Losses in Induction Motor

The various power losses in an induction motor can be classified as,

i) Constant losses

ii) Variable losses

i) Constant losses :

These can be further classified as core losses and mechanical losses.

Core losses occur in stator core and rotor core. These are also called iron losses. These losses include eddy current losses and hysteresis losses. The eddy current losses are minimised by using laminated construction while hysteresis losses are minimised by selecting high grade silicon steel as the material for stator and rotor.

The iron losses depends on the frequency. The stator frequency is always supply frequency hence stator iron losses are dominate. As against this in rotor circuit, the frequency is very small which is slip times the supply frequency. Hence rotor iron losses are very small and hence generally neglected, in the running condition.

The mechanical losses include frictional losses at the bearings and windings losses. The friction changes with speed but practically the drop in speed is very small hence these losses are assumed to be the part of constant losses.

ii) Variable losses :

This include the copper losses in stator and rotor winding due to current flowing in the winding. As current changes as load changes as load changes, these losses are said to be variable losses.

Generally stator iron losses are combined with stator copper losses at a particular load to specify total stator losses at particular load condition.

Rotor copper loss = $3 I_{2r}^2 R_2$ ....Analysed separately

where $\qquad$ I_{2r} = Rotor current per phase at a particular load

$\qquad\qquad$ R_2 = Rotor resistance per phase

Power Flow in an Induction Motor

Induction motor converts an electrical power supplies to it into mechanical power. The various stages in this conversion is called power flow in an inductor motor.

The three phase supply given to the stator is the net electrical input to the motor. If motor power factor is cos Φ and V_L, I_L are line values of supply voltage and current drawn, then net electrical supplied to the motor can be calculated as,

$$P_{in} = \sqrt{3}\ V_L\ I_L\ \cos\phi$$

Where P_{in} = Net input electrical power.

This is nothing but the stator input.

The part of this power is utilised to supply the losses in the stator which are stator core as well as copper losses.

The remaining power is delivered to the rotor magnetically through the air gap with the help of rotating magnetic field. This is called rotor input denoted as P_2.

So $$P_2 = P_{in} - \text{stator losses (core + copper)}$$

The rotor is not able to convert its entire input to the mechanical as it has to supply rotor losses. The rotor losses are dominantly copper losses as rotor iron losses are very small and hence generally neglected. So rotor losses are rotor copper losses denoted as P_c.

So $$P_c = 3 \times I_{2r}^2 \times R_2$$

where I_{2r} = Rotor current per phase in running condition R_2 = Rotor resistance per phase.

After supplying these losses, the remaining part of P_2 is converted into mechanical which is called gross mechanical power developed by the motor denoted as P_m.

$$\therefore \quad P_m = P_2 - P_c$$

Now this power, motor tries to deliver to the load connected to the shaft. But during this mechanical transmission, part of P_m is utilised to provide mechanical losses like friction and windage.

And finally the power is available to the load at the shaft. This is called net output of the motor denoted as P_{out}. This is also called shaft power.

