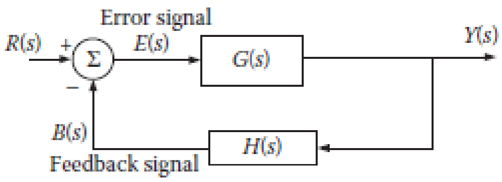
TRANSFER FUNCTIONS

A transfer function is the equation that represents the ratio of output to input in a system. It may be written across a block or across a complete system. Figure 8.8 shows the block diagram for a simple system where the output signal is directly fed back to the summing junction. In this system, R(s), Y(s), G(s), and H(s) represent the input, output, system dynamics plus any controller, and feedback multiplier, respectively.



$$Y(s) = E(s)G(s)$$
$$B(s) = Y(s)H(s) = E(s)G(s)H(s)$$

The transfer function for each block is simply the ratio of its output to its input. We define the following transfer functions:

• **Open-loop transfer function.** The ratio of the feedback signal to the actuating error signal while the feedback loop is open, although the sensor still reads the output. Here, the sensor is used to read the output and report it as the feedback signal. Therefore:

$$OLTF = \frac{B(s)}{E(s)} = \frac{E(s)G(s)H(s)}{E(s)} = G(s)H(s) \tag{8.22}$$

As you see, if the feedback loop is disconnected from the summing junction, the feedback signal is, in fact, a function of $G(s)H(s)$.

- ***Feed-forward transfer function.*** The ratio of the output to the actuating error signal, or:

$$FFTF = \frac{Y(s)}{E(s)} = G(s) \tag{8.23}$$

If the feedback function is unity, the open-loop and feed-forward transfer functions are the same.

- ***Closed-loop transfer function.*** The ratio of output to input for the system. For the system in Figure 8.8, the closed-loop transfer function is:

$$Y(s) = G(s)E(s)$$
$$E(s) = R(s) - B(s) = R(s) - Y(s)H(s)$$

Eliminating $E(s)$, we get:

$$Y(s) = G(s)[R(s) - Y(s)H(s)]$$
$$Y(s)[1 + G(s)H(s)] = G(s)R(s)$$

Consequently, the closed-loop transfer function is:

$$CLTF = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{8.24}$$

Assuming that both $G(s)$ and $H(s)$ can be represented in ratios of polynomials as $G(s) = \frac{N_G(s)}{D_G(s)}$ and $H(s) = \frac{N_H(s)}{D_H(s)}$, Eq. (8.24) can be written as:

$$CLTF = \frac{G(s)}{1 + G(s)H(s)} = \frac{N_G D_H}{N_G N_H + D_G D_H} \tag{8.25}$$

This form of the closed-loop transfer function can assist in quickly composing the equation if the numerators and denominators of $G(s)$ and $H(s)$ are known.