3.2 RL DC Circuits:

When we consider independent sources acting on a circuit, part of the response will resemble the nature of the particular source (or *forcing function*) used; this part of the response, called the *particular solution*, the *steady-state response*, or the ***forced response,*** will be "complemented'' by the complementary response produced in the source-free circuit. The complete response of the circuit will then be given by the sum of the complementary function and the particular solution. In other words, the complete response is the sum of the natural response and the forced response. The source-free response may be called the *natural* response, the *transient* response, the *free* response, or the *complementary function*, but because of its more descriptive nature, we will most often call it the *natural* response. We begin our study of transient analysis by considering the simple series RL circuit

$$Ri + v_L = Ri + L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

Our goal is an expression for $i(t)$ which satisfies this equation *and* also has the value $I(0)$ at $t = 0$.

integrating each side of the equation. The variables in Eq. [1] are i and t, and it is apparent that the equation may be multiplied by dt, divided by i, an arranged with the variables separated:

Applying KVL around the loop in Fig.

$$v_L + v_R = 0$$

But $v_L = L\,di/dt$ and $v_R = iR$. Thus,

$$L\frac{di}{dt} + Ri = 0$$

or

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

Rearranging terms and integrating gives

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L} dt$$

$$\ln i \Big|_{I_0}^{i(t)} = -\frac{Rt}{L}\Big|_0^t \quad \Longrightarrow \quad \ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0$$

or

$$\ln \frac{i(t)}{I_0} = -\frac{Rt}{L}$$

Taking the powers of e, we have

$$i(t) = I_0 e^{-Rt/L}$$

This shows that the natural response of the RL circuit is an exponential decay of the initial current. The current response is shown in Fig.

It is evident from Eq. that the time constant for the RL circuit is

$$\boxed{\tau = \frac{L}{R}}$$

with τ again having the unit of seconds. Thus, Eq. may be written as

$$\boxed{i(t) = I_0 e^{-t/\tau}}$$

With the current in Eq. , we can find the voltage across the resistor as

$$v_R(t) = iR = I_0 R e^{-t/\tau}$$

The power dissipated in the resistor is

$$p = v_R i = I_0^2 R e^{-2t/\tau}$$

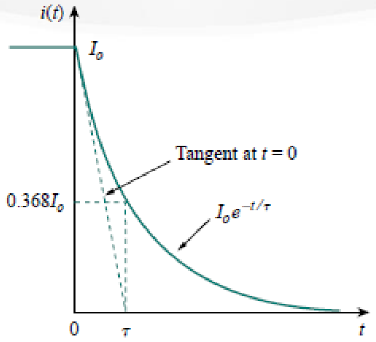

Fig. 3.2.1 The current response of RL circuit.

[Source: "Fundamentals of Electric Circuits" by charles K. Alexander, page: 244]

Problem 1:

In the circuit shown in Fig. 3.2.2, find io, vo, and i for all time, assuming that the switch was open for a long time.

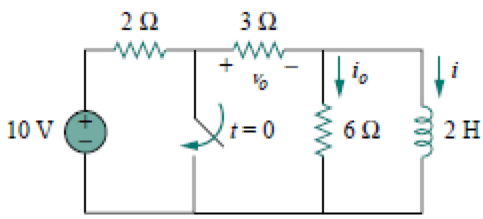

Fig. 3.2.2 For pbm 1.

[Source: "Fundamentals of Electric Circuits" by charles K. Alexander, page: 248]

Solution:

It is better to first find the inductor current i and then obtain other quantities from it.

For $t < 0$, the switch is open. Since the inductor acts like a short circuit to dc, the 6-Ω resistor is short-circuited, so that we have the circuit shown in Fig. 3.2.3 Hence, $i_o = 0$, and

$$i(t) = \frac{10}{2+3} = 2 \text{ A}, \qquad t < 0$$

$$v_o(t) = 3i(t) = 6 \text{ V}, \qquad t < 0$$

Thus, $i(0) = 2$.

For $t > 0$, the switch is closed, so that the voltage source is short- circuited. We now have a source-free RL circuit as shown in Fig. 3.2.3 b At the inductor terminals,

$$R_{\text{Th}} = 3 \parallel 6 = 2 \ \Omega$$

so that the time constant is

$$\tau = \frac{L}{R_{\text{Th}}} = 1 \text{ s}$$

Hence,

$$i(t) = i(0)e^{-t/\tau} = 2e^{-t} \text{ A}, \qquad t > 0$$

Since the inductor is in parallel with the 6-Ω and 3-Ω resistors,

$$v_o(t) = -v_L = -L\frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t} \text{ V}, \qquad t > 0$$

and

$$i_o(t) = \frac{v_L}{6} = -\frac{2}{3}e^{-t} \text{ A}, \qquad t > 0$$

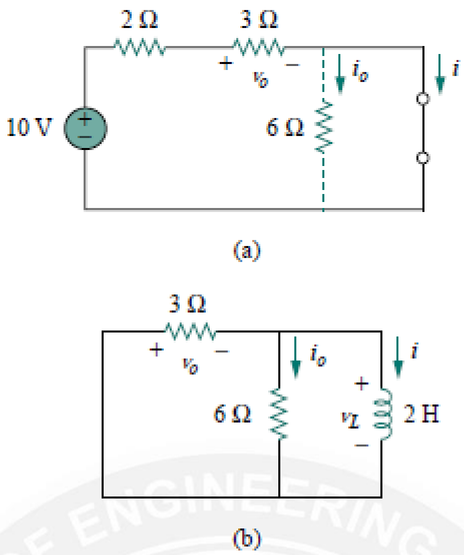

Fig. 3.2.3 For pbm 1.

[Source: "Fundamentals of Electric Circuits" by charles K. Alexander, page: 248]

Thus, for all time,

$$i_o(t) = \begin{cases} 0 \text{ A}, & t < 0 \\ -\frac{2}{3}e^{-t} \text{ A}, & t > 0 \end{cases}, \qquad v_o(t) = \begin{cases} 6 \text{ V}, & t < 0 \\ 4e^{-t} \text{ V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2 \text{ A}, & t < 0 \\ 2e^{-t} \text{ A}, & t \geq 0 \end{cases}$$

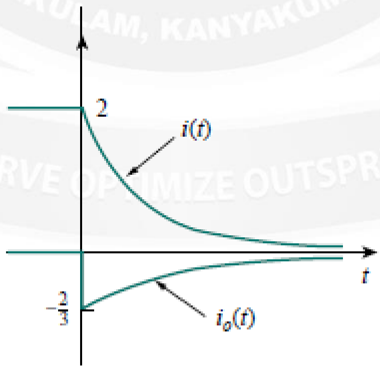

Fig. 3.2.4 Plot for I and i0.

[Source: "Fundamentals of Electric Circuits" by charles K. Alexander, page: 249]