

1.4 PRESSURE MEASUREMENTS BY MANOMETERS

MANOMETER

A manometer is an instrument that uses a column of liquid to measure pressure, although the term is currently often used to mean any pressure instrument.

Two types of manometer, such as

1. Simple manometer
2. Differential manometer

The U type manometer, which is considered as a primary pressure standard, derives pressure utilizing the following equation:

$$P = P_2 - P_1 = h \rho g$$

Where:

P = Differential pressure

P_1 = Pressure applied to the low pressure connection

P_2 = Pressure applied to the high pressure connection

h = is the height differential of the liquid columns between the two legs of the manometer

ρ = mass density of the fluid within the columns

g = acceleration of gravity

SIMPLE MANOMETER

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are:

1. Piezometer
2. U tube manometer
3. Single Column manometer

PIEZOMETER

A piezometer is either a device used to measure liquid pressure in a system by measuring the height to which a column of the liquid rises against gravity, or a device which measures the pressure (more precisely, the piezometric head) of groundwater at a specific point. A piezometer is designed to measure static pressures, and thus differs from a pitot tube by not being pointed into the fluid flow.

It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$= \rho \times g \times h \frac{\text{N}}{\text{m}^2}.$$

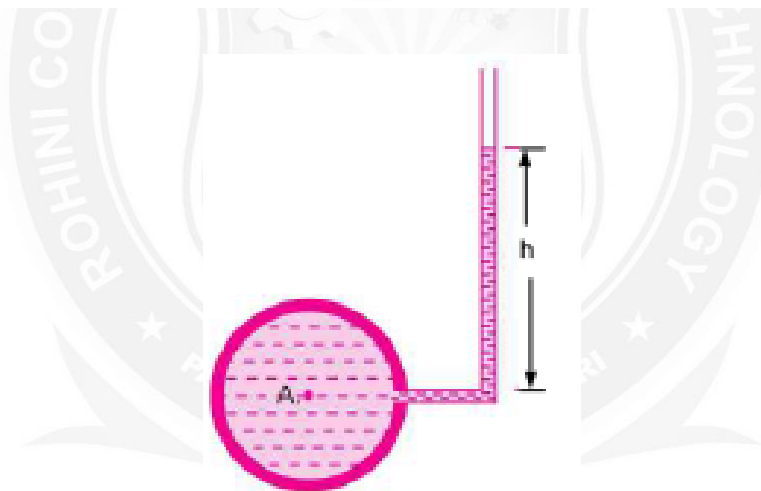


Figure 1.5.1 Piezometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 43]

U TUBE MANOMETER

Manometers are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere.

Manometer is needed for measuring large gauge pressures. It is basically the modified form of the piezometric tube.

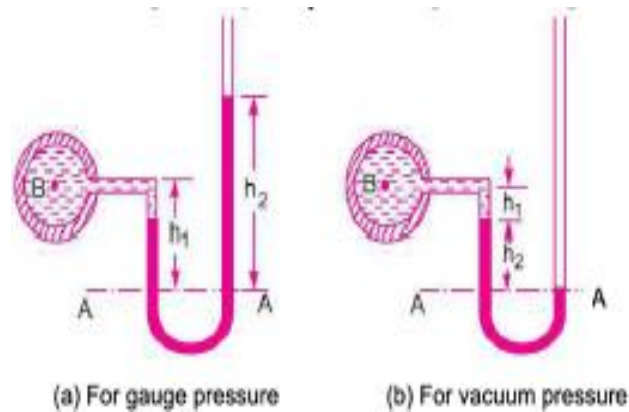


Figure 1.5.2 U Tube Manometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 43]

(a) For Gauge Pressure. Let B is the point at which pressure is to be measured, whose value is p . The datum line is $A-A$.

Let h_1 = Height of light liquid above the datum line
 h_2 = Height of heavy liquid above the datum line
 S_1 = Sp. gr. of light liquid
 ρ_1 = Density of light liquid = $1000 \times S_1$
 S_2 = Sp. gr. of heavy liquid
 ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line $A-A$ in the left column and in the right column of U-tube manometer should be same.

