2.3 INFLUENCE LINE FOR SHEARING FORCE, BENDING MOMENT AND SUPPORT REACTION COMPONENTS OF CONTINUOUS BEAMS

Influence lines

An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

Uses of influence line diagrams

(i) Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and

(ii) Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

The principle on which indirect model analysis is based

The indirect model analysis is based on the Muller Breslau principle.

Muller Breslau principle has lead to a simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.

To get the influence line for any force quantity,

(i) remove the resistant due to the force,

(ii) apply a unit displacement in the direction of the

(iii) plot the resulting displacement diagram. This diagram is the influence line for the force.

Similitude

Similitude means similarity between two objects namely the model and the prototype with regard to their physical characteristics:

a. Geometric similitude is similarity of form
b. Kinematic similitude is similarity of motion
c. Dynamic and/or mechanical similitude is similarity of masses and/or forces.

Example:

Determine the influence line for RA for continuous beam shown in fig. compute the IL ordinate at 1m intervals

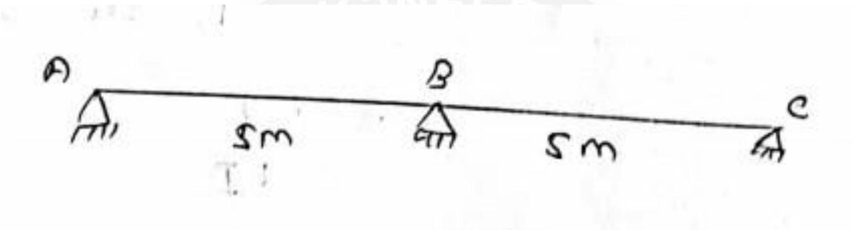

Fig.2.3.1

Solution

I) Remove support A

ii) Apply a unit force at A and compute the deflection at any"x" on the CB and BA

iii) divide these deflection by the displacement at A

Elastic curve due to RA =1

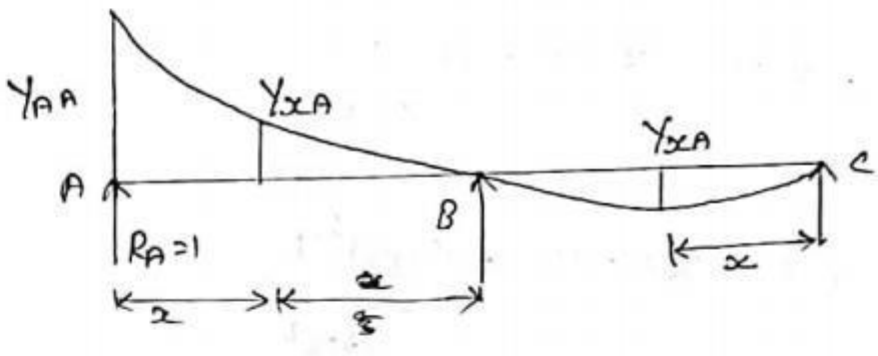

Fig.2.3.2

Taking moment about C

$RA \times 10 + RB \times 5 = 0$

$10 + RB\ 5 = 0$

$RB = -10/5$

$= -2$

$RA + RB + RC = 0$

$1 - 2 - RC = 0$

$RC = 1$

$Mx = -EI\ d^2y/dx^2$

$Mx = Rc\ x + RB\ (x-5)$

$1x - 2(x-5) = -EI\ d^2y/dx^2$

$-x + 2X - 10 = EI\ d^2y/dx^2$

Integrate on both side

$EI\ d^2y/dx^2 = -x^2/2 + 2x^2/2 - 10x + C1$

$EI\ dy/dx = -x^2/2 + x^2 - 10x + C1$ ________(1)

Again integrating on both side

$EI\ y = (-x^3/6) + (x^3/3) - (10x^2/2) + C1x + C2$

$EI\ y = (-x^3/6) + (x^3/3) - (5x^2) + C1x + C2$ _______(2)

Apply condition

$x=0$, $Y=0$

$EI\ y = (-x^3/6) + (x^3/3) - (5x^2) + C1x + C2$

$C2 = 0$

$X=5$ $Y=0$

$EI\ y = (-x^3/6) + (x^3/3) - (5x^2) + C1x + C2$

$0 = (-5^3/6) + (5^3/3) - (5(5^2) + C1\ (5) + 0$

$0 = -104.16 \times C1 \times 5$

$C1 = 20.83$

Apply C1 and C2

$Y_{XA} = (-x^3/6) + (x^3/3) - (5x^2) + C1x + C2$

$= 1/EI\ (-x^3/6) + (x^3/3) - (5x^2) + 20.86\ x + 0$

At $x = 10$

$Y_{AA} = 1/EI\ (-10^3/6) + (10^3/3) - (5(10^2) + 20.86(10) + 0$

$= -\ 125.033$

$Mx \quad = -EI\ d^2y/dx^2$

$1x + 2(x-5) \quad = EI\ d^2y/dx^2$

$EI\ d^2y/dx^2 \quad = -x + 2\ (x-5)$

Integrate on both sides

$EI\ dy/dx \quad = -x^2/2 + C1 + 2(x-5)^2/2$

$EI\ dy/dx \quad = -x^2/2 + (x-5)^2 + C1$ ________(1)

