TERZAGHI'SONE DIMENSIONAL CONSOLIDATION THEORY – GOERNING DIFFERENTIAL EQUATION:

Consolidation:

A Consolidation is a gradual process of reduction of volume. Under static loading, reduction in volume of a soil due to squeezing out of water from soil.

It is a process which occurs in nature when the saturated soil deposits are subjected to static loading caused by the weight of the building and other structures.

The theoretical concepts of the consolidation process was developed by Terzaghi.(1923).

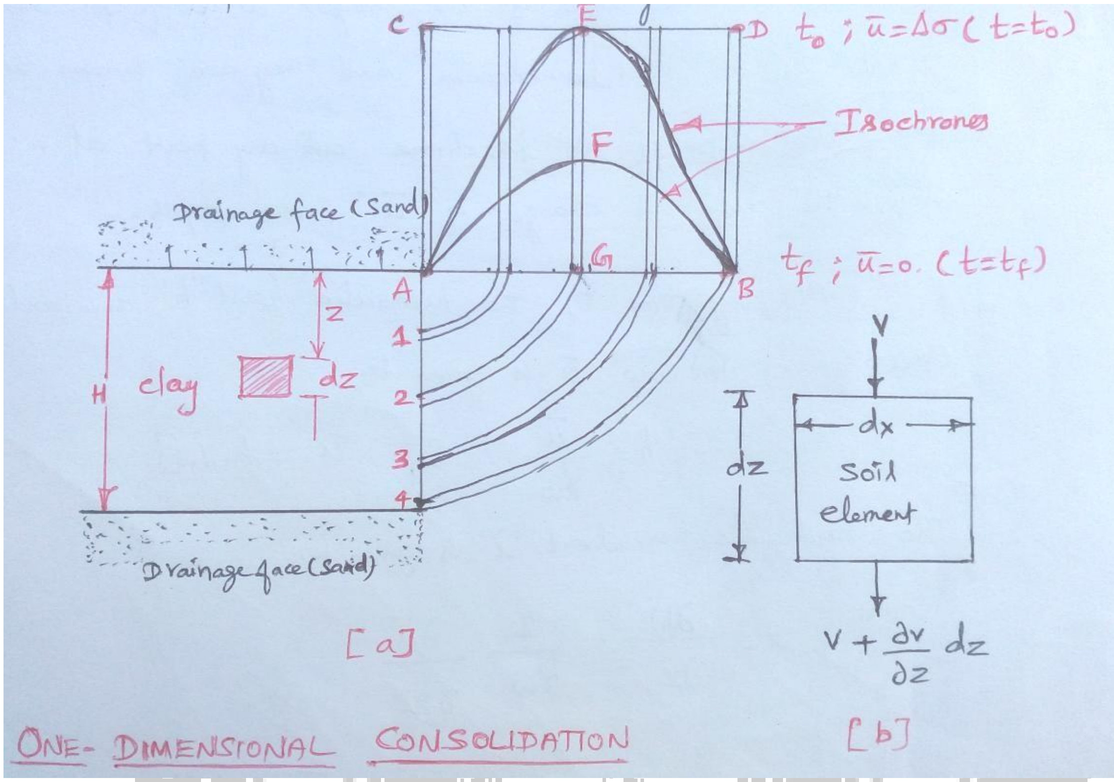
TERZAGHI'S THEORYOF ONE DIMENSIONAL CONSOLIDATION:

The theoretical concept of the consolidation process was developed by Terzaghi (1923).

The following assumption are made:

1) The soil is homogeneous and fully saturated.
2) Soil particles and water are incompressible (γ & γw are constant)
3) The compression (load) and depression (flow) are one – dimensional (vertical)
4) Strains are small
5) Davey's law is valid at all hydraulic gradients.
6) The co – efficient of permeability 'k' and the co-efficient of volume co permeability 'm' remains constant throughout the process.
7) There is a unique relationship, independent of time, between 'e' and 'σ' (effective stress)
8) The time taken for consolidation is entirely depends upon the permeability of soil.
9) Excess pore water drains out only in the vertical direction.

ONE – DIMENSIONAL CONSOLIDATION



Let, a Saturated layer of thickness 'H' lies between two layers of sand which serve as two drainage faces.

When the clay layer is subjected to a pressure increment $\Delta\sigma,$ it is first borne by pore water so that at initial time 'to' excess pore pressure, $\overline{u} = \Delta,$ at all points along the depth of day layer and it is plotted as line 'CD' in fig. (a)

Drainage of pore water into the sand layers starts and excess pore pressure at top and bottom boundaries of clay layer drops down to zero and remains so at all times, during the consolidation process.

