

3.4 DETERMINATION OF CLOSED LOOP RESPONSE FROM OPEN LOOP RESPONSE

M and N circles

Peak magnitude

$$M_r = 20 \log \left| \frac{C(j\omega)}{R(j\omega)} \right| \text{ dB}$$

where, 3 dB is considered good.

M-CIRCLES

$$M(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)}$$

$$G(j\omega) = X + jY$$

$$M(j\omega) = \frac{X + jY}{1 + X + jY} = \frac{\sqrt{X^2 + Y^2} \angle \tan^{-1} \left(\frac{Y}{X} \right)}{\sqrt{(1 + X)^2 + Y^2} \angle \tan^{-1} \left(\frac{Y}{1 + X} \right)}$$

$$= \frac{\sqrt{X^2 + Y^2}}{\sqrt{(1 + X)^2 + Y^2}} \angle \tan^{-1} \left(\frac{Y}{X} \right) - \tan^{-1} \left(\frac{Y}{1 + X} \right)$$

Let, M = Magnitude of M(j ω)

$$|M(j\omega)| = \frac{\sqrt{X^2 + Y^2}}{\sqrt{(1 + X)^2 + Y^2}}$$

$$M^2(1 + X)^2 + M^2Y^2 = X^2 + Y^2$$

$$X^2(1 - M^2) + (1 - M^2)Y^2 - 2M^2X = M^2$$

$$X^2 + Y^2 - 2 \frac{M^2}{(1 - M^2)} X = \frac{M^2}{(1 - M^2)}$$

Adding $\left(\frac{M^2}{(1 - M^2)} \right)^2$ on both sides, we get,

$$\left(X - \frac{M^2}{(1 - M^2)} \right)^2 + Y^2 = \left(\frac{M}{(1 - M^2)} \right)^2$$

The above equation represents a family of circles with its

$$\text{centre at } \left(\frac{M^2}{(1 - M^2)}, 0 \right) \text{ and radius } \frac{M}{(1 - M^2)}$$

Family of M-circles corresponding to the closed loop magnitudes, M of a unit feedback system is given by the figure 3.4.1.

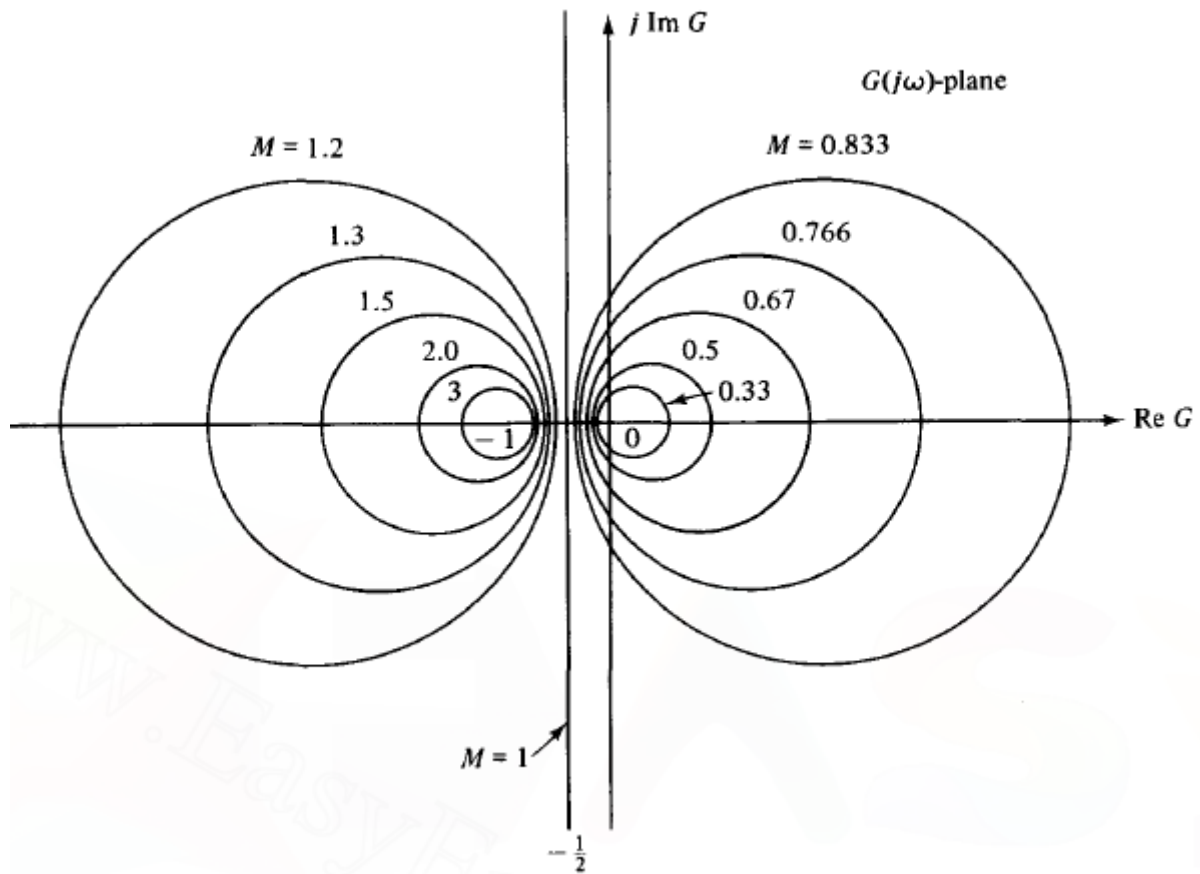


Figure 3.4.1 Constant M-circles in the polar co-ordinates

[Source: "Automatic Control Systems" by Benjamin C. Kuo, Page: 487]

