4.1 CONCEPT OF STATE VARIABLES

State space analysis is an excellent method for the design and analysis of control systems. The conventional and old method for the design and analysis of control systems is the transfer function method. The transfer function method for design and analysis had many drawbacks.

Drawbacks of transfer function model analysis:

a. Transfer function is defined under zero initial conditions
b. Transfer function is applicable to linear time invariant systems
c. Transfer function analysis is restricted to single input and single output systems
d. Does not provide information regarding the internal state of the system

Advantages of state variable analysis:

- It can be applied to linear system
- It can be applied to non-linear system
- It can be applied to time varying system
- It can be applied to time invariant system
- It can be applied to multiple input multiple output system
- Its gives idea about the internal state of the system

A state variable is one of the set of variables that are used to describe the mathematical "state" of a dynamical system. Intuitively, the state of a system describes enough about the system to determine its future behaviour in the absence of any external forces affecting the system. The state variable analysis can be applied for any type of systems. In this method of analysis, it is not necessary that the state variables represent physical quantities of the system, but variables that do not represent physical quantities and those that are neither measurable nor observable may be chosen as state variables.

STATE SPACE FORMULATION

State:

The state of a dynamic system is the minimal set of variables called state variables such that the knowledge of these variables at time t = t_0 (initial condition), together with the knowledge of input for $t \geq t_0$, completely determines the behaviour of the system for any time $t > t_0$. (or) A set of variables which describes the system at any time instant are called state variables. In the state variable formulation of a system, in general, a system consists of m-inputs, p-outputs and n-state variables. The state space representation of the system may be visualized as shown in figure 4.1.1.

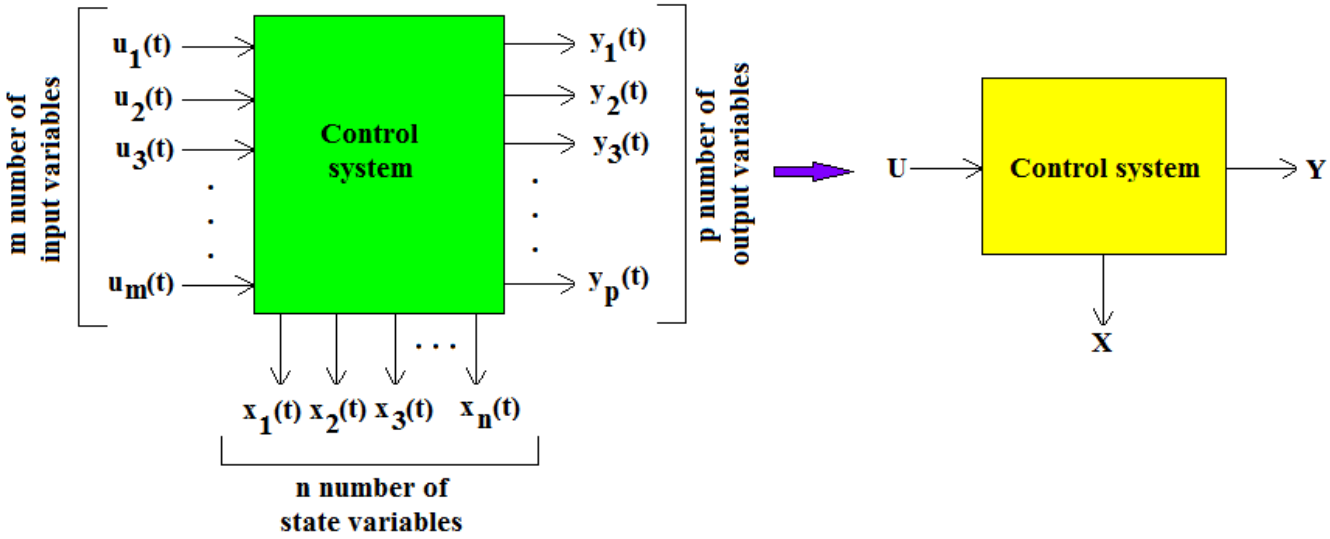

Figure 4.1.1 State space representation of a system

[Source: "Control Systems" by A. Nagoor Kani, Page: 5.2]

Let us consider a multi input & multi output (MIMO) system is having

m inputs: $u_1(t), u_2(t), \ldots\ldots, u_m(t)$

p number of outputs: $y_1(t), y_2(t), \ldots\ldots, y_p(t)$

n number of state variables: $x_1(t), x_2(t), \ldots\ldots, x_n(t)$

The different variables may be represented by the vectors (column matrix) as shown below:

Input vector

$$U(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$$

Output vector

$$Y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}$$

State variable vector

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

State vector:

If n state variables are needed to completely describe the behaviour of a given system, then these n state variables can be considered the n components of a vector X. Such a vector is called a state vector.

State space:

The n-dimensional space whose co-ordinate axes consists of the x_1 axis, x_2 axis……. x_n axis, where x_1, x_2, ..., x_n are state variables is called a state space.