BERNOULLI'S EQUATION - LINEAR MOMENTUM EQUATION

It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass equal to the change in the momentum of the flow per unit time in that direction. The force acting on a fluid mass „ m „ is given by Newton's second law of motion.

$$F = m \times a$$

Where 'a' is the acceleration acting in the same direction as force

But $a = \frac{dv}{dt}$

$$F = m\frac{dv}{dt} = \frac{d\ mv}{dt} \quad \text{(Since m is a constant and can be taken inside differential)}$$

$$F = \frac{d\ mv}{dt} \quad \_\_\_\_\_\_\_\_ \quad \text{is known as the momentum principle.}$$

F. dt = d(mv) Is known as the impulse momentum equation.

It states that the impulse of a force F acting on a fluid mass m in a short interval of time dt is equal to the change of momentum d(mv) in the direction of force.

Force exerted by a flowing fluid on a pipe-bend:

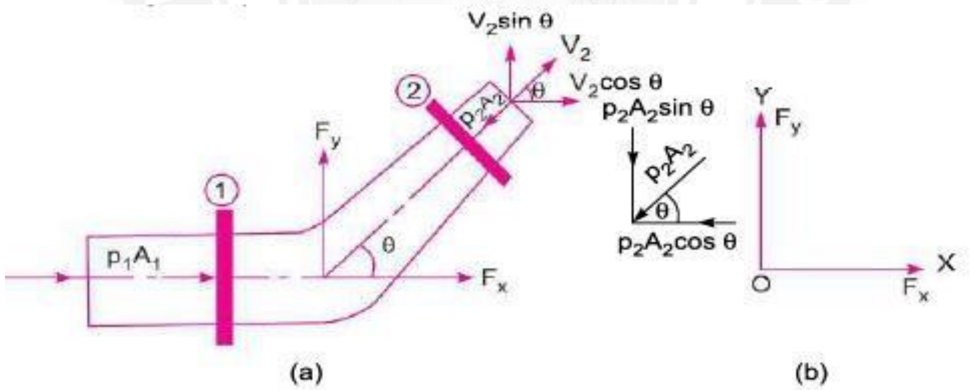


Figure 2.7.1 Forces on Bend

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 289]

The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two sections (1) and (2) as above Let v1 = Velocity of flow at section (1)

P1= Pressure intensity at section (1)

A1 = Area of cross-section of pipe at section (1)

And V2, P2, A2 are corresponding values of Velocity, Pressure, Area at section (2)

Let Fx and Fy be the components of the forces exerted by the flowing fluid on the bend in *x* and *y* directions respectively. Then the force exerted by the bend on the fluid in the directions of *x* and *y* will be equal to FX and FY but in the opposite directions.

Hence the component of the force exerted by the bend on the fluid in the *x* – direction = - F*x* and in the direction of *y* = - F*y*. The other external forces acting on the fluid are p1 A1 and p2 A2 on the sections (1) and (2) respectively.

Then the momentum equation in x-direction is given by

Net force acting on the fluid in the direction of x = Rate of change of momentum in x – direction

$$p_1 A_1 - p_2 A_2 \cos\theta - F_x = \text{(Mass per second) (Change of velocity)}$$

$$= \rho Q \text{ (Final velocity in x-direction – Initial velocity in x-direction)}$$

$$= \rho Q (V_2 \cos\theta - V_1)$$

$$F_x = \rho Q (V_1 - V_2 \cos\theta) + p_1 A_1 - p_2 A_2 \cos\theta \text{ ----------------- (1)}$$

Similarly the momentum equation in y-direction gives

$$0 - p_2 A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0)$$

$$F_y = \rho Q (-V_2 \sin\theta) - p_2 A_2 \sin\theta \text{ ---------------------- (2)}$$

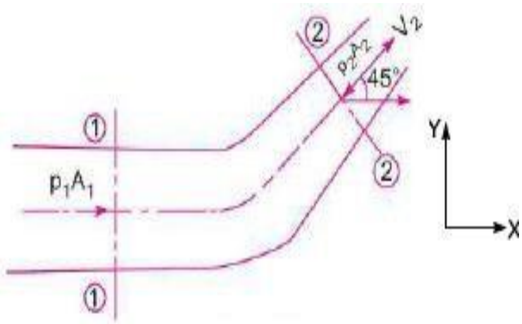
Now the resultant force (FR) acting on the bend

$$F_R = Fx^2 + Fy^2$$

And the angle made by the resultant force with the horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x}$$

PROBLEM 1. A 45° reducing bend is connected to a pipe line, the diameters at inlet and out let of the bend being 600mm and 300mm respectively. Find the force exerted by the water on the bend, if the intensity of pressure at the inlet to the bend is 8.829N/cm2 and rate of flow of water is 600 lts/sec.



Solution. Given :

Angle of bend, $\theta = 45°$

Dia. at inlet, $D_1 = 600 \text{ mm} = 0.6 \text{ m}$

∴ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.6)^2$

$= 0.2827 \text{ m}^2$

Dia. at outlet, $D_2 = 300 \text{ mm} = 0.30 \text{ m}$

∴ Area, $A_2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}^2$

Pressure at inlet, $p_1 = 8.829 \text{ N/cm}^2 = 8.829 \times 10^4 \text{ N/m}^2$

$Q = 600 \text{ lit/s} = 0.6 \text{ m}^3\text{/s}$

$$V_1 = \frac{Q}{A_1} = \frac{0.6}{.2827} = 2.122 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{.07068} = 8.488 \text{ m/s.}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \quad \text{or} \quad \frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.122^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{8.488^2}{2 \times 9.81}$$

$$9 + .2295 = p_2/\rho g + 3.672$$

$$\therefore \quad \frac{p_2}{\rho g} = 9.2295 - 3.672 = 5.5575 \text{ m of water}$$

$$\therefore \quad p_2 = 5.5575 \times 1000 \times 9.81 \text{ N/m}^2 = 5.45 \times 10^4 \text{ N/m}^2$$

Forces on the bend in x- and y-directions

$$F_x = \rho Q [V_1 - V_2 \cos \theta] + p_1 A_1 - p_2 A_2 \cos \theta$$

$$= 1000 \times 0.6 [2.122 - 8.488 \cos 45°] + 8.829 \times 10^4 \times .2827 - 5.45 \times 10^4 \times .07068 \times \cos 45°$$

$$= -2327.9 + 24959.6 - 2720.3 = 24959.6 - 5048.2$$

$$= 19911.4 \text{ N}$$

and

$$F_y = \rho Q [- V_2 \sin \theta] - p_2 A_2 \sin \theta$$

$$= 1000 \times 0.6 [- 8.488 \sin 45°] - 5.45 \times 10^4 \times .07068 \times \sin 45°$$

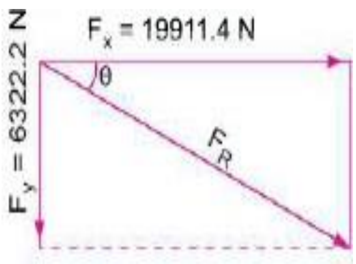
$$= - 3601.1 - 2721.1 = - 6322.2 \text{ N}$$

–ve sign means F_y is acting in the downward direction

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(19911.4)^2 + (-6322.2)^2}$$

= 20890.9 N. Ans.



The angle made by resultant force with x-axis is given by
:quation

$$\tan \theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4} = 0.3175$$

$$\therefore \qquad \theta = \tan^{-1} .3175 = \mathbf{17^\circ\ 36'.\ Ans.}$$