N-CIRCLES

$$\angle M(j\omega) = \alpha = \frac{\angle G(j\omega)}{\angle(1 + G(j\omega))}$$

$$\alpha = \tan^{-1} \frac{Y}{X} - \tan^{-1} \frac{Y}{1 + X}$$

$$\tan \alpha = N = \tan \left(\tan^{-1} \frac{Y}{X} - \tan^{-1} \frac{Y}{1 + X} \right)$$

We know,

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$N = \left(\frac{Y}{X^2 + X + Y^2} \right)$$

$$\left(X + \frac{1}{2} \right)^2 + \left(Y - \frac{1}{2N} \right)^2 = \frac{1}{4} + \left(\frac{1}{2N} \right)^2$$

The above equation represents the family of circles with its

$$\text{Centre at } \left(-\frac{1}{2}, \frac{1}{2N} \right) \text{ and radius } \sqrt{\frac{1}{4} + \left(\frac{1}{2N} \right)^2}$$

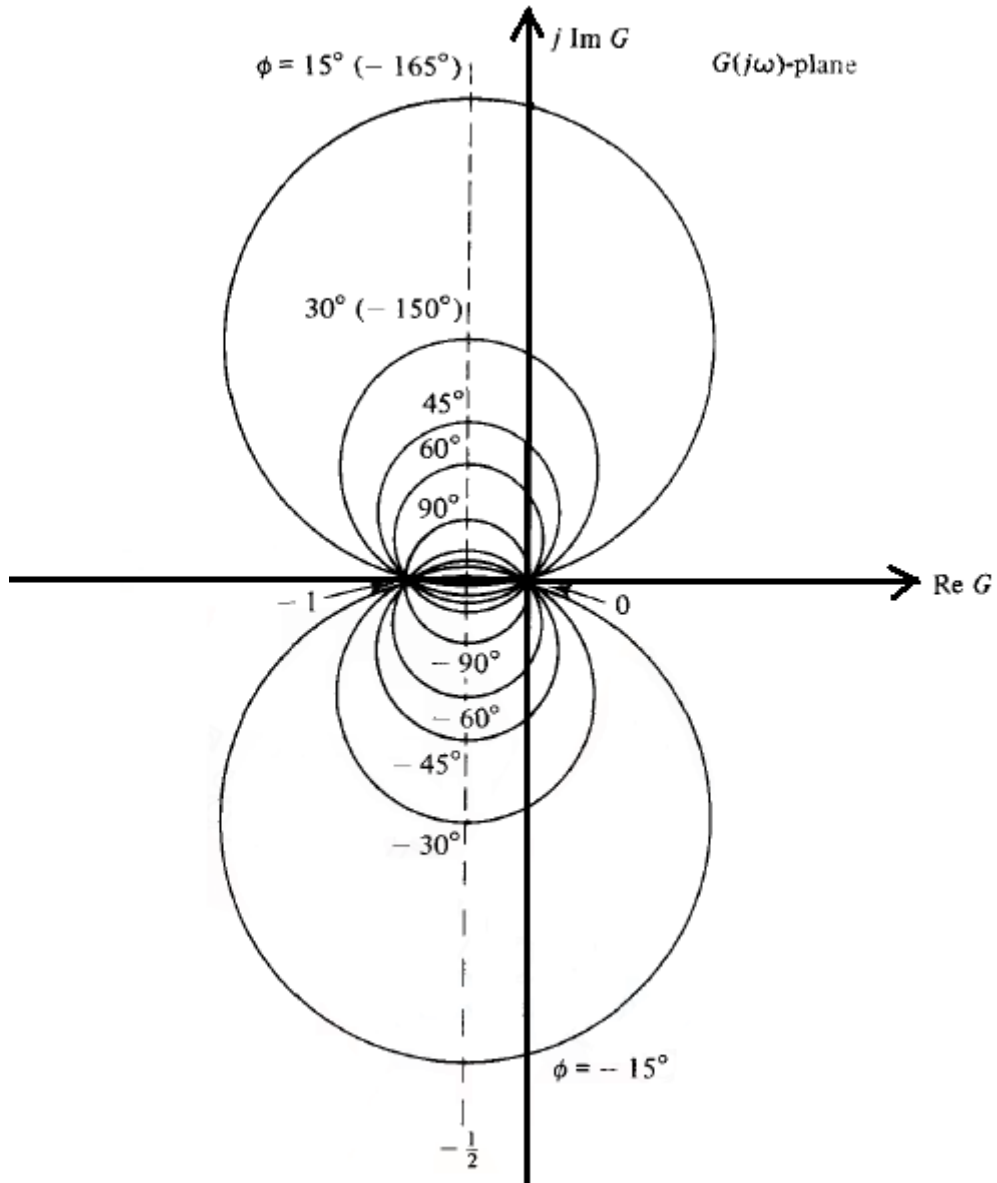


Figure 3.4.2 Constant N-circles in the polar co-ordinates

[Source: "Automatic Control Systems" by Benjamin C. Kuo, Page: 490]