Pressure above $A-A$ in the left column = $p + \rho_1 \times g \times h_1$

Pressure above $A-A$ in the right column = $\rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$\therefore p = (\rho_2 g h_2 - \rho_1 \times g \times h_1).$

(b) For Vacuum Pressure. For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (b). Then

Pressure above $A-A$ in the left column = $\rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above $A-A$ = 0

$\therefore \rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore p = -(\rho_2 g h_2 + \rho_1 g h_1).$

Single Column Manometer

Single column manometer is a modified form of a U-tube manometer in which one side is a large reservoir and the other side is a small tube, open to the atmosphere.

There are two types of single column manometer:

1. Vertical single column manometer.
2. Inclined single column manometer.

1.Vertical single column Manometer

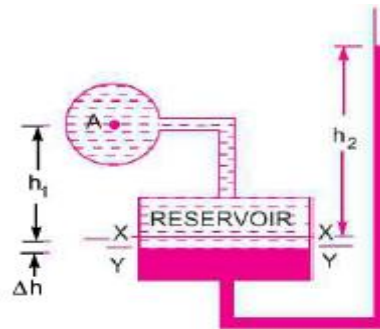


Figure 1.5.3 Vertical single column Manometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 49]

Let Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above X-X

p_A = Pressure at A, which is to be measured

A = Cross-sectional area of the reservoir

a = Cross-sectional area of the right limb

S_1 = Sp. gr. of liquid in pipe

S_2 = Sp. gr. of heavy liquid in reservoir and right limb

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore A \times \Delta h = a \times h_2$$

$$\therefore \Delta h = \frac{a \times h_2}{A} \quad \dots(i)$$

$$p_A = \frac{a \times h_2}{A} [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

$$A \gg a$$

Then:
$$p_A = h_2 \rho_2 g - h_1 \rho_1 g$$

2. Inclined single column Manometer

This manometer is more sensitive. Due to the inclination the distance moved by the heavy liquid in the right limb will be more.

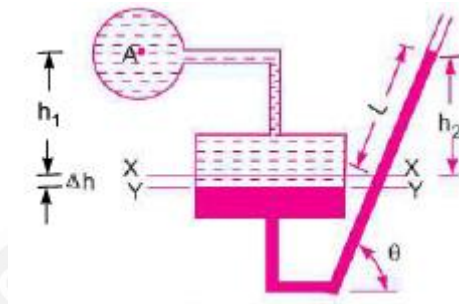


Figure 1.5.4 Inclined single column Manometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 49]

Let L = Length of heavy liquid moved in right limb from X-X
 θ = Inclination of right limb with horizontal
 h_2 = Vertical rise of heavy liquid in right limb from X-X = $L \times \sin \theta$

From the eq.

$$p_A = h_2 \rho_2 g - h_1 \rho_1 g$$

By substituting the value of h_2 , We get:

$$p_A = \sin \theta \times \rho_2 g - h_1 \rho_1 g.$$

DIFFERENTIAL MANOMETER

Differential Manometers are devices used for measuring the difference of pressure between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, which difference of pressure is to be measure.

Most commonly types of differential manometers are:

- 1.U-tube differential manometer.
- 2.Inverted U-tube differential manometer

1.U-tube differential Manometer

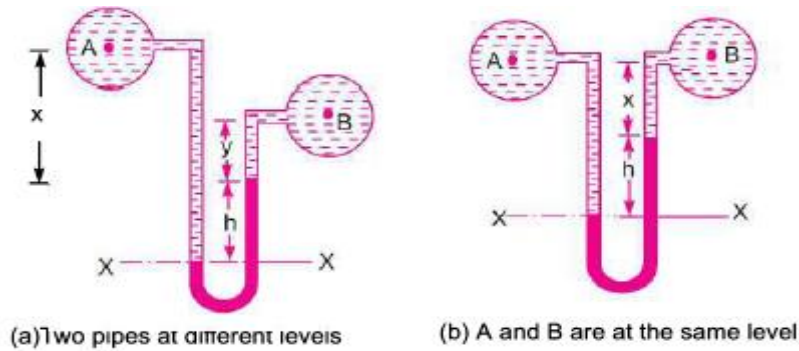


Figure 1.5.5 U-tube differential Manometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 51]

In Fig. (a), the two points A and B are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are p_A and p_B .