At the end of process, say at $t = t_f$ the excess pore pressure will have been completely dissipated, so that $\overline{u} = 0$ at all points and is represented by the line 'AB' in fig. (a)

At any intermediate time $t,$ between 't_0' and 't_f', part of consolidation pressure '$\Delta\sigma$' is transferred to soil particles so that $\Delta\sigma == \Delta\sigma' + u$

The distribution of EXCESS PORE PRESSURE '$\overline{u}$' at any intermediate time 't' is represented by a curve such as AFB in fig. (a)

A number of such curves representing excess instants of time $t = t_1, t_2, \dots \dots$ can be drawn and they are known as Isochrones.

The slope of an isochrones art any point at a given time gives the rate of change of '$\bar{u}$' with depth.

At any time t, the hydraulic head 'h' corresponding to the excess pore pressure $\bar{u}$ is given by,

$$h = \frac{\bar{u}}{\gamma_w} \quad [\because \bar{u} = h\gamma_w]$$

The hydraulic gradient 'i' is given by

$$i = \frac{\partial h}{\partial z} = \frac{\partial \frac{\bar{u}}{\gamma_w}}{\partial z}$$

$$= \frac{1}{\gamma_w}\frac{\partial \bar{u}}{\partial z}$$

Applying Darcy's law, the velocity of flow of pore water due to this hydraulic gradient is givenby,

$$V = Ki = \frac{K}{\gamma_w} . \frac{\partial \bar{u}}{\partial z}$$

The rate of change of velocity along the depth of the layer is given by,

Multiply by$\frac{\partial}{\partial z}$ on both sides

$$\frac{\partial V}{\partial z} = \frac{K}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2} - - - - - - - - (A)$$

Let us consider a soil element, of size dx, dz and of width 'dy' perpendicular to the plane of figure. If,

$V \rightarrow$ velocity of water at entry

Velocity of water at exit = $V + \frac{\partial V}{\partial z} . dz$ as in indicated fig. (b)

The quantity of water entering the soil element in unit time

$= V . dx . dy$ (velocity x Area = Q)

The quantity of water leaving the soil element in unit time

$= \left(V + \frac{\partial V}{\partial Z}.dz\right).dx.dy \quad (\therefore Q = Av)$

Hence, the net quantity of water squeezed out of the soil element in unit time is given by,

$\Delta q = \left(V + \frac{\partial V}{\partial Z}.dz\right).dx.dy - \text{V . dx . dy}$

$\Delta q = V.dx.dy + \frac{\partial V}{\partial Z}.dz.dx.dy - \text{V . dx . dy}$

$\Delta q = \frac{\partial V}{\partial Z}.dz.dx.dy ------(1)$

The decrease in the volume of soil element is equal to the volume of water squeezed out.

Also, we have, $\Delta v = -m_v \; v_0 \, \Delta\sigma'$

$V_0 =$ volume of soil element at time, 't_0' $= dx\,dy\,dz$

∴ change in volume per unit time is given by,

$$\frac{\partial \Delta V}{\partial t} = -M_V.V_0.\frac{\partial \Delta\sigma'}{\partial t}$$

$$\frac{\partial \Delta V}{\partial t} = -M_V.dx.dy.dz.\frac{\partial \Delta\sigma'}{\partial t} ----- (2)$$

Comparing(1)&(2)

$$\frac{\partial V}{\partial Z}.dz.dx.dy = -M_V.dx.dy.dz.\frac{\partial \Delta\sigma'}{\partial t}$$

$$\frac{\partial V}{\partial Z} = -M_V \frac{\partial \Delta\sigma'}{\partial t} -------(3)$$

$$\frac{\partial \Delta\sigma'}{\partial t} = \frac{1}{-M_V}\frac{\partial V}{\partial Z} ----- (3)$$

Now, $\Delta\sigma = \Delta\sigma' + \overline{U}$

$\Delta\sigma' + \overline{U} = 0$ where, '$\Delta\sigma$' is constant

$$\Delta\sigma' = -\bar{U} ------(4)$$

Diff eqn (4) w.r.t time

$$\frac{\partial}{\partial t}\Delta\sigma' = -\frac{\partial \bar{u}}{\partial t} ----(5)$$