Again integrate on both sides

$EI\ y \quad = (-x^3/6) + (x-5)^3/\ 3 + C1x + C2$ ______(2)

Apply condition

I)x=0 , Y=0

$EI\ y \quad = (-x^3/6) + (x-5)^3/\ 3 + C1x + C2$

$C2 \quad = 0$

ii(X=5 Y=0

$EI\ y \quad = (-x^3/6) + (x-5)^3/\ 3 + C1x + C2$

$0 \quad = (-5^3/6) + (5-5)^3/\ 3 + C1(5) + C2$

$0 \quad = -20.83 + 0 + 5C1 + 0$

$C1 \quad = 20.83/5$

$C1 \quad = 4.167$

Apply c1 and c2

$$Y_{XA} = (-x^3/6) + (x-5)^3/3 + C1x + C2$$

$$= 1/EI\ [(-x^3/6) + 4.167x + (x-5)^3/3]$$

At x= 10

$$Y_{AA} = 1/EI\ [(-x^3/6) + 4.167x + (x-5)^3/3]$$

$$= 1/EI\ [(-10^3/6) + 4.167(10) + (10-5)^3/3]$$

$$= 1/EI\ [-83.33]$$

$$\text{ILO at x} = Y_{XA} / Y_{AA}$$

$$= 1/EI\ [(-x^3/6) + 4.167x + (x-5)^3/3] / (-83.33)$$

$$= [(-x^3/6) + 4.167x + (x-5)^3/3] \times (1/83.33)$$

Ordinate at ILD for RA

x(m)	Support C	1	2	3	4	support B 5	6	7	8	9	support 10
ILO RA	0	-0.048	-0.084	0.096	-0.072	0	0.128	0.128	0.304	0.516	1

Table. 2.3.1 Ordinate at ILD for RA

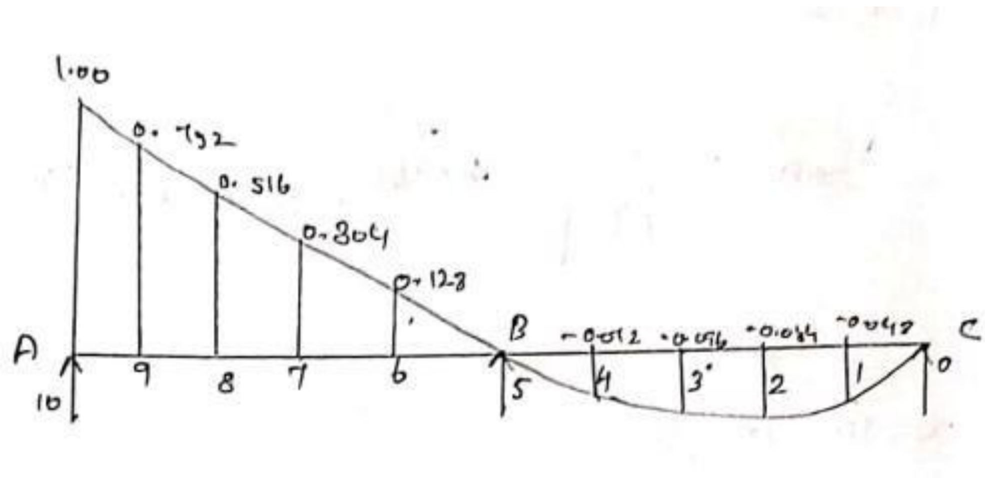

Fig.2.3.3 Ordinate at ILD for RA

Example:

Using Muller Breslau principle, draw the influence line for bending moment at the mid-point of span AB of the continuous beam ABC shown in fig, determine the influence line ordinate at suitable intervals and plot them.