- Let
- h = Difference of mercury level in the U-tube.
 - y = Distance of the centre of B, from the mercury level in the right limb.
 - x = Distance of the centre of A, from the mercury level in the right limb.
 - ρ_1 = Density of liquid at A.
 - ρ_2 = Density of liquid at B.
 - ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X.

Pressure above X-X in the left limb = $\rho_1 g(h + x) + p_A$

where p_A = pressure at A.

Pressure above X-X in the right limb = $\rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 g y + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

$$\therefore \quad \text{Difference of pressure at A and B} = h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

In Fig. (b), the two points A and B are at the same level and contains the same liquid of density ρ_1 . Then

Pressure above X-X in right limb = $\rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above X-X in left limb = $\rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 g x + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1).$$

2. Inverted U-tube differential Manometer

It consists of inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring differences of low pressures.

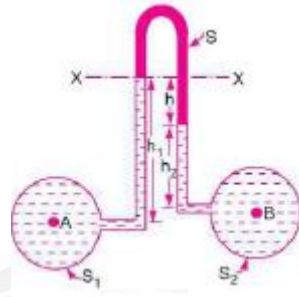


Figure 1.5.6 Inverted U-tube differential Manometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 53]

Let the pressure at A is more than the pressure at B.

- Let
- h_1 = Height of liquid in left limb below the datum line
 - h_2 = Height of liquid in right limb
 - h = Difference of light liquid
 - ρ_1 = Density of liquid at A
 - ρ_2 = Density of liquid at B
 - ρ_s = Density of light liquid
 - p_A = Pressure at A
 - p_B = Pressure at B.

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or

$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h.$$

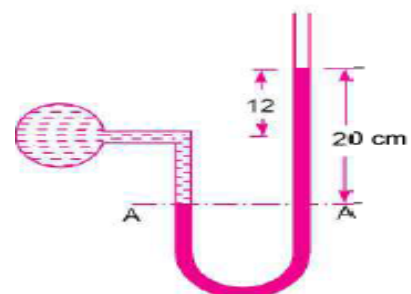
Problem 1: The right limb of a simple U – tube manometer containing mercury is open to the atmosphere, while the left limb is connected to a pipe in which a fluid of sp.gr.0.9 is flowing. The centre of pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe, if the difference of mercury level in the two limbs is 20 cm.

Given, Sp.gr. of liquid S1= 0.9

Density of fluid $\rho_1 = S_1 \times 1000 = 0.9 \times 1000$
 $= 900 \text{ kg/ m }^3$

Sp.gr. of mercury S2 = 13.6

Density of mercury $\rho_2 = 13.6 \times 1000 = 13600$
 kg/m^3



Difference of mercury level $h_2 = 20\text{cm} = 0.2\text{m}$

Height of the fluid from A – A $h_1 = 20 - 12 = 8\text{cm} = 0.08\text{ m}$

Let 'P' be the pressure of fluid in pipe

Equating pressure at A – A, we get $p + \rho_1 g h_1 = \rho_2 g h_2$

$$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times 0.2$$

$$p = 13.6 \times 1000 \times 9.81 \times 0.2 - 900 \times 9.81 \times 0.08$$

$$p = 26683 - 706$$

$$p = 25977 \text{ N/m}^2$$

$$p = 2.597 \text{ N/cm}^2$$

Pressure of fluid = 2.597 N/ cm²

Problem2: A simple U – tube manometer containing mercury is connected to a pipe in which a fluid of sp.gr. 0.8 And having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40cm. and the height of the fluid in the left tube from the centre of pipe is 15cm below.

Given,

Sp.gr of fluid $S_1 = 0.8$

Sp.gr. of mercury $S_2 = 13.6$

Density of the fluid $= S_1 \times 1000 = 0.8 \times 1000 = 800$

Density of mercury $= 13.6 \times 1000$

Difference of mercury level $h_2 = 40\text{cm} = 0.4\text{m}$

Height of the liquid in the left limb $= 15\text{cm} = 0.15\text{m}$

Let the pressure in the pipe $= p$

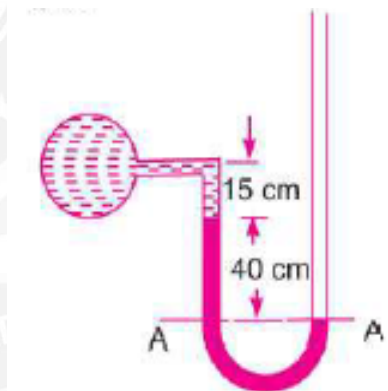
Equating pressures above datum line A—A

$$\rho_2 g h_2 + \rho_1 g h_1 + P = 0$$

$$P = - [\rho_2 g h_2 + \rho_1 g h_1] = - [13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$

