2.2 ELECTRIC FILED AND EQUIPOTENTIAL PLOTS

ELECTRIC FILED OR ELECTRIC FIELD INTENSITY:

The electric field or electric field intensity is defined as the electric force per unit charge .It is given by

$$E = \frac{F}{q}$$

According to coulomb's law

$$F = \frac{Qq}{4\pi\varepsilon r^2}$$

Electric Filed

$$E = \frac{F}{q}$$

Substitute F value in above equation

$$E = \frac{\frac{Qq}{4\pi\varepsilon r^2}}{q}$$

$$E = \frac{Qq}{4\pi\varepsilon r^2 q}$$

$$E = \frac{Q}{4\pi\varepsilon r^2}\ V/m$$

The another unit of electric field is $Volts/meter$

ELECTRIC DIPOLE:

An electric dipole is formed when two point charges of equal magnitude but opposite sign separated by a small distance.

An electric dipole or simply dipole is nothing but two equal and opposite charges are separated by a very small distance. The product of charge and spacing is called **Electric Dipole Moment.**

Let Q_1 and Q_2 be the two charges separated by a small distance d.The product of charge Q and spacing d is called dipole moment.

Let $\boldsymbol{P}$ **be any point at distance of** $\boldsymbol{r_1}, \boldsymbol{r_2}$ **and** $\boldsymbol{r}$ from$+\boldsymbol{Q}$,$-\boldsymbol{Q}$ and midpoint of dipole respectively as shown in figure 2.2.1.

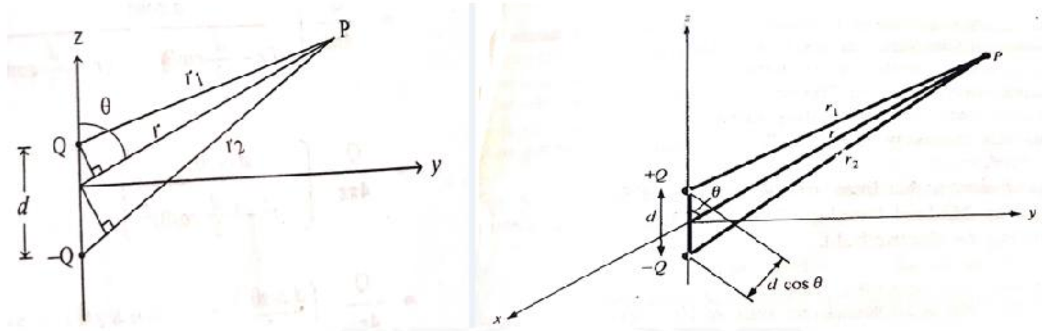

Figure 2.2.1 An Electric DIpole

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-145]

$$\boldsymbol{n} = \boldsymbol{Qd}$$

Potential at $\boldsymbol{P}$ due to $+\boldsymbol{Q}$ is

$$\boldsymbol{V_1} = \frac{+\boldsymbol{Q}}{4\pi\varepsilon \boldsymbol{r_1}}$$

$$\boldsymbol{V_1} = \frac{\boldsymbol{Q}}{4\pi\varepsilon \boldsymbol{r_1}}$$

Potential at $\boldsymbol{P}$ due to -$\boldsymbol{Q}$ is

$$\boldsymbol{V_2} = \frac{-\boldsymbol{Q}}{4\pi\varepsilon \boldsymbol{r_2}}$$

The resultant potential at $\boldsymbol{P}$

$$\boldsymbol{V} = \boldsymbol{V_1} + \boldsymbol{V_2}$$

Substitute $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$ in above expression

$$\boldsymbol{V} = \frac{\boldsymbol{Q}}{4\pi\varepsilon \boldsymbol{r_1}} + \frac{-\boldsymbol{Q}}{4\pi\varepsilon \boldsymbol{r_2}}$$

Take $\frac{\boldsymbol{Q}}{4\pi\varepsilon}$ common as outside

$$V = \frac{Q}{4\pi\varepsilon r_1} + \frac{-Q}{4\pi\varepsilon r_2}$$

$$V = \frac{Q}{4\pi\varepsilon r_1} + \frac{-Q}{4\pi\varepsilon r_2}$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} + \frac{-1}{r_2}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

If the point P is too far from the dipole, the distance r_1 and r_2 are written as.

$$r_1 = r - \frac{d}{2}\cos\theta$$

$$r_2 = r + \frac{d}{2}\cos\theta$$

Potential at P due to dipole

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Substitute r_1 and r_2 in above expression

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\left(\frac{1}{r - \frac{d}{2}\cos\theta}\right) - \left(\frac{1}{r + \frac{d}{2}\cos\theta}\right)\right]$$

Take LCM

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{\left(r + \frac{d}{2}\cos\theta\right) - \left(r - \frac{d}{2}\cos\theta\right)}{\left(r - \frac{d}{2}\cos\theta\right) \times \left(r + \frac{d}{2}\cos\theta\right)}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{r + \frac{d}{2}\cos\theta - r + \frac{d}{2}\cos\theta}{\left(r - \frac{d}{2}\cos\theta\right) \times \left(r + \frac{d}{2}\cos\theta\right)}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{r + \frac{d}{2}\cos\theta - r + \frac{d}{2}\cos\theta}{\left(r - \frac{d}{2}\cos\theta\right) \times \left(r + \frac{d}{2}\cos\theta\right)}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{+\frac{d}{2}\cos\theta + \frac{d}{2}\cos\theta}{\left(r - \frac{d}{2}\cos\theta\right) \times \left(r + \frac{d}{2}\cos\theta\right)}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{\frac{2d}{2}\cos\theta}{\left(r - \frac{d}{2}\cos\theta\right) \times \left(r + \frac{d}{2}\cos\theta\right)}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{d\cos\theta}{\left(r - \frac{d}{2}\cos\theta\right) \times \left(r + \frac{d}{2}\cos\theta\right)}\right]$$