Sub the above value in equation (3)

$$-\frac{\partial \bar{u}}{\partial t} = \frac{1}{-M_V}\frac{\partial V}{\partial Z} ------ (6)$$

$$\frac{\partial V}{\partial Z} = M_V \frac{\partial \bar{u}}{\partial t} ----(7)$$

We know that,

$$\frac{\partial V}{\partial z} = \frac{K}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2} ----- (8)$$

Equate (7)&(8)

$$M_V \frac{\partial \bar{u}}{\partial t} = \frac{K}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2} \frac{\partial \bar{u}}{\partial t} = \frac{K}{M_V.\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2}$$

$$Replace\ \frac{K}{M_V.\gamma_w} = C_V$$

$$\frac{\partial \bar{u}}{\partial t} = C_V \frac{\partial^2 \bar{u}}{\partial z^2}$$

$c_v \rightarrow$ co - efficient of consolidation

The above equation is the basic differential equation of consolidation which relates the rate of dissipation of excess pore pressure with the rate of expulsion of pore water from a unit volume of soil.

The co-efficient of consolidation c_v as defined in above equation indicates. The combined effects of permeability and compressibility of soil on the rate of volume change.

$k \rightarrow m/$ sec ; $m_v \rightarrow m^2/KN$; $\gamma w \rightarrow KN/m^2$; $c_v \rightarrow m^2/sec$

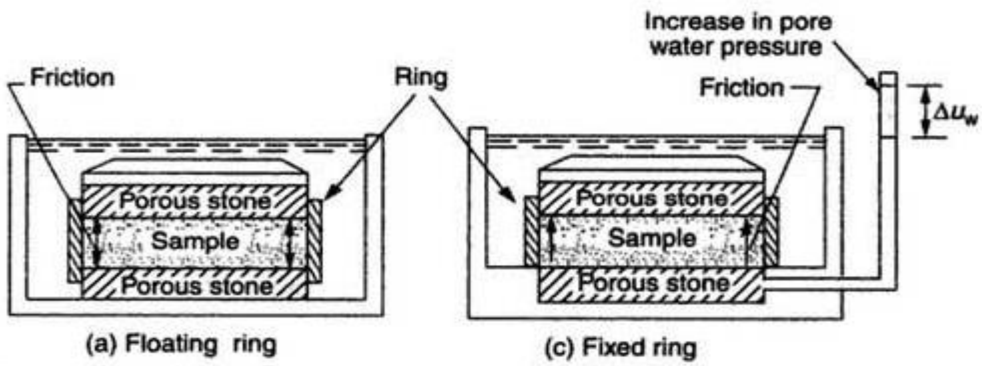
Consolidation Test:

Apparatus Required:

- Consolidometer or oedometer
  - Consolidation ring
  - Two porous stones
  - Two filter papers
  - Loading pad
- Dial gauge (accuracy of 0.002mm)s
- Stop watch
- Knife or spatula or fine metal wires
- Weighing balance (accuracy of 0.01g)
- Vernier calipers
- Oven
- Water reservoir

Procedure:

1. First step is to collect the soil specimen using consolidation metal ring. The ring should be clean and dried and its weight, inner diameter and height are measured using weighing balance and calipers respectively.
2. The soil specimen should project about 10 mm on either side of metal ring .Now press the metal ring into the soil sample using hands and it is.
3. Depending on the type of ring (fixed or floating), choose the correct size of the porous stone.
4. Place the ring and the specimen centrally on the saturated bottom porous stone and place the upper saturated porous stone, followed by the loading cap.



Types of consolidometers

5. Now trim the excess soil content on top and bottom of the rings using Knife or spatula or fine metal wires. This excess soil can be used to measure the water content of soil sample.
6. Make sure that the ring should not contain any soil on its outer part and weight the metal ring with soil specimen
7. Take two porous stones and saturate them by boiling (15 minutes) or by submerging (4 to 8 hours) in distilled water.
8. Assemble the consolidometer. Place the parts of consolidometer from bottom to top in the order beginning with bottom porous stone, filter paper, specimen ring, filter paper and top porous stone.
9. Water reservoir is connected to the mounted assembly to saturate the soil. The water level in the water reservoir should be of same level as the soil specimen.
10. Now apply the initial trail load which should not allow any swelling in the soil. In general 5 kN/m^2 initial load applied for ordinary soils and 2.5 kN/m^2 is applied for very soft soils.
11. Leave the load until there is no change in dial gauge reading or for 24 hours and note down the final reading of dial gauge for initial load.
12. First load increment of 10 kN/m^2 is applied and start the stop watch immediately and note down the readings of dial gauge at various time intervals. In general, readings are taken at 0.25, 1, 2.5, 4, 6.25, 9, 16, 25, 30 minutes, 1, 2, 4, 8, 24 hrs.
13. In general primary consolidation of soil (90% of consolidation) is reached within 24 hours. Hence readings are noted up to 24 hours.
14. Next apply the second load increment of 20 kN/m^2 and repeat same procedure as said in 14 th step.
15. Similarly apply the load increments 50, 100, 200, 400 and 800 kN/m^2 and repeat the same procedure and note down the readings.
16. When values of last load increment are noted, now reduce the load to ¼ of the last load value and leave it for 24 hours. At this point note down the dial gauge reading. Reduce the load again and again and repeat the procedure until the load gets 10 kN/m^2. At every point note down the final gauge readings.
17. Now remove the assembly from loading frame and dismantle it.

