Problems:

1. The symmetrical components of a phase –a voltage in a 3-phase unbalanced system are 0 Va0 =10∠180 V, 0 Va1 = 50∠0 V and 0 Va2 = 20∠90 V. Determine the phase voltages Va ,Vb and Vc.

Solution:

The phase voltages of $Va, VbandV$c

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$$

$$V_c = V_{a0} + aV_{a1} + a^2 V_{a2}$$

$$V_{a0} = 10\angle 180^0 = -10 + j0 \text{ V}$$

$$V_{a1} = 50\angle 0^0 = 50 + j0 \text{ V}$$

$$V_{a2} = 20\angle 90^0 = 0 + j20 \text{ V}$$

a=1∠120^0 a^2= 1∠240^0

$$a^2 V_{a1} = 1\angle 240^0 \times 50\angle 0^0 = 50\angle 240^0 = -25 - \text{j}43.30$$

$$aV_{a1} = 1\angle 120^0 \times 50\angle 0^0 = 50\angle 120^0 = -25 + \text{j}43.30$$

$$a^2 V_{a2} = 1\angle 240^0 \times 20\angle 90^0 = 20\angle 233 = 17.32 - \text{j}10$$

$$aV_{a2} = 1\angle 120^0 \times 20\angle 90^0 = 20\angle 210^0 = -17.32 - \text{j}10$$

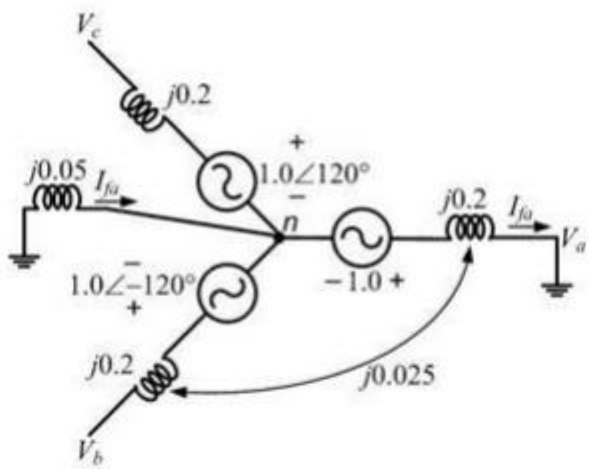
$$V_a = V_{a0} + V_{a1} + V_{a2} = (-10 + j0) + (50 + j0) + (0 + j20) = 40 + j20 = 44.72\angle 27^0\, V$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2} = (-10 + j0) + (-25 - \text{j}43.30) + (-17.32 - \text{j}10) = -52.32 - j53.90$$
$$= 74.69\angle -134^0 \text{ V}$$

$$V_c = V_{a0} + aV_{a1} + a^2 V_{a2} = (-25 - \text{j}43.30) + (-25 + \text{j}43.30) + 17.32 - \text{j}10 = -17.68 + \text{j}33.3$$

$$= 37.70\ \angle -118^0\, V$$

2. A three-phase Y-connected synchronous generator is running unloaded with rated voltage when a 1LG fault occurs at its terminals. The generator is rated 20 kV, 220 MVA, with subsynchronous reactance of 0.2 per unit. Assume that the subtransient mutual reactance between the windings is 0.025 per unit. The neutral of the generator is grounded through a 0.05 per unit reactance. The equivalent circuit of the generator is shown in Fig. We have to find out the negative and zero sequence reactances.



Since the generator is unloaded the internal emfs are Ean = 1.0 Ebn = 1.0∠−1200 Ecn = 1.0∠120°

Since no current flows in phases b and c, once the fault occurs, we have from Fig.

$$I_{fa} = \frac{1}{j(0.2 + 0.05)} = 2 - j4.0$$

Then we also have $Vn = -Xn$

$$Ifa = -0.2$$

From Fig. we get $Va = 0$

$$Vb = Ebn + Vn + j0.025Ifa = -0.6 - j0.866 = 1.0536\angle - 124.72°$$
$$Vc = Ecn + Vn + j0.025Ifa = -0.6 + j0.866 = 1.0536\angle 124.72°$$

Therefore

$$V_{a012} = C\begin{bmatrix} 0 \\ 1.0536\angle -124.72^0 \\ 1.0536\angle 124.72^0 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 0.7 \\ -0.3 \end{bmatrix}$$

$$I_{fa1} = \frac{E_{an} - V_{a1}}{Z_1} = \frac{1-0.7}{j0.225} = -j1.333$$

$$I_{fa0} = I_{fa1} = I_{fa2}$$

$$Z_{go} = \frac{-V_{a0}}{I_{a0}} - 3Z_n = j(0.3-0.15) = j0.15$$

$$Z_2 = \frac{-V_{a2}}{I_{a2}} = j0.225$$

$$I_{fa0} = \frac{1}{j(0.225+0.225+0.15+3\times 0.05)} = -j1.333$$

3. Let us consider the same generator as given in Example 1. Assume that the generator is unloaded when a bolted (Zf = 0) short circuit occurs between phases b and c. Then we get from I fb = - I fc . Also since the generator is unloaded, we have I fa = 0.