Formula $A^2 - B^2 = (A + B)(A - B)$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{d\cos\theta}{\left((r)^2 - \left(\frac{d}{2}\cos\theta\right)^2\right)}\right]$$

$$\frac{d}{2} \ll r^2$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{d\cos\theta}{((r)^2)}\right]$$

$$V = \frac{Qd\cos\theta}{4\pi\varepsilon r^2}$$

Substitute Qd expression in above equation

$$n = Qd$$

$$V = \frac{m\cos\theta}{4\pi\varepsilon r^2}$$

This shows that the potential is directly proportional to the dipole moment and inversely proportional to the square of the distance.

EQUIPOTENTIAL PLOTS OR EQUIPOTENTIAL SURFACE:

In an electric field ,there are many points at which the electric potential is same. This is because,the potential is a scalar quantity which depends on the distance between the point at which potential is to be obtained and the location of the charge. There can be number of points which can be located at the same distance from the charge. All such points are at the same electric potential. If the surface is imagined, joining all such points which are at the same potential, then such a surface is called **equipotential surface.**

Any surface on which the potential is the same thought is known as an **equipotential surface.**

The intersection of an eqipotential surface and a plane results in a path or line known as **equipotential line.** No work is done in moving a charge from one point to another along an eqipotential line or surface ($\boldsymbol{V_A - V_B = 0}$) and hence

$$\int_L \boldsymbol{Edl = 0}$$

On the line surface. The lines of force or flux lines (or the direction of $\boldsymbol{E}$) are always normal to equipotential surface .Equipotential surface for point charge and a dipole are shown in figure 2.2.2 .The direction of $\boldsymbol{E}$ is everywhere normal to the equipotential lines. Consider a point charge located at the origin of a sphere. The potential at a point which is at a radial distance $\boldsymbol{r}$ from the point charge is given by

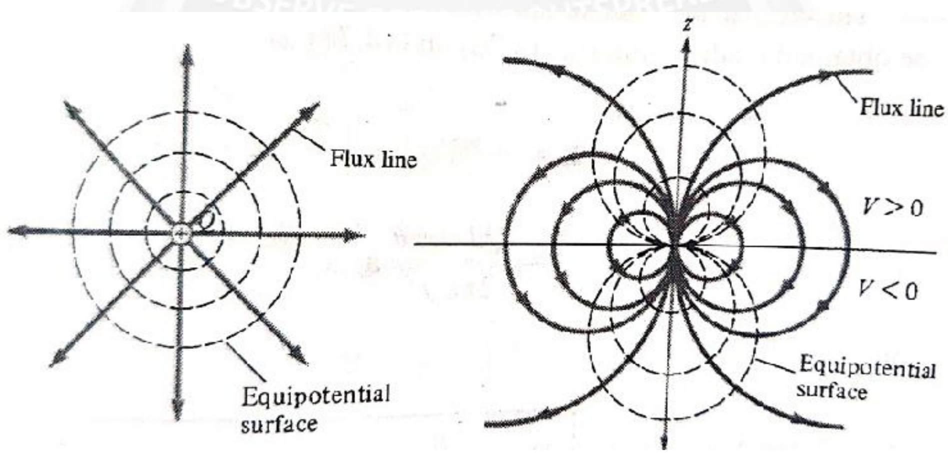

Figure 2.2.2 Equipotential surface for a point charge and an electric dipole

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-146]

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

At all points which are at a distance r from Q the potential at a point which is at a radial distance r from the point charge is given by.

Similarly at $r = r_1, r = r_2$ there exists other equipotential surfaces, in an electric field of point charge ,in the form of concentric sphere as shown in figure 2.2.3

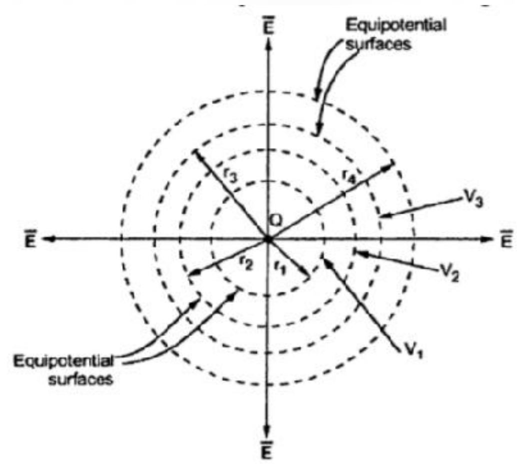

Figure 2.2.3 Equipotential Surfaces

[Source: " Electromagnetic Theory'' by U.A.Bakshi, page-4-27]

V is inversely proportional to distance .Thus V_1 at equipotential surface at $r = r_1$ is highest and it goes on decreasing, as the distance r increasing. Thus $V_1 > V_2 > V_3$.As we move away from the charge, The E decreases the hence potential of equipotential surfaces goes on decreasing. While potential of equipotential surface goes on decreasing. While potential of equipotential surface goes on increasing as we move against the direction of electric field.

For a uniform electric field E the equipotential surfaces are perpendicular to E and are equispaces for fixed increment of voltages. Thus if we move a charge along a circular path of radius r_1 as shown in a_φ direction, the work done is zero. This is because E and dL are perpendicular .Thus E and equipotential surface are at right angles each other.

For non uniform filed, the field lines tends to diverge in the direction of decreasing ***E*****.**Hence equipotential surfaces are still perpendicular to ***E*** but are not equispaced,for fixed increment of voltages.