

F_{AE}=48.75kN(Tension)

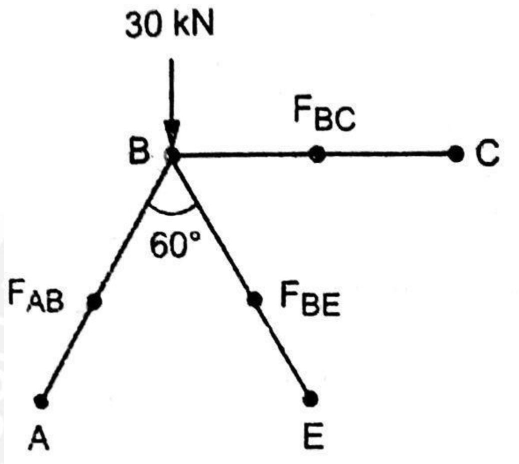

Fig2.5.f

Consider the joint B.

Assume the forces acting on joint B are tensile forces. If we get negative value, force in that member is compressive.

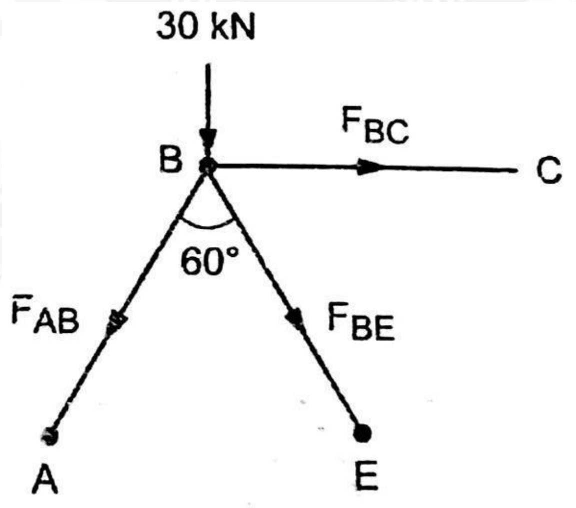

Fig2.5.f

At joint B

Resolving the force vertically, Sum of vertical forces=0

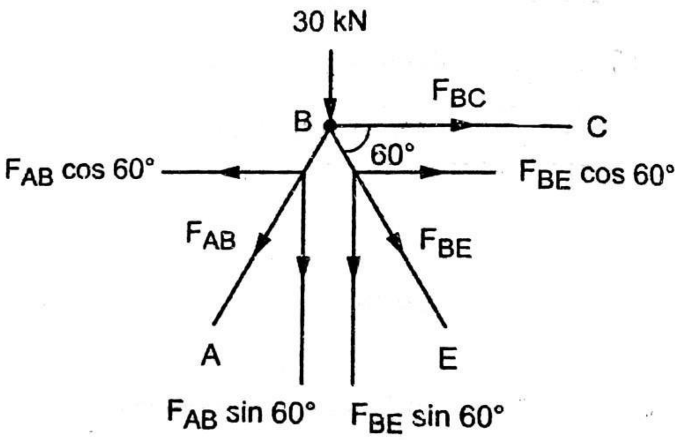

Fig5.5.g

$30kN – F_{AB} \sin 60^0 – F_{BE} \sin 60^0 = 0$

$-30 + 17.5 \times \sin 60^0 = F_{BE} \sin 60^0$

F_{BE}=-17.1kN(Compression)

Resolving the force horizontally,

Sum of horizontal forces = 0

$F_{BC} + F_{BE} \cos 60^0 - F_{AB} \cos 60^0 = 0$ F_{BC} -

17.1(0.5) +17.1(0.5) = 0

F_{BC}= -0.2kN(Compression)

Consider the joint C:

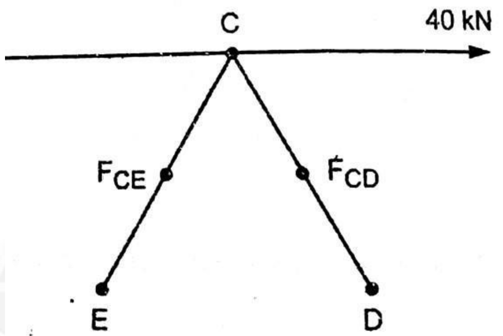

Fig2.5.h

AssumetheforcesactingonCaretensileforces.Ifwegetnegativevalue,the force on that member is compressive.

