CRITICAL CLEARING ANGLE AND CRITICAL CLEARING TIME IN TRANSIENT STABILITY.

If a fault occurs in a system, δ begins to increase under the influence of positive accelerating power, and the system will become unstable If δ becomes very large. There is a critical angle within which the fault must be cleared if the system is to remain stable and the equal-area criterion is to be satisfied. This angle is known as the critical clearing angle. Consider the system of Fig. operating with mechanical input Pi at stady angle δ0. (Pi=Pe) as shown by the point 'a' on the power angle diagram of Fig.
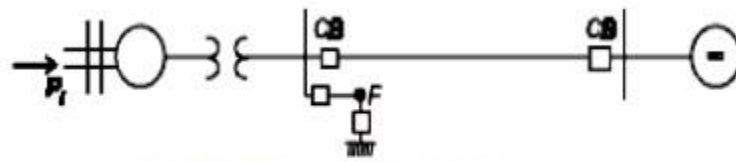

Fig. 11.9: Single machine infinite bus system.

Now if a three phase short circuit occurs at the point F o the outgoing radial line, the terminal voltage goes to zero and hence the electrical power output of the generator instantly reduces to aero, i.e., Pe = 0 and the state point drops to 'b'. The acceleration area A1 starts to increase while the state point moves along bc. At time tc corresponding clearing angle δo the fault is cleared by the opening of the line circuit breaker. tc is called clearing time and δe is called clearing angle. After the fault is cleared, the system again becomes healthy and transmits poerPe = Pmaxsin δ, i.e., the state point shit to 'd' on the power angle curve. The rotor now decelerates and the decelerating area A2 begins to increase while the state point moves along de.

For stability, the clearing angle, δo must be such that area A1 = area A2.
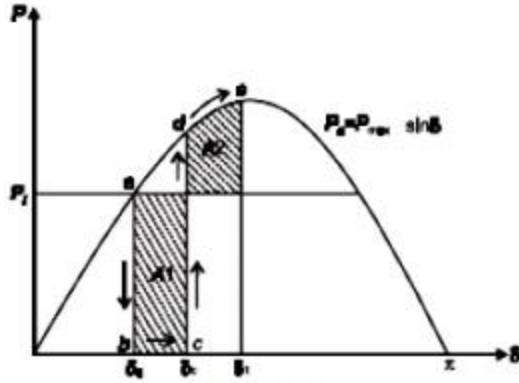Expressing area A1 = area A2 mathematically, we have

Fig. 11.10: P_e – δ characteristic.

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$$P_i\,(\delta_c - \delta_0) = \int_{\delta_c}^{\delta_1}(P_e - P_i)d\delta$$

$\therefore$
$$P_i\,(\delta_c - \delta_0) = \int_{\delta_c}^{\delta_1} P_{max}\,\sin\delta.d\delta \; - P_i\,(\delta_1 - \delta_c)$$

$\therefore$
$$P_i\delta_c - P_i\,\delta_0 = P_{max}\,(-\cos\delta_1 + \cos\delta_c) - P_i\delta_1 + P_i\delta_c$$

$\therefore$
$$P_{max}\,(\cos\delta_c - \cos\delta_1) = P_i(\delta_1 - \delta_0) \qquad\qquad ...(11.47)$$

Also

$$P_i = P_{max}\,\sin\delta_0 \qquad\qquad ...(11.48)$$

Using eqns. (11.47) and (11.48) we get

$$P_{max}\,(\cos\delta_c - \cos\delta_1) = P_{max}(\delta_1 - \delta_0)\,\sin\delta_0$$

$\therefore$
$$\cos\delta_c = \cos\delta_1 + (\delta_1 - \delta_0)\,\sin\delta_0 \qquad\qquad ...(11.49)$$

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- To reiterate, with reference to Fig. 11.10, the various angle in eqn. (11.49) are δe = clearing angle; $\delta 0$ = initial power angle; and $\delta 1$ = power angle to which the rotor advances (or overshoots) beyond δe.
- In order to determine the clearing time, we re-write eqn.(11.20), with Pe=0, since we have a three phase fault,

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_i \qquad \qquad ...(11.50)$$

Integrating eqn. (11.50) twice and utilizing the fact that when $t = 0$, $\frac{d\delta}{dt} = 0$ yields

$$\delta = \frac{\pi f P_i}{2H} t^2 + \delta_0 \qquad \qquad ...(11.51)$$

If t_c is a clearing time corresponding to a clearing angle δ_c, then we obtain from eqn. (11.51),

$$\delta_c = \frac{\pi f P_i}{2H} t_c^2 + \delta_0$$

$\therefore$
$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f P_i}} \qquad \qquad ...(11.52)$$

Note that δ_c can be obtained from eqn. (11.49). As the clearing of the faulty line is delayed, A_1 increases and so does δ_1 to find $A_2 = A_1$ till $\delta_1 = \delta_m$ as shown in Fig. 11.11. For a clearing
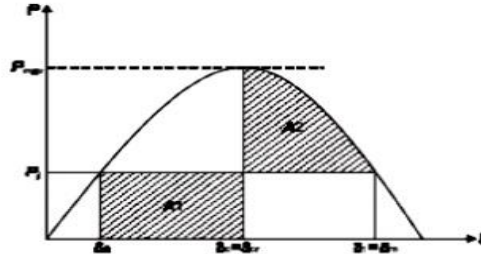

Fig. 11.11: Critical clearing angle.

Angle (or clearing time) larger than this value, the system would be unstable. The maximum allowable value of the clearing angle and clearing time for the system to remain stable are known as critical clearing angle and critical clearing time respectively.

From Fig. 11.11, $\delta_m = \pi - \delta_0$, we have upon substitution into eqn. (11.49)

$$\cos\delta_{cr} = \cos\delta_m + (\delta_m - \delta_0) \sin\delta_0$$

$\therefore$
$$\cos\delta_{cr} = \cos\delta_m + (\pi - \delta_0 - \delta_0) \sin\delta_0$$

$\therefore$
$$\cos\delta_{cr} = \cos(\pi - \delta_0) + (\pi - 2\delta_0) \sin\delta_0$$

$\therefore$
$$\cos\delta_{cr} = (\pi - 2\delta_0) \sin\delta_0 - \cos\delta_0$$

$\therefore$
$$\delta_{cr} = \cos^{-1}[(\pi - 2\delta_0) \sin\delta_0 - \cos\delta_0] \qquad \qquad ...(11.53)$$

Using eqn. (11.52), critical clearing time can be written as:

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_i}} \qquad \qquad ...(11.54)$$