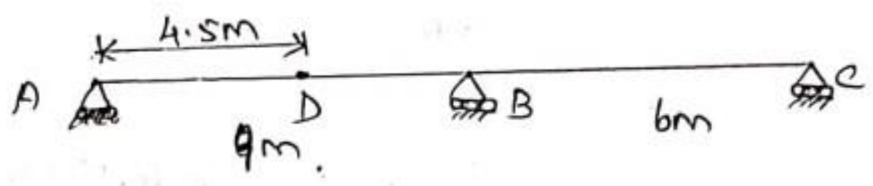

Fig.2.3.4

Solution

To get the influence line for MD

I)Introduce a hinge at D

ii) Apply a unit bending moment at D

iii) Determine the deflection Y_{XD} and slope Q_{DD} at D

iv) Y_{XD} / Q_{DD} is the influence line ordinate at any x

Bending moment at any x is

$$M_x = -EI\, d^2y/dx^2$$

$$0.333x - 0.555\,(x - 6) = -EI\, d^2y/dx^2$$

$$EI\, d^2y/dx^2 = -\,0.333x + 0.555\,(x - 6)$$

Integrate on both sides

$$EI\, dy/dx = (-\,0.333x^2/2) + (0.555\,(x - 6)^2/2)$$

$$EI\, dy/dx = -\,0.1665x^2 + 0.2775(x - 6)^2 + C1 \_\_\_\_\_\_\_ (1)$$

Again integrate on both sides

$$EI\, Y = (-\,0.1665x^3/3) + (0.2775(x - 6)^3/3) + C1x + C2$$

$$EI\, Y = -\,0.555\,x^3 + 0.925\,(x - 6)^3 + C1x + C2 \_\_\_\_\_\_\_\_\_\_\_ (2)$$

Find RA,RB,RC,RD1 and RD2

$$M = 1 \text{ at D}$$

$$RA \times 4.5 = 1$$

$$RA = 1/4.5$$

$$= 0.222\ \text{KN}$$

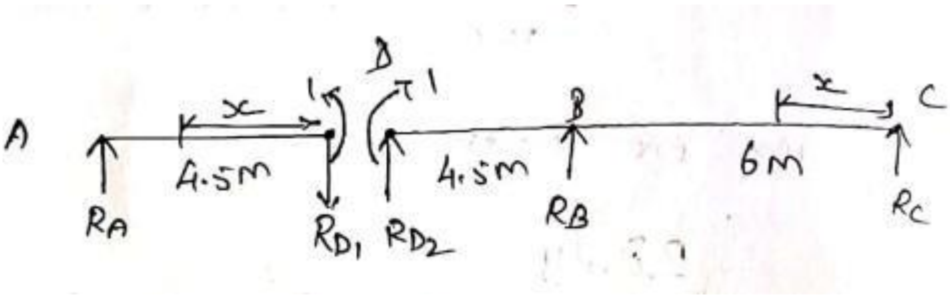

Fig.2.3.5

R D1 = 0.222 ↓

R D2 = 0.222↑

Taking moment about C

0.222×10.5 +1 +RBx6 = 0

RB = - 0.555 KN

RA + RB+ RC = 0

0.222 - 0.555 + Rc =0

RC = 0.333 KN

Two regions AD and DBC will be considered separatively (because of discontinuity at D)

Boundary condition

I) x=O y= 0

$$EI\,Y = -0.555\,x^3 + 0.925\,(x-6)^3 + C1x + C2$$

(2) C2 =0

ii)X =6 Y = 0

(2) $EI\,Y = -0.555\,x^3 + 0.925\,(x-6)^3 + C1x + C2$

$0 = -0.555(6)^3 + 0.925 + C1\ 6 + 0$

$0 = -11.988 + C1\ 6$

$C1 = 2$

Apply C1 and C2 in slope of deflection value

(1) $EI\ dy/dx = -0.1665x^2 + 0.2775(x - 6)^2 + C1$

X = 10.5

$Q_{DC} = dy/dx$

$= 1/EI\ [-0.1665(10.5)^2 + 0.2775(10.5 - 6)^2 + 2$

$= 1/EI\ (-10.73)$

Apply (2)

$= 1/EI\ [-0.555(10.5)^3 + 0.925\ (10.5 - 6)^3 + 2\ (10.5) + 0$

$= 1/EI\ (-34.8)$

For the Zone AD

$Mx = 1 - 0.222\ x$

$EI\ d^2y/dx^2 = 0.222x - 1$

Integrate on both sides

$EI\ dy/dx = 0.222\ x^2/2 - X + C3$

$EI\ dy/dx = 0.111\ x^2 - X + C3$______(3)

