DC Response of RLC Series Circuit :

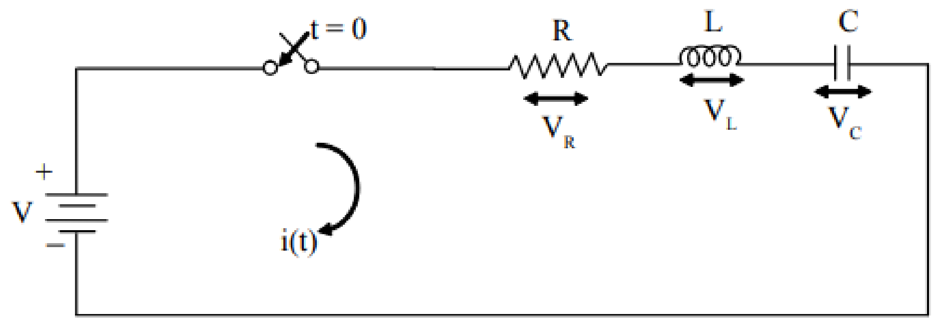

Fig. 4.10 RLC series circuit

The RLC series circuit is shown above which is excited by a DC source. Assume that at t = 0, the switch S is closed. While closing the switch, the voltage drop across the capacitor and current flowing through the inductor is zero.

Applying KVL to the circuit,

$$V = V_R(t) + V_L(t) + V_C(t)$$

$$V = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt \qquad \text{---------(13)}$$

By applying laplace transform to equation (13), we get

$$\frac{V}{S} = RI(s) + L\left[SI(s) - I(0)\right] + \frac{1}{C}\left[\frac{I(s)}{S} + \frac{q(0)}{S}\right]$$

Assume I(0) = q(0) = 0

$$\therefore \frac{V}{S} = RI(S) + LSI(S) + \frac{I(S)}{CS}$$

$$= I(S)\left[R + LS + \frac{1}{CS}\right]$$

$$\frac{V}{S} = I(S)\left[\frac{RCS + LS^2C + 1}{CS}\right] = I(S)\left[\frac{S^2LC + RCS + 1}{CS}\right]$$

$$V = I(S)\left[\frac{S^2LC + SRC + 1}{C}\right]$$

$$V = I(S)\frac{LC}{C}\left[S^2 + \frac{SR}{L} + {}^1\!/_{LC}\right]$$

$$\therefore \quad I(S) = \frac{V}{L\left[S^2 + \frac{R}{L}S + {}^1\!/_{LC}\right]} \qquad \text{---------(14)}$$

The roots of the denominator for equation (14)

$$S_1, S_2 = \frac{-R/L \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2\times 1}$$

$$= \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$S_1, S_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad \text{----------(15)}$$

Equation (15) can be represented by

$$S_1, S_2 = \alpha \pm \beta$$

where, $\alpha = \frac{-R}{2L}$ and $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Here, $S_1 = \alpha + \beta$, $\qquad S_2 = \alpha - \beta$

The natural frequency, $\omega_n = \frac{1}{\sqrt{LC}}$

Equation (14) can be written as

$$I(s) = \frac{V/L}{(s-s_1)(s-s_2)}$$

i.e, $$I(s) = \frac{V/L}{[s-(\alpha+\beta)][s-(\alpha-\beta)]} \quad \text{---------(16)}$$

There are three possibilities.

Case 1 : when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

The two roots are real and distinct. The denominator has the roots $(\alpha+\beta)$ and $(\alpha-\beta)$ and we may write

$$I(s) = \frac{k_1}{s-(\alpha+\beta)} + \frac{k_2}{s-(\alpha-\beta)}$$

Taking inverse laplace transform,

$$i(t) = k_1 e^{(\alpha+\beta)t} + k_2 e^{(\alpha-\beta)t}$$

$$= e^{\alpha t}\left[k_1 e^{\beta t} + k_2 e^{-\beta t}\right] \quad \text{----------(17)}$$

The value of k_1 and k_2 can be find by using partial fraction method. The current is said to be overdamped as in below fig. 4.11.

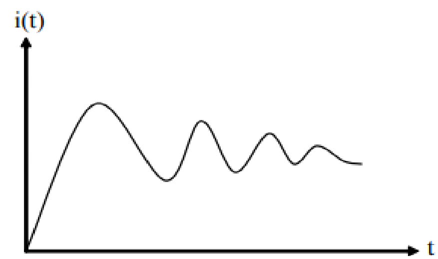

Fig. 3.11 Overdamped response

Case 2 : When $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

The root are equal and the oscillation in the circuit are just eliminated. The solution is the critically damped case.

$$I(S) = \frac{k_1}{(s-\alpha)^2} + \frac{k_2}{(s-\alpha)}$$

Taking inverse laplace transform,

$$i(t) = k_1 t\, e^{\alpha t} + k_2\, e^{\alpha t} = e^{\alpha t}\left[k_1 t + k_2\right]$$

The current response of i(t) for critically damped case is shown below.

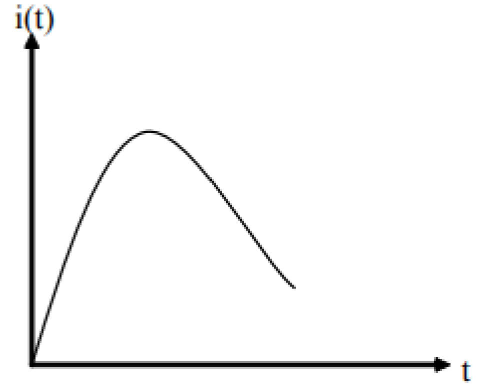

Fig. 3.12 Critically damped response

Case 3 :

$$\text{When } \left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

The roots are complex conjugate and the circuit is under damped as shown below.

$$I(S) = \frac{k_1}{s-(\alpha+j\beta)} + \frac{k_2}{s-(\alpha-j\beta)}$$

Taking inverse laplace transform,

$$i(t) = k_1 e^{(\alpha+j\beta)t} + k_2 e^{(\alpha-j\beta)t}$$

$$= e^{\alpha t}\left[k_1 e^{j\beta t} + k_2 e^{-j\beta t}\right]$$

Where, k_1 and k_2 are complex and are also conjugate of one another.

$$k_2 = k_1^*$$

$\therefore$ i can be rewritten as, $i = e^{\alpha t}\left[A\cos\beta t + B\sin\beta t\right]$

This solution shows that the current is oscillatory and at the same time decays in a short time as $\alpha = -R/2L$ is always negative.

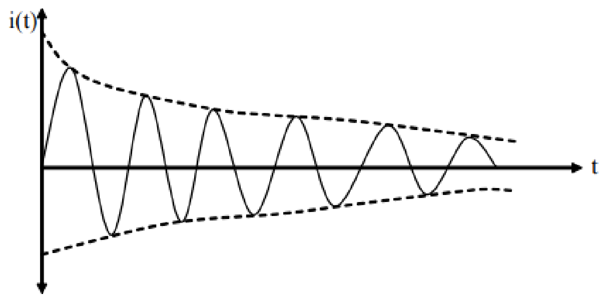

Fig. 3.13 Oscillatory response

Note : When the terms $\left(\frac{R}{2L}\right)^2$ and $\frac{1}{LC}$ are equal the oscillations are just eliminated and this condition is called "critical damping".