3.6 ENERGY TRANSFER OF A WAVE

The mechanical energy is transferred through the vibration of the string.

As a wave propagates in a medium, it transports energy. It means a vibrating string has more energy than a string that is not vibrating.

Consider string under uniform tension T

m- mass per unit length

If a small element of the string of length dx is considered, then its mass is mdx.

As the string is vibrating, then the kinetic energy of this small element is

dk $= \frac{1}{2}$m(velocit$y)^2$

dk $= \frac{1}{2}$m.dx. $\left(\frac{dy}{dt}\right)^2$ --------------(1)

dy- vertical displacement

x- direction of propagation of the wave.

When the string is displaced from the equilibrium, a segment associated with interval dx has length dl.

dl $= \sqrt{(dx)^2 + (dy)^2}$

$= dx\sqrt{1 + (\frac{dy}{dx})^2}$

$= dx[1 + \frac{1}{2} (\frac{dy}{dx})^2]$ ----------------(2)

Thus under tension a small segment of the string has expanded by an amount

$\Delta l = dl - dx = \frac{1}{2} (\frac{dy}{dx})^2 dz$

So work is done for this expansion and it is stored as potential energy.

If the string is vibrating with displacement

$y(x,t) = A\cos(\omega t - kx)$ ----------------(3)

Therefore, the potential energy is

$dU = T. \Delta l$

$= \frac{1}{2}T(\frac{dy}{dx})^2dx$---------------(4)

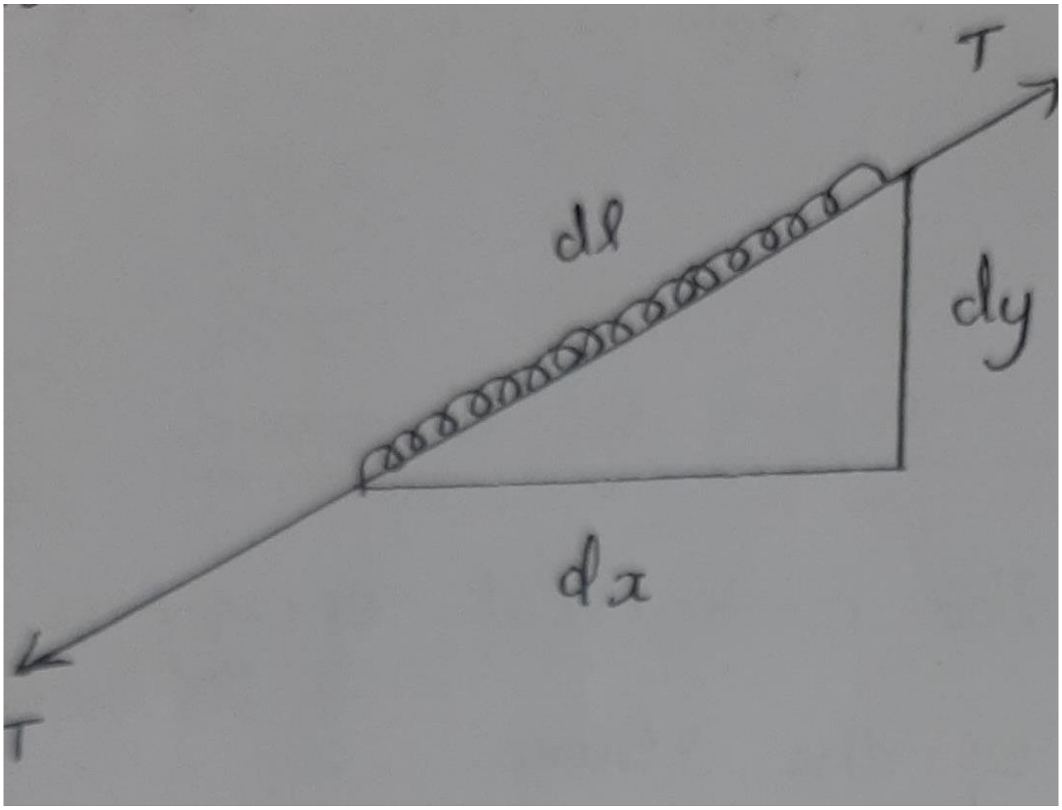

Fig: An element of a string under the action of a wave.

Differentiating (3) w.r.t to "x"

$\frac{dy}{dx}$ = Asin(ωt – kx)

$(\frac{dy}{dx})^2 = K^2A^2sin^2(\omega t - kx)$----------------(5)

Sub (5) in (4)

dU = $\frac{1}{2}TK^2A^2sin^2(\omega t - kx)dx$

$= \frac{1}{2}m\omega^2 \quad A^2sin^2(\omega t - kx)dx$------------(6)

[$TK^2 = T\frac{\omega^2}{v^2} = T\frac{\omega^2}{\frac{T}{m}} = m\omega^2$]

Differentating (3) w.r.t 't'

$\frac{dy}{dt}$ = - ωA sin(ωt – kx)

$(\frac{dy}{dt})^2 = \omega^2 A^2 sin^2(\omega t - kx)$---------(7)

Sub (7) in (1)

$dK = \frac{1}{2} m\omega^2 \quad A^2 sin^2(\omega t - kx)dx$-----------(8)

comparing (6) and (8)

dU = dK----------------------(9)

The total energy is

dE = dU + dK

= 2dK

$dE = m\omega^2 \quad A^2 sin^2(\omega t - kx)dx$ -----------------(10)

The quantity $(\frac{dE}{dx})$ is called the linear energy density.

At any point ,for example x=0 the average value of $sin^2(\omega t) = \frac{1}{2}$

$\frac{dE}{dx} = \frac{1}{2}$ m $(A\omega)^2$------------------------------(11)

This relation is called as average density.

As the average power transmitted by the wave is

$\vec{P} = (\frac{dE}{dt})$

Eqn (11) becomes

$dE = \frac{1}{2}$ m $(A\omega)^2 dx$------------(12)

$\vec{P} = \frac{1}{2}$ m $(A\omega)^2 (\frac{dx}{dt})$

$\vec{P} = \frac{1}{2}$ m $(A\omega)^2 v$-------------------(13)

Wher $v = \frac{dx}{dt}$ is the wave velocity.

Eqn (13) states that wave power is directly proportional to the speed or velocity v at which energy moves along the wave.