$$\therefore \quad P_{out} = P_m - \text{Mechanical losses.}$$

The rating of the motor is specified in terms of value of P_{out} whenload condition is full load condition. The above stages can be shown diagrammatically called power flow diagram of an induction motor.
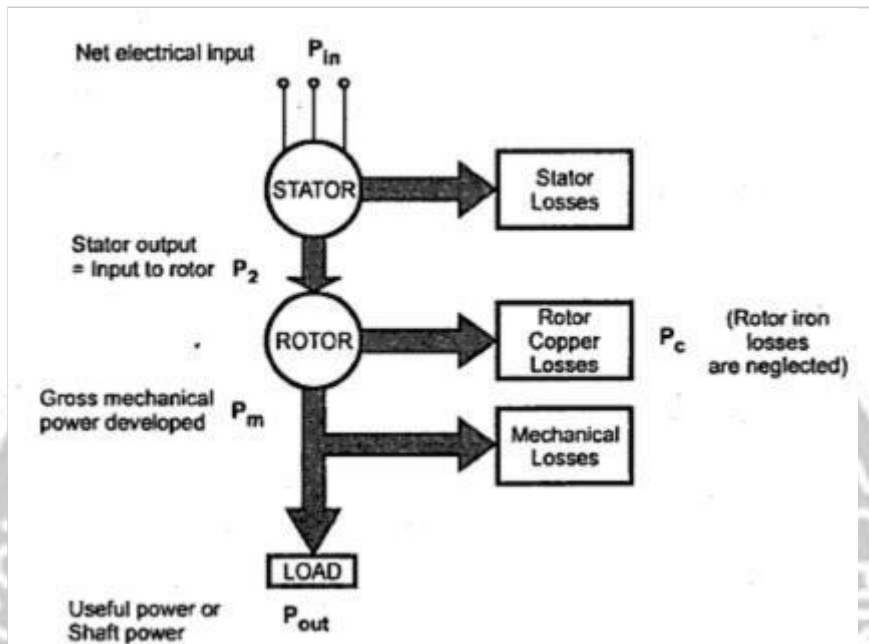
This is shown in the Fig.1.

Fig. 3.23 Power flow diagram

From the power flow diagram we can define,

$$\text{Rotor efficiency} = \frac{\text{rotor output}}{\text{rotor input}} = \frac{\text{gross mechanical power developed}}{\text{rotor input}}$$

$$= P_m / P_2$$

$$\text{Net motor efficiency} = \frac{\text{net output at shaft}}{\text{net electrical input to motor}} = \frac{P_{out}}{P_{in}}$$

Relation between P2, Pc, and Pm

The rotor input P_2, rotor copper loss P_c and gross mechanical power developed P_m are related through the slip s. Let us derive this relationship.

Let   T = Gross torque developed by motor in N-m.

We know that the torque and power are related by the relation,

$$P = T \times \omega$$

where P = Power   and ω = angular speed

$$= (2\pi N)/60 \text{ , } N = \text{speed in r.p.m.}$$

Now input to the rotor P_2 is from stator side through rotating magnetic field which is rotating at synchronous speed N_s.

So torque developed by the rotor can be expressed interms of power input and angular speed at which power is inputted i.e. ω_s as,

$$P_2 = T \times \omega_s \text{ where } \omega_s = (2\pi N_s)/60 \text{ rad/sec}$$
$$P_2 = T \times (2\pi N_s)/60 \quad \text{where } N_s \text{ is in r.p.m} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

The rotor tries to deliver this torque to the load. So rotor output is gross mechanical power developed P_m and torque T. But rotor gives output at speed N and not N_s. So from output side P_m and T can be related through angular speed ω and not ω_s.

$$P_m = T \times \omega \qquad \text{where } \omega = (2\pi N)/60$$
$$P_m = T \times (2\pi N)/60 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

The difference between P_2 and P_m is rotor copper loss P_c. $P_c = P_2 -$
$$P_m = T \times (2\pi N_s/60) - T \times (2\pi N/60)$$
$$P_c = T \times (2\pi/60)(N_s - N) = \text{rotor copper loss} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

Dividing (3) by (1),

$$\frac{P_c}{P_2} = \frac{T \times \frac{2\pi}{60}(N_s - N)}{T \times \frac{2\pi}{60} \times N_s} = \frac{N_s - N}{N_s}$$

$P_c/P_2 = s \qquad$ as $(N_s - N)/N_s = $ slip s

Rotor copper loss $P_c = s \times$ Rotor input P_2

Thus total rotor copper loss is slip times the rotor input.

Now $\qquad\qquad\qquad P_2 - P_c = P_m \quad P_2$
$$- sP_2 = P_m$$
$$(1 - s)P_2 = P_m$$

Thus gross mechanical power developed is (1 - s) times the rotor input The relationship can be expressed in the ratio from as,

$$P_2 : P_c : P_m \quad \text{is} \quad 1 : s : 1 - s$$

The ratio of any two quantities on left hand side is same as the ratio of corresponding two sides on the right hand side.

For example, $\qquad \dfrac{P_c}{P_m} = \dfrac{s}{1-s}, \quad \dfrac{P_2}{P_c} = \dfrac{1}{s}$ and so on.