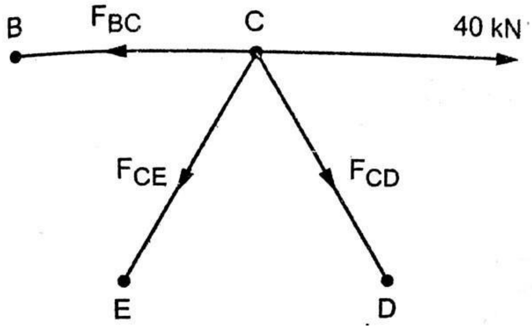

Fig2.5.i

At joint B:

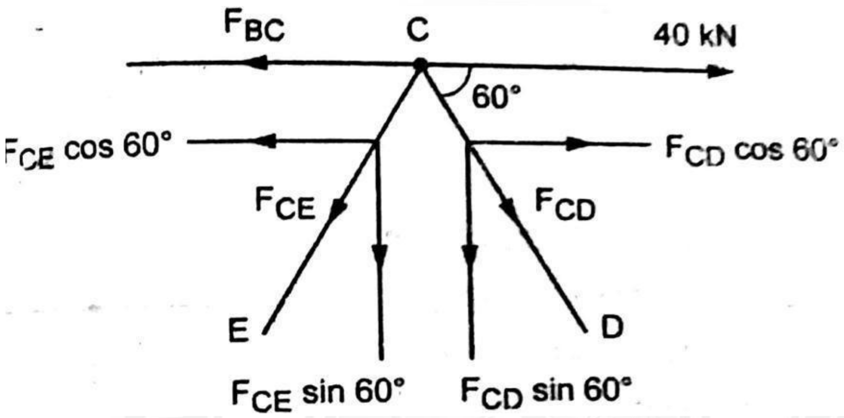

Fig2.5.j

Resolving the force vertically,

Sum of vertical forces=0

$-F_{CE}\sin 60^o - F_{CD}\sin 60^o = 0$

$\mathbf{F_{CE} = - F_{CD}}$ …..(A)

Resolving the force horizontally,

Sum of horizontal forces=0

$-F_{BC}+40 - F_{CE}\cos 60^o + F_{CD}\cos 60^o = 0$

$0.2+40 + 2F_{CD}\cos 60^o = 0$

(Since, $F_{CE}=-F_{CD}$)

$\mathbf{F_{CD}=-40.2\ kN}$(Compression)

Apply in (A),

$\mathbf{F_{CE}=40.2kN}$(Tension)

Consider the joint D:

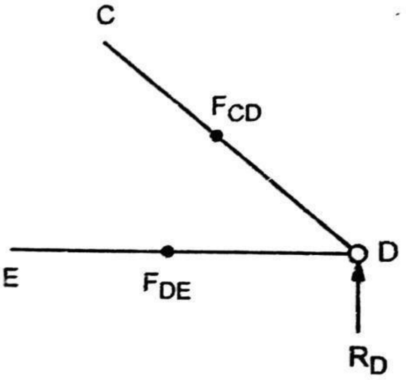

Fig2.5.k

AssumetheforcesactingonjointDaretensileforcesifwegetnegativevalue,the force in that member is compressive.

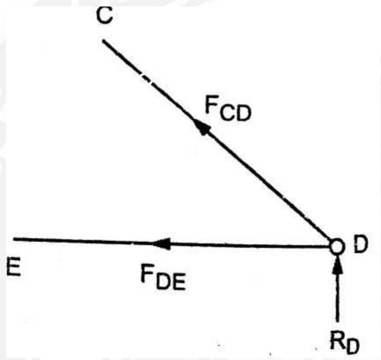

Fig2.5.l

Resolving the force horizontally,

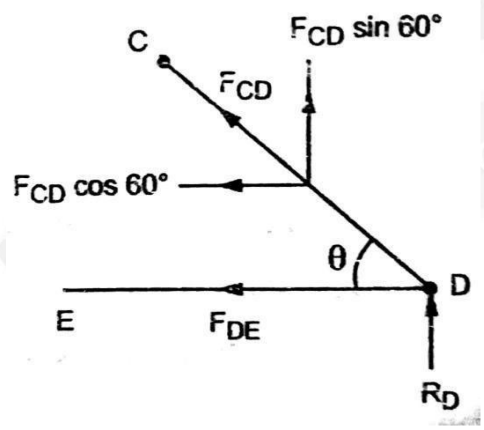

Fig2.5.m

Sum of horizontal forces=0

$-F_{DE} - F_{CD}\cos 60^0=0$

$-F_{DE}+40.2\cos 60^0= 0$

$\mathbf{F_{DE}=20.1kN}$(Tension)

Sl.No.	Member	Force (kN)	Nature of force

1	AB	-17.5	Compression
2	AE	48.75	Tension
3	BE	-17.1	Compression
4	BC	-0.2	Compression
5	CD	-40.2	Compression
6	CE	40.2	Tension
7	DE	20.1	Tension