Again integrate on both side

$$EI\,Y = 0.111\ x^3 / 3) - (x^2 /2) + C3\ x + C4$$

$$EI\,Y = 0.037\ x^3 - (x^2 /2) + C3\ x + C4 \_\_\_\_\_ (4)$$

Bounday condition

i) x = 0   y= -34.82/EI

(4)

$$EI\,Y = 0.037\ x^3 - (x^2 /2) + C3\ x + C4$$

$$-34.82\ EI /EI = 0.037(0) -0 + C4$$

$$C4 = - 34.82$$

ii) X= 4.5 Y = 0

(4)

$$EI\,Y = 0.037\ x^3 - (x^2 /2) + C3\ x + C4$$

$$0 = 0.037\ (4.5\ )^3 - (4.5^2 /2) + C3\ (4.5) - 34.82$$

$$0 = - 41.57 + 4.5\ C3$$

$$C3 = 9.24$$

Apply c3 and C4 in (3)

(3)

$$EI\ dy/dx = 0.111\ x^2 - X + C3$$

$$EI\ dy/dx = 0.111\ x^2 - X + 9.24$$

$$Q_{DA} = dy/dx$$

$$= 9.24 / EI$$

At $x = 0$

(4)

$$EI\,Y = 0.037\,x^3 - (x^2/2) + C3\,x + C4$$

$$EI\,y = 0.037\,x^3 - (x^2/2) + 9.24\,x - 34.82$$

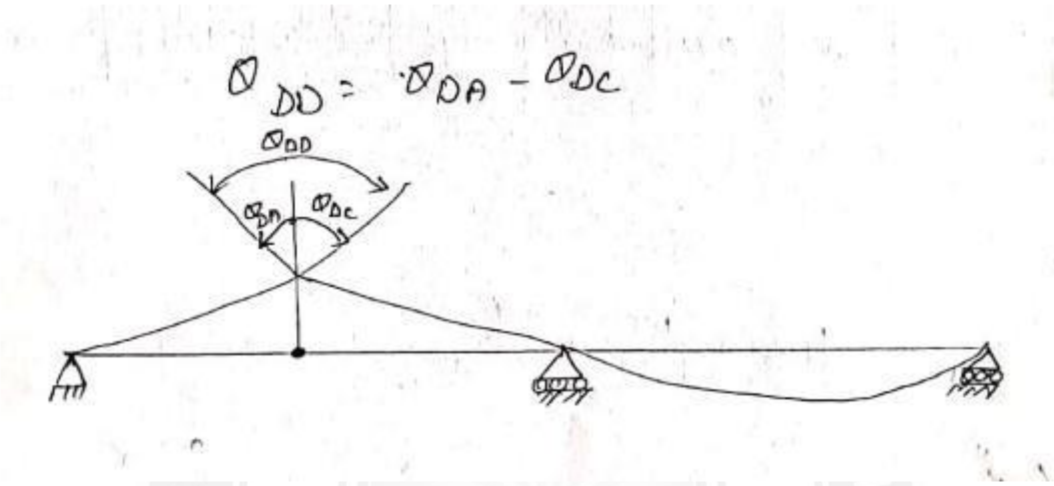

Fig.2.3.6

$$Q_{DD} = Q_{DA} - Q_{Dc}$$

$$= 9.24/EI + 10.738/EI$$

$$= 19.978/EI$$

For the region CD

ILO for MD

$$= y_{XD} / Q_{DD}\,[0.33x^3/6 + 2x + 0.555(x-6)^3/6] / (19.978)$$

For the region D

ILO for MD

$$= [(0.222\,x^3/6) - (x^2/2) + 9.24\,x - 34.82] / (19.978)$$

Influence line ordinate

x(m)	0	3	6	9	10.5	12	15
ILO	0	0.225	0.0	-0.999	-1.743	-1.099	0

Table. 2.3.2 Ordinate at ILD

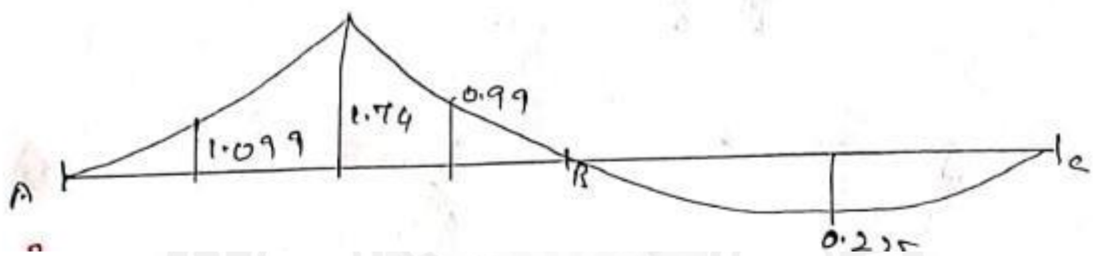

Fig.2.3.7 Influence line ordinate