The Lossless Medium for Electromagnetic Waves

- A lossless medium is a medium with zero conductivity and finite permeability and permittivity.
- When an electromagnetic wave propagates through a lossless medium, the amplitude of its electric field or magnetic field remains constant throughout the propagation.
- The properties of the lossless medium affect the speed of propagation, and it is reduced by a factor of 1/ (√***μ***r***ε***r) compared to the speed of the electromagnetic waves in the vacuum.

- The Lossless Medium for Electromagnetic Waves
- A lossless medium for electromagnetic waves is a medium with zero conductivity (***σ***) and finite permeability (***μ***) and permittivity(***ε***). It can be described using the equation below:

$$\text{Lossless media} \Rightarrow \quad (\sigma=0,\ \mu=\mu_r\mu_o,\ \epsilon=\epsilon_o\epsilon_r) \qquad (1)$$

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- Consider a plane wave propagating in the y-direction, with electric and magnetic fields mutually perpendicular to each other and to the wave propagation. A set of wave equations are used to define the electromagnetic waves:

$$\frac{d^2E_{zs}}{dy^2} - \gamma^2 E_{zs} = 0 \qquad (2)$$

$$\frac{d^2H_{xs}}{dy^2} - \gamma^2 H_{xs} = 0 \qquad (3)$$

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- The general solution to the linear, homogenous second order differential equations can be given in equations 4 and 5.

$$E_{zs}(y) = E_1 e^{\gamma y} + E_2 e^{-\gamma y}$$
$$= E_1 e^{\alpha y} e^{j\beta y} + E_2 e^{-\alpha y} e^{-j\beta y} \qquad (4)$$

$$H_{xs}(y) = H_1 e^{\gamma y} + H_2 e^{-\gamma y}$$
$$= H_1 e^{\alpha y} e^{j\beta y} + H_2 e^{-\alpha y} e^{-j\beta y} \qquad (5)$$

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- From the above equations, we can conclude that other than the field amplitudes, E_1, E_2, H_1, and H_2, the electromagnetic wave characteristics of the fields are identical. Characteristics of the electromagnetic waves defined by the general field solutions can be determined by investigating the corresponding instantaneous electric or magnetic fields.

Taking this into consideration, the time-domain solution of the wave equation in terms of the electric field can be represented by equation 6.

$$E_z(y,t) = \underbrace{E_1 e^{\alpha y}\cos(\omega t + \beta y)}_{\substack{\text{Amplitude} = E_1 e^{\alpha y} \\ \text{Phase} = \omega t + \beta y}} + \underbrace{E_2 e^{-\alpha y}\cos(\omega t - \beta y)}_{\substack{\text{Amplitude} = E_2 e^{-\alpha y} \\ \text{Phase} = \omega t - \beta y}} \quad (6)$$

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- In the equation, **α** is the attenuation constant, **β** is the phase constant, and **ω** is the angular velocity of propagation.
- The Attenuation and Phase Constants
- The attenuation constant is the measure of the attenuation of the amplitude of the fields in electromagnetic wave propagation. It can be described by equation 7. As the wave moves forward, its phase may undergo variations, and the factor that indicates this variation is called the phase constant and is shown in equation 8.

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]} \quad (7)$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]} \quad (8)$$

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- By substituting **σ**=0 in the above equations, we obtain the following:
- $\alpha = 0$ (9)
- $\beta = \omega\sqrt{\mu\varepsilon}$ (10)
- Equation 9 is the mathematical representation of no attenuation in the lossless medium. When an electromagnetic wave propagates through a lossless medium, the amplitude of its electric field or magnetic field remains constant throughout the propagation. There is no reduction to the amplitudes of the wave in the course of propagation through the lossless medium. In a lossless medium, electromagnetic waves can be called non-decaying waves.
- The phase of the electromagnetic wave changes in a lossless medium, and is influenced by the velocity of wave propagation and the characteristics of the medium. Equation 10 shows that as the value of the relative permeability and relative permittivity of the medium increases, the phase constant also increases. The angular velocity of the wave matters when coming to the phase variations of the electromagnetic wave.
- Now, let's discuss the effect of a lossless medium on the velocity of wave propagation.
- The Velocity of Wave Propagation

$$u = \frac{\omega}{\beta} \quad (11)$$

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- The velocity, u, of the electromagnetic wave is given by equation 11. Since we already know what the value of **β** is, we can rewrite the velocity as equation 12.

$$u = \frac{1}{\sqrt{\mu\epsilon}} \qquad (12)$$

- From the equation, we can see that the properties of the lossless medium affect the speed of propagation, and it is reduced by a factor of $1/(\sqrt{\mu_r\varepsilon_r})$ compared to the speed of the electromagnetic waves in the vacuum.