This relationship is very important and very frequently required to solve the problems on the power flow diagram.

Key Point : The torque produced by rotor is gross mechanical torque and due to mechanical losses entire torque can not be available to drive load. The load torque is net output torque called shaft torque or useful torque and is denoted as T_{sh}. It is related to P_{out} as,

$$T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{\left(\frac{2\pi N}{60}\right)}$$

and $T_{sh} < T$ due to mechanical losses.

Derivation of k in Torque Equation

We have seen earlier that

$$T = (k \, s \, E_2^2 \, R_2)/(R_2^2 + (s \, X_2)^2)$$

and it mentioned that $k = 3/(2\pi \, n_s)$. Let us see its proof. The rotor copper losses can be expressed as,

$$P_c = 3 \times I_{2r}^2 \times R_2$$

but $I_{2r} = (s \, E_2)/\sqrt{(R_2^2 + (s \, X_2)^2)}$, hence substituting above

$$P_c = 3 \times \left[\frac{s \, E_2}{\sqrt{R_2^2 + (sX_2)^2}} \right]^2 \times R_2$$

$$\therefore \quad P_c = \frac{3 s^2 \, E_2^2 \, R_2}{R_2^2 + (sX_2)^2} \; .$$

Now as per $\quad P_2 : P_c : P_m \text{ is} \quad 1 : s : 1\text{-s}$,

$$P_c/P_m = s/(1-s)$$

Now $\quad P_m = T \times \omega$

$$= T \times (2\pi N/60)$$

$$\therefore \quad T \times \frac{2\pi N}{60} = \frac{(1-s) \, 3 \, s \, E_2^2 \, R_2}{R_2^2 + (sX_2)^2}$$

$$\therefore \quad T = \frac{60}{2\pi N} \times \frac{(1-s) \, 3 \, s \, E_2^2 \, R_2}{R_2^2 + (sX_2)^2}$$

Now $N = N_s (1-s)$ from definition of slip, substituting in above,

$$\therefore \quad T = \frac{60}{2\pi N_s (1-s)} \times \frac{(1-s) \, 3 \, s \, E_2^2 \, R_2}{R_2^2 + (sX_2)^2}$$

$$= \frac{3}{2\pi \left(\frac{N_s}{60}\right)} \times \frac{s \, E_2^2 \, R_2}{R_2^2 + (sX_2)^2}$$

but $\qquad N_s/60 = n_s$ in r.p.m.

So substituting in the above equation,

$$T = \frac{3}{2\pi n_s} \times \frac{s E_2{}^2 R_2}{R_2{}^2 + (sX_2)^2}$$

Comparing the two torque equations we can write,

$$k = \frac{3}{2\pi n_s}$$ where n_s is in r.p.s.

Efficiency of an Induction Motor

The ratio of net power available at the shaft (P_{out}) and the net electrical power input (P_{in}) to the motor is called as overall efficiency of an induction motor.

$$\therefore \quad \% \eta = \frac{P_{out}}{P_{in}} \times 100$$

The maximum efficiency occurs when variable losses becomes equal to constant losses. When motor is on no load, current drawn by the motor is small. Hence efficiency is low. As load increases, current increases so copper losses also increases. When such variable losses achieve the same value as that of constant losses, efficiency attains its maximum value. If load is increased further, variable losses becomes greater than constant losses hence deviating from condition for maximum, efficiency starts decreasing. Hence the nature of the curve of efficiency against output power of the motor is shown in the Fig. 1.
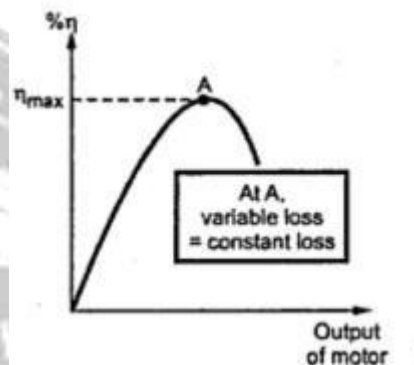

Fig. 3.24 Efficiency curve for an induction motor