5.7 Culmann's Method:

Culmann (1866) considered a simple failure mechanism of a slope of homogeneous soil with plane failure surface passing through the toe of the slope.

Let AB be any probable slip plane. The wedge ADB is in equilibrium under the action of three force:

i)weight of the wedge $W = \frac{1}{2}AB.h.\gamma = \frac{1}{2}Lh\gamma ----(1)$

ii)The cohesive force C along the surface AB, resisting motion=$C_m L$

iii)The reaction R, inclined at angle φ_m to the normal

Now,$AD = \frac{h}{\sin(i-\theta)} = \frac{H}{\sin i}$

Hence $h = \frac{H\sin(i-\theta)}{\sin i}$

Substitute h value in equation 1

$$W = \frac{1}{2}AB.h.\gamma = \frac{1}{2}L\gamma\frac{H\sin(i-\theta)}{\sin i} ------(2)$$

If C and φ are the appropriate shear strength parameter,the shear strength (τ_f) along the slip plane

$$\tau_f = C.L + W\cos\theta.\tan\varphi ------(3)$$

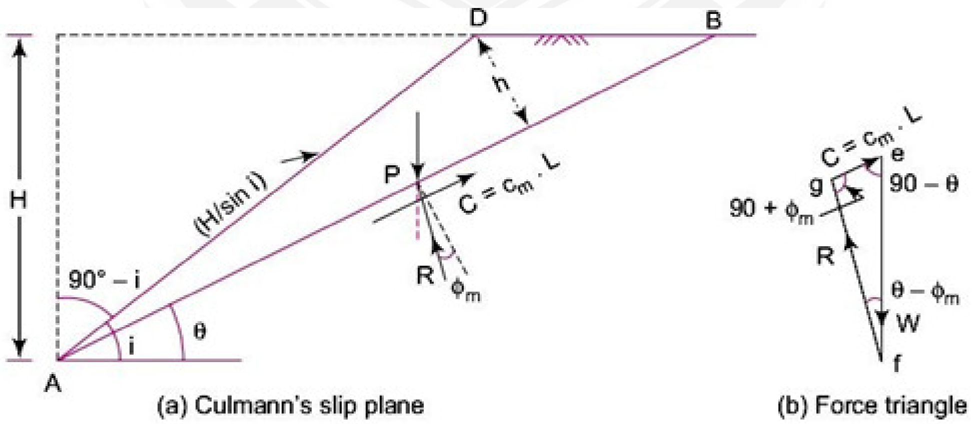

Fig5.27 Culmann's method using planar failure surface

The weight component parallel to the plane AC causing sliding is $\tau = W\sin\theta$

Factor of safety$F = \frac{\tau_f}{\tau} = \frac{cL+W\cos\theta\tan\varphi}{W\sin\theta}$

Substituting the value of W in the above equation

$$F = \frac{C + \frac{1}{2}\gamma H[\sin(i-\theta)/\sin i]\cos\theta\tan\varphi}{\frac{1}{2}\gamma H\left[\frac{\sin(i-\theta)}{\sin i}\right]\sin\theta} ------(4)$$

Let C_m be the mobilised cohesion $= \frac{C}{F}$ and φ_m=angle of mobilised friction $= \tan^{-1}\left(\frac{\tan\varphi}{F}\right)$

Considering AB to be the failure plane,

$$\frac{C_m L}{\sin(\theta-\varphi_m)} = \frac{W}{\sin(90+\varphi_m)} = \frac{W}{\cos\varphi_m}$$

Substituting for the weight W, we get $\frac{C_m L}{\sin(\theta-\varphi_m)} = \frac{\frac{1}{2}L\gamma H\sin(i-\theta)}{\sin i\cos\varphi_m}$

$$\frac{C_m}{\gamma H} = \frac{1}{2}\left[\frac{\sin(\theta-\varphi_m)\sin(i-\theta)}{\sin i\cos\varphi_m}\right]$$

$$= \frac{1}{2}\operatorname{cosec} i\sec\varphi_m\sin(i-\theta)\sin(\theta-\varphi_m)$$

where $\frac{C_m}{\gamma H} = S_n$ *is a stability number*

$$S_n = \frac{1}{2}\operatorname{cosec} i\sec\varphi_m\sin(i-\theta)\sin(\theta-\varphi_m) ----(5)$$

For failure to occur, the stability number(S_n) has to be maximum.The plane is which ($\theta=\theta_c$) the S_n becomes maximum.

$$\frac{d(S_n)}{d\theta} = \frac{d}{d\theta}[\sin(i-\theta)\sin(\theta-\varphi_m)] = 0$$

$$or \cos(\theta-\varphi_m)\sin(i-\theta) - \sin(\theta-\varphi_m)\cos(i-\theta) = 0$$

$$or \tan(\theta-\varphi_m) = \tan(i-\theta)$$

$$or\ \theta-\varphi_m = i-\theta$$

Making $\theta=\theta_c$ = critical angle,we get $\theta_c = \frac{1}{2}(i+\varphi_m) ----- (6)$

This gives the angle of inclination of critical slip plane

Substituting θ_c in eqn 5

$$\left(\frac{C_m}{\gamma H}\right)_{max} = \frac{1}{2}\operatorname{cosec} i\sec\varphi_m\sin\left(i-\frac{i+\varphi_m}{2}\right)\sin\left(\frac{i+\varphi_m}{2}-\varphi_m\right)$$

$$= \frac{1}{2}\operatorname{cosec} i\sec\varphi_m\left(\frac{1-\cos(i-\varphi_m)}{2}\right)$$

$$\left(\frac{C_m}{\gamma H}\right)_{max} = \frac{1-\cos\,(i-\varphi_m)}{4 sinicos\varphi_m}$$

$$H_{max} = \frac{C_m 4 sinicos\varphi_m}{\gamma(1-\cos(i-\varphi_m))} ----(7)$$

In the above equation C_m and φ_m are known corresponding to a factor of safety F and the Culmann's method is suitable for very steep slopes.