$$= 53366.4 + 1177.2 = -54543.6 \text{ N/m}^2$$

$$\mathbf{P = - 5.454 \text{ N/cm}^2}$$



Problem 3: A single column manometer is connected to the pipe containing liquid of sp.gr.0.9. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube of manometer. sp.gr. of mercury is 13.6. Height of the liquid from the centre of pipe is 20cm and difference in level of mercury is 40cm.

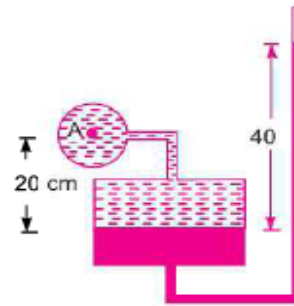
Given,

Sp.gr. of liquid in pipe $S_1 = 0.9$

Density $\rho_1 = 900 \text{ kg/m}^3$

Sp.gr. of heavy liquid $S_2 = 13.6$

Density $\rho_2 = 13600$



$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of the liquid $h_1 = 20\text{cm} = 0.2\text{m}$

Rise of mercury in the right limb $h_2 = 40\text{cm} = 0.4\text{m}$

$$p_A = \frac{a}{A} h_2 [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

$$\begin{aligned} &= \frac{1}{100} \times 0.4 [13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81 \\ &= \frac{0.4}{100} [133416 - 8829] + 53366.4 - 1765.8 \\ &= 533.664 + 53366.4 - 1765.8 \text{ N/m}^2 = 52134 \text{ N/m}^2 = \mathbf{5.21 \text{ N/cm}^2}. \text{ Ans.} \end{aligned}$$

Pressure in pipe $A = 5.21 \text{ N/cm}^2$

Problem 4: A pipe contains an oil of sp.gr.0.9. A differential manometer is connected at the two points A and B shows a difference in mercury level at 15cm. find the difference of pressure at the two points.

Given:

Sp.gr. of oil $S_1 = 0.9$: density $\rho_1 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Difference of level in the mercury $h = 15\text{cm} = 0.15 \text{ m}$

Sp.gr. of mercury = 13.6, Density = $13.6 \times 1000 = 13600 \text{ kg/m}^3$

The difference of pressure $p_A - p_B = g \times h \times (\rho_g - \rho_1)$

$$= 9.81 \times 0.15 (13600 - 900)$$

$$\mathbf{p_A - p_B = 18688 \text{ N/m}^2}$$

Problem 5: A differential manometer is connected at two points A and B. At B air pressure is 9.81 N/cm^2 . Find absolute pressure at A.

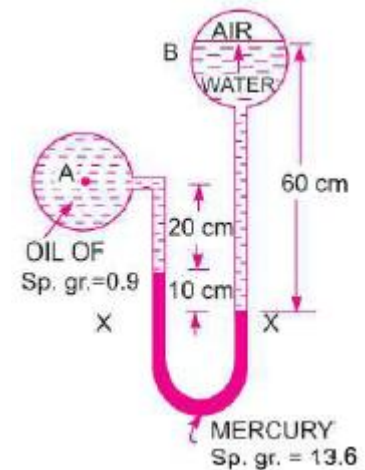
Given:

Density of air $= 0.9 \times 1000 = 900 \text{ kg/m}^3$

Density of mercury $= 13.6 \times 10^3 \text{ kg/m}^3$.

