

Problems:

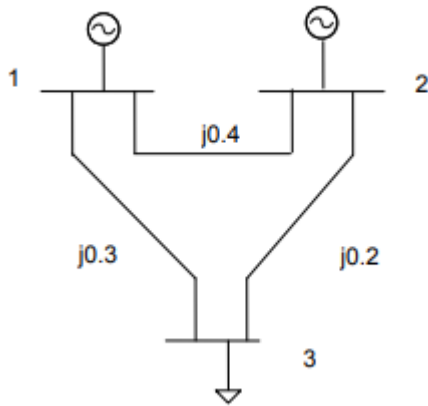
1) Fig. shows a three bus power system.

Bus 1 : Slack bus, $V = 1.05/00$ p.u.

Bus 2 : PV bus, $V = 1.0$ p.u. $P_g = 3$ p.u.

Bus 3 : PQ bus, $P_L = 4$ p.u., $Q_L = 2$ p.u.

Carry out one iteration of load flow solution by Gauss Seidel method.



Neglect limits on reactive power generation.

Solution:

Admittance of each line

$$Y_{12} = \frac{1}{Z_{12}} = \frac{1}{j0.4} = -j2.5 \text{ p.u.}$$

$$Y_{13} = \frac{1}{Z_{13}} = \frac{1}{j0.3} = -j3.333 \text{ p.u.}$$

$$Y_{23} = \frac{1}{Z_{23}} = \frac{1}{j0.2} = -j5 \text{ p.u.}$$

$$Y_{11} = y_{12} + y_{13} = -j2.5 - j3.333 = -j5.833 \text{ p.u.}$$

$$Y_{22} = y_{12} + y_{23} = -j2.5 - j5 = -j7.5 \text{ p.u.}$$

$$Y_{33} = y_{13} + y_{23} = -j3.333 - j5 = -j8.333 \text{ p.u.}$$

$$Y_{12} = Y_{21} = -y_{12} = -(-j2.5) = j2.5 \text{ p.u.}$$

$$Y_{13} = Y_{31} = -y_{13} = -(-j3.33) = j3.33 \text{ p.u.}$$

$$Y_{23} = Y_{32} = -y_{23} = -(-j5) = j5 \text{ p.u.}$$

The admittance matrix is given as

$$Y_{\text{bus}} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} \\ -Y_{21} & Y_{22} & -Y_{23} \\ -Y_{31} & -Y_{32} & Y_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -j5.833 & j2.5 & j3.33 \\ j2.5 & -j7.5 & j5 \\ j3.33 & j5 & -j8.333 \end{bmatrix}$$

Assume initial voltages to all buses

$$V_1(0) = 1.05 \angle 0^\circ = 1.05 + j0 \text{ p.u}$$

$$V_2(0) = 1.0 + j0 \text{ p.u}$$

$$V_3(0) = 1.0 + j0 \text{ p.u}$$

Bus 1 is a slack bus

$$V_1(1) = 1.05 \angle 0^\circ = 1.05 + j0 \text{ p.u}$$

Bus 2 is a generator bus

To calculate reactive power

$$Q_{p,cal}^{k+1} = (-1) \times \text{Im} \left\{ (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^n Y_{pq} V_q^k \right] \right\}$$

$$Q_{2,cal}^1 = (-1) \times \text{Im} \{ (V_2^0)^* [Y_{21} V_1^1 + Y_{22} V_2^0 + Y_{23} V_3^0] \}$$

$$= (-1) \times \text{Im} (1 - j0) [(j2.5)(1.05 + j0) + (-j7.5)(1 + j0) + (j5)(1 + j0)]$$

$$Q_{2,cal}^1 = -0.125 \text{ p.u}$$

The phase of bus -2 voltage in first iteration is given by phase of $V_{p,temp}$

K+1

When $p=3$ $Q_2^1 = -0.125 \text{ p.u}$ and $k=0$

$$V_{p,temp}^{k+1} = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^n Y_{pq} V_q^k \right]$$

$$V_{2,temp}^{0+1} = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - \sum_{q=1}^{2-1} Y_{2q} V_q^{0+1} - \sum_{q=2+1}^3 Y_{2q} V_q^0 \right]$$

$$V_{2,temp}^1 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 \right]$$

$$= \frac{1}{-j7.5} \left[\frac{3 + j0.125}{1 - j0} - (j2.5)(1.05 + j0) - (j5)(1 + j0) \right]$$

