Ampere's Circuital Law:

Ampere's circuital law states that the line integral of the magnetic field $\vec{H}$ (circulation of H) around a closed path is the net current enclosed by this path. Mathematically,

$$\oint \vec{H}.d\vec{l} = I_{enc} \qquad \text{.......................................(4.8)}$$

The total current I_{enc} can be written as,

$$I_{enc} = \int_S \vec{J}.d\vec{s} \qquad \text{.......................................(4.9)}$$

By applying Stoke's theorem, we can write

$$\oint \vec{H}.d\vec{l} = \int_S \nabla \times \vec{H}.d\vec{s}$$

$$\therefore \int_S \nabla \times \vec{H}.d\vec{s} = \int_S \vec{J}.d\vec{s}$$

$$\therefore \nabla \times \vec{H} = \vec{J} \qquad \text{.......................................(4.10)}$$

which is the Ampere's law in the point form.

Applications of Ampere's law:

We illustrate the application of Ampere's Law with some examples.

Example 4.2: We compute magnetic field due to an infinitely long thin current carrying conductor as shown in Fig. 4.5. Using Ampere's Law, we consider the close path to be a circle of radius ρ as shown in the Fig. 4.5.

If we consider a small current element $Id\vec{l}(= Idz\hat{a}_z)$, $d\vec{H}$ is perpendicular to the plane containing both $d\vec{l}$ and $\vec{R}(= \rho\hat{a}_\rho)$. Therefore only component of $\vec{H}$ that will be present is H_ϕ,i.e., $\vec{H} = H_\phi\hat{a}_\phi$.

By applying Ampere's law we can write,

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi \int_0^{2\pi} H_\phi \rho d\phi = H_\phi \rho 2\pi = I \quad \text{.......................................(4.11)}$$

Therefore, $\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$ which is same as equation (4.7)

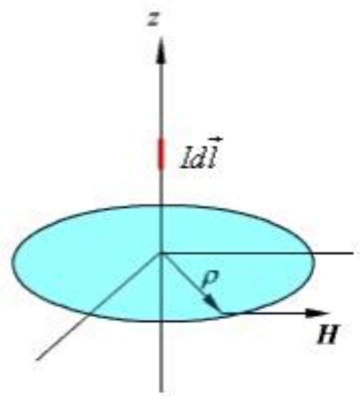

Fig. 4.5: Magnetic field due to an infinite thin current carrying conductor

Example 4.3: We consider the cross section of an infinitely long coaxial conductor, the inner conductor carrying a current I and outer conductor carrying current - I as shown in figure 4.6.

We compute the magnetic field as a function of ρ as follows:

In the region $0 \leq \rho \leq R_1$

$$I_{enc} = I\frac{\rho^2}{R_1^2} \quad \text{.......................................(4.12)}$$

$$H_\phi = \frac{I_{enc}}{2\pi\rho} = \frac{I\rho}{2\pi a^2} \quad \text{.............................(4.13)}$$

In the region $R_1 \leq \rho \leq R_2$

$$I_{enc} = I$$

$$H_\phi = \frac{I}{2\pi\rho} \quad \text{.......................................(4.14)}$$

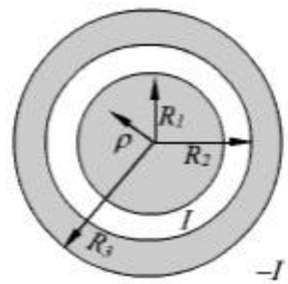

Fig. 4.6: Coaxial conductor carrying equal and opposite currents

In the region $R_2 \leq \rho \leq R_3$

$$I_{enc} = I - I\frac{\rho^2 - R_2^{\,2}}{R_3^{\,2} - R_2^{\,2}} \quad \text{.......................................(4.15)}$$

$$H_\phi = \frac{I}{2\pi\rho}\frac{R_3^{\,2} - \rho^2}{R_3^{\,2} - R_2^{\,2}} \quad \text{.......................................(4.16)}$$

In the region $\rho > R_3$

$$I_{nec} = 0 \qquad H_\phi = 0 \quad \text{.......................................(4.17)}$$