Let pressure at A is p_A

Taking datum as X – X



Pressure above X – X in the right limb

$$= 1000 \times 9.81 \times 0.6 + p_B = 5886 + 98100 = 103986$$

Pressure above X – X in the left limb

$$= 13.6 \times 10^3 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + p_A$$

$$= 13341.6 + 1765.8 + p_A$$

Equating the two pressures heads

$$103986 = 13341.6 + 1765.8 + p_A$$

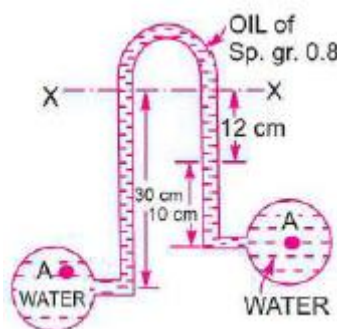
$$= 15107.4 + p_A$$

$$p_A = 103986 - 15107.4$$

$$= 88878.6 \text{ N/m}^2$$

$$p_A = 8.887 \text{ N/cm}^2$$

Problem 6: Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp.gr. 0.8 is connected. The pressure head in the pipe A is 2m of water. Find the pressure in the pipe B for the manometer readings shown in fig.



Given:

Pressure head at $A = \frac{p_A}{\rho g} = 2 \text{ m of water}$

$\therefore p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620 \text{ N/m}^2$

Pressure below X – X in the left limb

$$= p_A - \rho_1 g h_1$$

$$= 19620 - 1000 \times 9.81 \times 0.3$$

$$= 16677 \text{ N/m}^2$$

Pressure below X – X in the right limb

$$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$$

$$= p_B - 981 - 941.76 = p_B - 1922.76$$

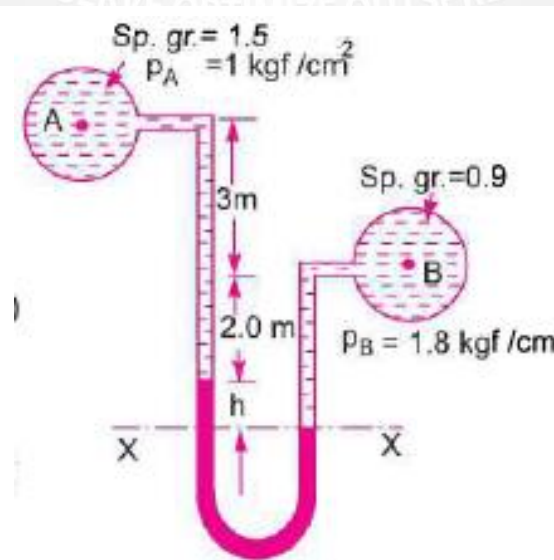
Equating the two pressures, we get,

$$16677 = p_B - 1922.76$$

$$p_B = 16677 + 1922.76$$

$$\mathbf{p_B = 18599.76 \text{ N/m}^2}$$

Problem 7: A differential manometer is connected at two points A and B of two pipes. The pipe A contains liquid of sp.gr. = 1.5 while pipe B contains liquid of sp.gr. = 0.9. The pressures at A and B are 1 kgf/cm^2 and 1.80 Kg f/cm^2 respectively. Find the difference in mercury level in the differential manometer.



Sp.gr. of liquid at A $S_1 = 1.5$

Sp.gr. of liquid at B $S_2 = 0.9$

Pressure at A $p_A = 1 \text{ kgf/cm}^2 = 1 \times 10^4 \times \text{kg/m}^2 = 1 \times 10^4 \times 9.81 \text{ N/m}^2$

Pressure at B $p_B = 1.8 \text{ kgf/cm}^2 = 1.8 \times 10^4 \times 9.81 \text{ N/m}^2$ [1kgf = 9.81 N]

Density of mercury $= 13.6 \times 1000 \text{ kg/m}^3$

Taking X – X as datum line

Pressure above X – X in left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81(2+3) + (9.81 \times 10^4)$$

Pressure above X – X in the right limb $= 900 \times 9.81(h + 2) + 1.8 \times 9.81 \times 10^4$

Equating the two pressures, we get

$$13.6 \times 1000 \times 9.81h + 1500 \times 9.81 \times 5 + 9.81 \times 10^4 = 900 \times 9.81(h + 2) + 1.8 \times 9.81 \times 10^4$$

Dividing both sides by 1000×9.81

$$13.6h + 7.5 + 10 = 0.9(h+2) + 18$$

$$(13.6 - 0.9)h = 1.8 + 18 - 17.5 = 19.8 - 17.5 = 2.3$$

$$h = 2.3 / 12.7 = 0.181\text{m}$$

$$\mathbf{h = 18.1 \text{ cm}}$$