18. Take out the specimen ring and wipe out the excess water and Weigh the specimen ring and note down.
19. Finally Put the specimen in oven and determine the dry weight of specimen.

SOME IMPORTANT TERMS:

Co-efficient of compressibility: (a_v)

It is defined as decrease in void ration per unit increase in effective stress.

$$a_V = \frac{\Delta e}{\Delta \sigma'}$$

Co-efficient of volume change: (m_v)

It is defined as the ratio of co-efficient of volume change to the initial void ratio.

$$m_v = \frac{a_v}{1 + e_0}$$

Compression Index: (C_c)

It is defined as the slope of the linear portion of the void ratio verses log $(\sigma')lot$

$$c_c = \frac{\Delta e}{\Delta \log P}$$

Re compression Index: (C_γ)

It is the compression of a soil which had already been loaded and unloaded.

$$c_Y = \frac{\Delta e}{\Delta \log P}$$

Degree of consolidation: (U)

It is the ration of initial settlement to the final settlement.

$$U = \frac{\rho}{\rho_f} x100 \;\; or \;\; U = \frac{\gamma}{\gamma_f} x100$$

Time factor (T_v)

It is a factor representing the consolidation

$$T_v = \frac{C_v t}{d^2}$$

d = drainage path for single drainage, d= H

for Double drainage, d = $H/_2$

H= Thickness of layer

Degree of consolidation (U) is a function of time factor (T_v)

i$e.$, (%) = $f(T_v)$

Co-efficient of consolidation : (C_v)

$$C_v = \frac{k}{m_v \gamma_w}$$

$$U < 60\%, T_V = \frac{\pi}{4}\left(\frac{U}{100}\right)^2$$

$$U < 60\%, T_V = -0.9332\, Log_{10}\left[1 - \frac{U}{100}\right] - 0.085$$

Calculation of void ratio sing consolidation data:

The void ratio at the end of each pressure increment can be calculated by two methods.

1) Height of solids method:

The void ratio is calculated from the following relation,

$$e = \frac{H - H_S}{H_S}$$

$$e = \frac{V_V}{V_S} = \frac{V - V_S}{V_S} = \frac{H - H_S}{H_S}$$

e = void ratio

H = Total thickness = $H_1 + \Delta H$

H_s = Height of soil solids.

H_1 = Initial thickness at '$\Delta\sigma$'

ΔH = compression (or) decrease in thickness under '$\Delta\sigma$'

$$\gamma_S = \frac{W_S}{V_S} = \frac{W_d}{H_S A} = G.\gamma_W$$

$$\frac{W_d}{AG.\gamma_W} = H_S$$

γ

2) Change in void ratio method:

Assuming the specimen is fully saturated, the final void ratio e_f at the end of the test is computed from the relation $\Delta e = \frac{1+e_f}{H_f} eH$

We know that,

$$C_v = \frac{(T_v)_{90} d^2}{t_{90}}$$

for 90% consolidation

$T_v = 1.7813 - 0.9332 \log_{10}(100 - \%)$

$T_v = 0.848$

$$\therefore C_v = \frac{0.848 d^2}{t_{90}}$$

Co-efficient of volume change:

1) Change in void ratio method:

In this method, the coefficient of volume change (m_v) is computed using the following equation.

$$m_v = \frac{\Delta e}{1 + e_0} \cdot \frac{1}{\Delta \sigma'}$$

Determination of co-efficient of consolidation:

The co-efficient of consolidation (C_v) is determined by methods based on the compassionbetween

i) The characteristics of the theoretical relation b/w 'T_v' and 'U'

ii) Relation b/w the elapsed time 't' and 'U' Obtained for soil specimen in laboratory test.

