Magnetostatics

In previous chapters we have seen that an electrostatic field is produced by static or stationary charges. The relationship of the steady magnetic field to its sources is much more complicated.

The source of steady magnetic field may be a permanent magnet, a direct current or an electric field changing with time. In this chapter we shall mainly consider the magnetic field produced by a direct current. The magnetic field produced due to time varying electric field will be discussed later. Historically, the link between the electric and magnetic field was established Oersted in 1820. Ampere and others extended the investigation of magnetic effect of electricity. There are two major laws governing the magnetostatic fields are:

- Biot-Savart Law
- Ampere's Law

ually, the magnetic field intensity is represented by the vector H(Bar). It is customary to represent the direction of the magnetic field intensity (or current) by a small circle with a dot or cross sign depending on whether the field (or current) is out of or into the page as shown in Fig. 4.1.

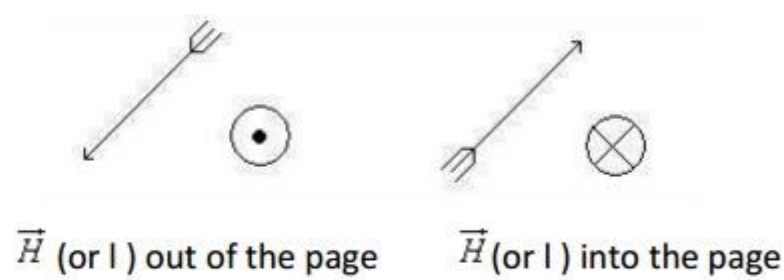


Fig. 4.1: Representation of magnetic field (or current)

Biot- Savart Law

This law relates the magnetic field intensity *dH* produced at a point due to a differential cur
element $Id\vec{l}$ as shown in Fig. 4.2.

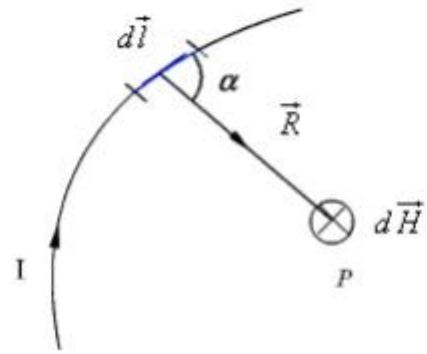


Fig. 4.2: Magnetic field intensity due to a current element

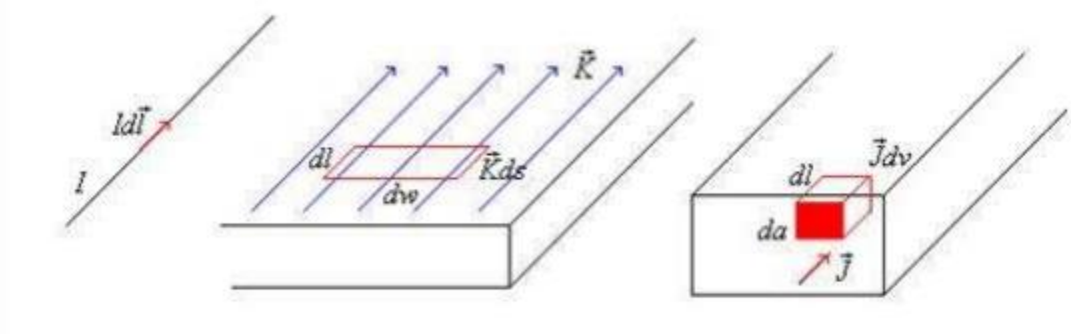
The magnetic field intensity $d\vec{H}$ at P can be written

$$d\vec{H} = \frac{Id\vec{l}\times\hat{a}_R}{4\pi R^2} = \frac{Id\vec{l}\times\vec{R}}{4\pi R^3} \quad \text{..........................(4}$$

$$dH = \frac{IdlSin\alpha}{4\pi R^2} \quad \text{...(4.1b)}$$

where $R = |\vec{R}|$ is the distance of the current element from the point

Similar to different charge distributions, we can have different current distribution such as line current, surface current and volume current. These different types of current densities are shown in Fig. 4.3.



Line Current   Surface Current   Volume Current

Fig. 4.3: Different types of current distributions

By denoting the surface current density as K (in amp/m) and volume current density as J (in amp/m2) we can write:

$Id\vec{l} = \vec{K}ds = \vec{J}dv$ ....(4.2)

(It may be noted that $I = Kdw = Jda$)

Employing Biot-Savart Law, we can now express the magnetic field intensity H. In term these current distributi

$\vec{H} = \int_l \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$ .... **for line current.**............................(4

$\vec{H} = \int_S \frac{Kd\vec{s} \times \vec{R}}{4\pi R^3}$ .... **for surface current** ....(4

$\vec{H} = \int_V \frac{\vec{J}dv \times \vec{R}}{4\pi R^3}$ .... **for volume current**.......................(4.3c)

To illustrate the application of Biot - Savart's Law, we consider the following example.

Example 4.1: We consider a finite length of a conductor carrying a current $\vec{I}$ placed along z
as shown in the Fig 4.4. We determine the magnetic field at point P due to this current carr
conductor.

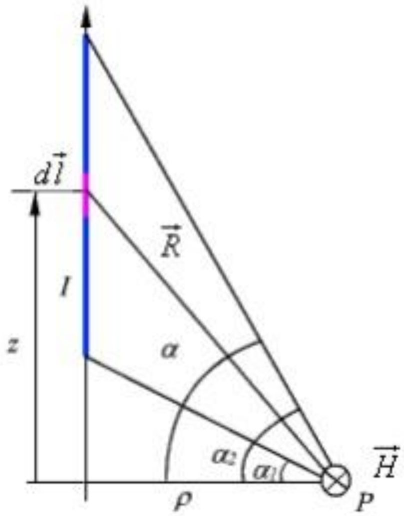


Fig. 4.4: Field at a point P due to a finite length current carrying conductor

With reference to Fig. 4.4, we find that

$$d\vec{l} = dz\,\hat{a}_z \quad and \quad \vec{R} = \rho\hat{a}_\rho - z\hat{a}_z$$ ....(4.4)

Applying Biot - Savart's law for the current element $\vec{I}\,d\vec{l}$

we can write,

$$\overrightarrow{dH} = \frac{Id\vec{l}\times\vec{R}}{4\pi R^3} = \frac{\rho dz\hat{a}_\phi}{4\pi[\rho^2 + z^2]^{3/2}}$$ ....(4.5)

Substituting $\frac{z}{\rho} = \tan\alpha$ we can write,

$$\vec{H} = \int_{\alpha_1}^{\alpha_2} \frac{I}{4\pi}\frac{\rho^2\sec^2\alpha d\alpha}{\rho^3\sec^3\alpha}\hat{a}_\phi = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\phi$$ ....(4.6)

We find that, for an infinitely long conductor carrying a current I , $\alpha_2 = 90^0$ and $\alpha_1 = -90^0$

Therefore, $\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$ ....(4.7)