Solution:

$Van = Ean = 1.0$

$Vbn = Ebn - j0.225Ifb = 1.\angle -120^{o} - j0.225Ifb$

$Vcn = Ecn - j0.225Ifc = 1.\angle 120^{o} + j0.225Ifb$

Also since V bn = V cn, we can combine the above two equations to get

$$I_{fb} = -I_{fc} = \frac{1\angle -120^0 - 1\angle 120^0}{j0.45} = -3.849$$

Then

$$I_{fa012} = C\begin{bmatrix} 0 \\ -3.849 \\ 3.849 \end{bmatrix} = \begin{bmatrix} 0 \\ -j2.2222 \\ j2.2222 \end{bmatrix}$$

We can also obtain the above equation as

$$I_{fa1} = -I_{fb2} = \frac{1}{j0.225 + j0.225} = -j2.222$$

Also since the neutral current I n is zero, we can write

V a = 1.0 and $Vb = Vc = Vbn = Vbn = -0.5$

Hence the sequence components of the line voltages are

$$V_{a012} = C\begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

Also note that

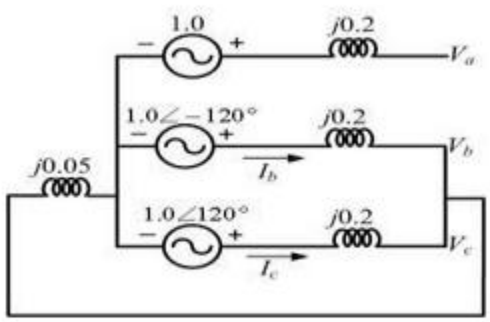
$$Va1 = 1.0 - j0.2251Ifa1$$
$$Va2 = -j0.2251Ifa2 = 0.5$$

4. Let us consider the same generator as given in Examples 1 and 2. Let us assume that the generator is operating without any load when a bolted 2LG fault occurs in phases b and c. The equivalent circuit for this fault is shown in Fig. From this figure we can write

$$E_{bn} + V_n = 1\angle -120^\circ + V_n = j0.2I_{fb} - j0.025I_{fc}$$

$$E_{cn} + V_n = 1\angle 120^\circ + V_n = j0.2I_{fc} - j0.025I_{fb}$$

$$V_n = -j0.05(I_{fb} + I_{fc})$$



Solution:

Equivalent circuit of the generator for a 2LG fault in phases b and c.

Combining the above three equations we can write the following vector-matrix form

$$j\begin{bmatrix} 0.25 & 0.025 \\ 0.025 & 0.25 \end{bmatrix}\begin{bmatrix} I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1\angle -120^\circ \\ 1\angle 120^\circ \end{bmatrix}$$

Solving the above equation we get

$$I_{fb} = -3.849 + j1.8182$$
$$I_{fc} = 3.849 + j1.8182$$

Hence we can also obtain the above values using (8)-(10).

$$Z1 = Z2 = j0.225, Z0 = j\,0.15 + 3 \times 0.05 = j0.3\ andZf = 0$$

$$I_{fa1} = \frac{1}{j0.225 + \left(\frac{j0.225 \times j0.3}{j0.525}\right)} = -j2.8283$$

$$I_{fa0} = -I_{fa1}\frac{j0.225}{j0.525} = j1.2121$$

Now the sequence components of the voltages are

$$Va1 = 1.0 - j0.225Ifa1 = 0.3636$$
$$Va2 = j0.225Ifa2 = 0.3636$$
$$Va0 = -j0.3Ifa0 = 0.3636$$

Also note from above Fig.

$$Va = Ean + Vn + j0.0225\ Ifb + Ifc = 1.0909$$

and Vb = Vc = 0.

Therefore

$$V_{a012} = C\begin{bmatrix} 1.0909 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.3636 \\ 0.3636 \\ 0.3636 \end{bmatrix}$$