Two commonly used methods are,

i) Square root of time fitting method.

ii) Logorithm of time fitting method.

1) Square root of time fitting method (or) $\sqrt{t}$ method (or) Taylors method

- ❖ Draw the curve between '$\sqrt{t}$' as x-axis, and the dial reading '-R' in y-axis. The initial dial reading 'R_0' corresponds to time $t = 0$ and $u = 0$
- ❖ The starting portion (line A) is produced back to meet y-axis at reading 'R_c'. From 'R_c' another line "B" is drawn that its magnitude is 1.15 times that of "line A"
- ❖ The intersection of 'line B' with the consolidation curve gives a point 'P' corresponding to 90% consolidation, whose dial reading and time may be designated as 'R_{90}' & '$\sqrt{t_{90}}$' respectively.
- ❖ Find the mid point between R_o and R_{100} to get R_{50}. Make it, find thecorresponding t_{50}

$$C_v = \frac{T_{50}}{t_{50}} d^2$$

for 50% *consolidation*

$$T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2$$

$$T_v = \frac{\pi}{4}\left(\frac{50}{100}\right)^2$$

Logarithm of time fitting method (or) log t method (or) casagrande's method:

❖ The curve is drawn between logt and dial reading ‘R’. Select any point on the initial portion of the same curve (A) extend that point along y- axis on x – axis find time t_1.

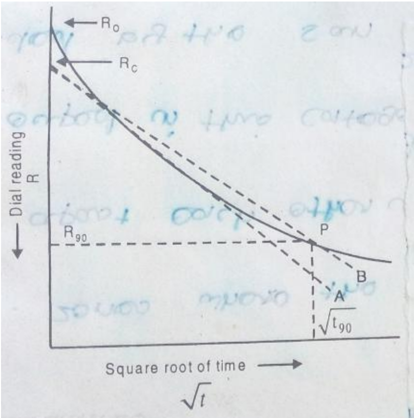


Fig. 11.9. R versus $\sqrt{t}$ (from laboratory data)

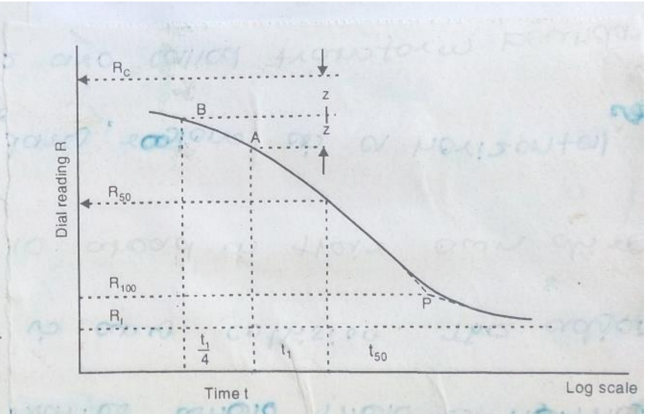


Fig. 11.11. R versus log *t* (from laboratory data)

❖ Then ‘t_1’ is multiplied by ‘4’ as ‘$4t_1$’ measured along x axis Extend that point on the curve in vertical direction (B).

❖ Measure the distance between ‘A’ & ‘B’ as ‘Z’, Mark this distance from first selected point (A) from the initial portion in vertical direction, to get initial dial reading R_0

❖ Draw the tangent from middle portion and bottom portion of the curve, these twomeets at a particular point note it as ‘P’

❖ Extend this point parallel to x- axis, it meets on y-axis, that point is R_{100}

$$C_v = \frac{T_{50}}{t_{50}} H^2 = \frac{0.197\ H^2}{t_{50}}$$

Basic definition	Formula
Coefficient of compressibility	$a_v = -\frac{\Delta e}{\Delta \sigma}$
Coefficient of volume compressibility or volume change	$m_v = \frac{-\Delta e}{1+e}\left(\frac{1}{\Delta \bar{\sigma}}\right)$
Compression index	$C_c = \frac{-\Delta e}{\log \frac{(\sigma_o + \Delta \sigma)}{\bar{\sigma}_o}}$
Coefficient of consolidation	$C_v = \frac{k}{m_v \times \gamma_w}$
Time factor(U<60)	$T_v = \frac{\pi}{4} U^2$
Time factor(U>60)	$T_v = 1.781 - 0.933 \log_{10} (100 - U\%)$