$$V_2^1 = \frac{1}{-j7.5} [3 - j7.5] = 1.077 \angle 21.8^\circ \text{ V}$$

$$\delta_2^1 = \angle V_{2,temp}^1 = 21.8^\circ \text{ V}$$

$$|V_2^1| = |V_2| \quad \text{spc} \angle \delta_2^1 = 1.0 \angle 21.8^\circ$$

$$|V_2^1| = 0.928429 + j0.3713 \text{ V}$$

Bus 3 Load Bus

The specified powers are load powers and so they considered as negative powers

$$P_3 = -P_L = -4$$

$$Q_3 = -Q_L = -2$$

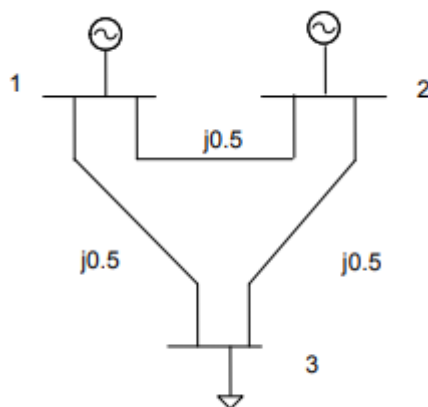
$$\begin{aligned}
 V_P^{k+1} &= \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^n Y_{pq} V_q^k \right] \\
 V_3^1 &= \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1^1 - Y_{32} V_2^1 \right] \\
 &= \frac{1}{-j8.333} \left[\frac{-4+j2}{1-j0} - (j3.33)(1.05 + j0) - (j5)(0.928429 + j0.37135) \right] \\
 V_3^1 &= 0.7806 \angle -19.24^\circ \\
 V_3^1 &= 0.737046 - j0.25724 \text{ p.u}
 \end{aligned}$$

- 2) Carry out one iteration of load flow analysis of the system given below by Gauss-Seidal method

Bus no	Bus type	P	Q	V p.u
1	Slack	-	-	1.02
2	P-V	0.8	$0.1 \leq Q \leq 1$	1
3	P-Q	1.0	0.4	-

Line reactance in p.u

Bus code	Impedance
1-2	j0.5
2-3	j0.5
3-1	j0.5



Solution:

$$Y_{12} = \frac{1}{Z_{12}} = \frac{1}{j0.5} = -j2 \text{ p.u.}$$

$$Y_{13} = \frac{1}{Z_{13}} = \frac{1}{j0.5} = -j2 \text{ p.u.}$$

$$Y_{23} = \frac{1}{Z_{23}} = \frac{1}{j0.5} = -j2 \text{ p.u.}$$

$$Y_{11} = Y_{12} + Y_{13} = -j2 - j2 = -j4 \text{ p.u.}$$

$$Y_{22} = Y_{12} + Y_{23} = -j2 - j2 = -j4 \text{ p.u.}$$

$$Y_{33} = Y_{13} + Y_{23} = -j2 - j2 = -j4 \text{ p.u.}$$

$$Y_{12} = Y_{21} = -Y_{12} = -(-j2) = j2 \text{ p.u.}$$

$$Y_{13} = Y_{31} = -Y_{13} = -(-j2) = j2 \text{ p.u.}$$

$$Y_{23} = Y_{32} = -Y_{23} = -(-j2) = j2 \text{ p.u.}$$

The admittance matrix is given as

$$Y_{bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} \\ -Y_{21} & Y_{22} & -Y_{23} \\ -Y_{31} & -Y_{32} & Y_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -j4 & j2 & j2 \\ j2 & -j4 & j2 \\ j2 & j2 & -j4 \end{bmatrix}$$

Assume initial voltages to all buses

$$V_1(0) = 1.02 \angle 0^\circ = 1.02 + j0 \text{ p.u.}$$

$$V_2(0) = 1.0 + j0 \text{ p.u.}$$

$$V_3(0) = 1.0 + j0 \text{ p.u.}$$

Bus 1 is a slack bus

$$V_1^{(1)} = 1.02 \angle 0^\circ = 1.02 + j0 \text{ p.u.}$$

Bus 2 is a generator bus

To calculate reactive power

$$Q_{p,cal}^{k+1} = (-1) \times \text{Im} \left\{ (V_p^k)^* \left[\sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} + \sum_{q=p}^n Y_{pq} V_q^k \right] \right\}$$

$$Q_{2,cal}^1 = (-1) \times \text{Im} \{ (V_2^0)^* [Y_{21} V_1^1 + Y_{22} V_2^0 + Y_{23} V_3^0] \}$$

$$= (-1) \times \text{Im} (1 - j0) [(j2)(1.02 + j0) + (-j4)(1 + j0) + (j2)(1 + j0)]$$

$$Q_{2,cal}^1 = -0.04 \text{ p.u.}$$

This value is not within the specified limit, so treat this bus as load bus

$$Q_2 = 0.1 \text{ p.u.}, P_2 = 0.3 \text{ p.u. and } V_2^0 = 1.0 + j0$$

$$\begin{aligned}
V_p^{k+1} &= \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^n Y_{pq} V_q^k \right] \\
&= \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 \right] \\
&= \frac{1}{-j4} \left[\frac{0.8 - j0.1}{1 - j0} - (j2)(1.02 + j0) - (j2)(1 + j0) \right] \\
&= \\
|V_2^1| &= \mathbf{1.035 + j0.2 = 1.054 \angle 10.93^\circ V}
\end{aligned}$$

Bus 3 Load Bus

The specified powers are load powers and so they considered as negative powers

$$P_3 = -P_L = -1$$

$$Q_3 = -Q_L = -0.4$$

$$\begin{aligned}
V_p^{k+1} &= \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^n Y_{pq} V_q^k \right] \\
V_3^1 &= \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1^1 - Y_{32} V_2^1 \right] \\
&= \frac{1}{-j4} \left[\frac{-1 + j0.4}{1 - j0} - (j2)(1.02 + j0) - (j2)(\mathbf{1.035 + j0.2}) \right]
\end{aligned}$$