Classical step-by-step solution of the swing equation

Numerical Solution of Swing Equation

There are several sophisticated methods for solving the swing equation. The step-by-step or point-by-point method is conventional, approximate but well tried and proven method. This method determines the changes in the rotor angular position during a short interval of time.

Consider the swing equation:

$$M \frac{d^2\delta}{dt^2} = P_S - P_{max} \sin \delta = P_A \qquad ...(7.52)$$

The solution δ(t) is obtained at discrete intervals of time with interval spread of Δt uniform throughout. Accelerating power, P_A and change in speed, which are continuous function of time and are described as below:

1. The accelerating power P_A computed at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered, as illustrated in Fig. 7.26.

2. The angular rotor velocity ω', i.e., dδ/dt (over and above synchronous velocity ω_0) is assumed to remain constant throughout any interval at the value computed for the middle of the interval, as illustrated in Fig. 7.26.

In Fig. 7.26 the numbering on t/Δt axis pertains to the end of intervals.

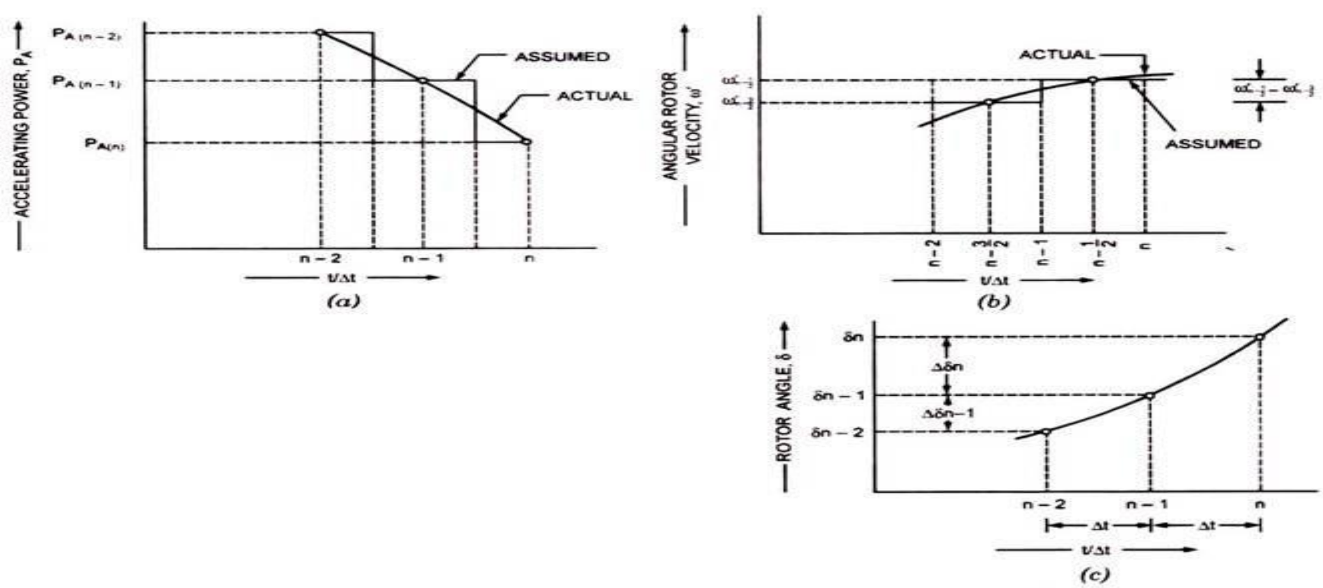

Fig. 7.26

The equation for accelerating power at the end of the $(n-1)^{th}$ interval or for nth interval can be written as

$$P_{A(n-1)} = P_S - P_{max} \sin \delta_{n-1} \qquad ...(7.53)$$

where δ_{n-1} has been earlier calculated.

The change in velocity caused due to $P_{A(n-1)}$ assumed to remain constant over Δt from $(n - 3/2)$ to $(n - 1/2)$,

$$\Delta\omega'_{n-1/2} = \omega'_{n-1/2} - \omega'_{n-3/2}$$

$$= \frac{\Delta t}{M} P_{A(n-1)} \qquad ...(7.54)$$

The change in rotor angle δ during $(n - 1)$th interval,

$$\Delta \delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \Delta t \omega'_{n-\frac{3}{2}} \qquad ...(7.55)$$

and during the nth interval, $\Delta \delta_n = \delta_n - \delta_{n-1} = \Delta t \omega'_{n-\frac{1}{2}}$ ...(7.56)

Subtracting Eq. (7.55) from Eq. (7.56) we have

$$\Delta\delta_n - \Delta\delta_{n-1} = \Delta t\left(\omega'_{n-\frac{1}{2}} - \omega'_{n-\frac{3}{2}}\right) = \frac{(\Delta t)^2}{M} P_{A(n-1)} \qquad \because \text{ from Eq. (7.54)}$$

$$\omega'_{n-\frac{1}{2}} - \omega'_{n-\frac{3}{2}} = \frac{(\Delta t)}{M} P_{A(n-1)}$$

$$\text{or} \quad \Delta\delta_n = \Delta\delta_{n-1} + \frac{P_{A(n-1)}}{M}(\Delta t)^2 \qquad ...(7.57)$$

$$\therefore \delta_n = \delta_{n-1} + \Delta\delta_n \qquad ...(7.58)$$

The above process of computation is repeated to obtain $P_A(n)$, $\Delta\delta_{n+1}$ and δ_{n+1}. The time solution in discrete form is thus carried out over the desired length of time, usually 0.05 second. Actual swing curve can be plotted by drawing a smooth curve through discrete values, as shown in Fig. 7.26.

The accuracy of the solution depends upon the time duration of the intervals. As the time interval is reduced the computed swing curve approaches the true. Usually $\Delta t = 0.05$ second provides good accuracy in results. The occurrence or removal of a fault or initiation of any switching action causes a discontinuity in accelerating power.

There are three possibilities of occurrence of discontinuity:

(i) The discontinuity occurs at the beginning of the nth interval,

(ii) The discontinuity occurs at the middle of an interval.

(iii) The discontinuity occurs at some time other than the beginning or the middle of an interval.

In the first case, the average of the values of accelerating power P_A before and after discontinuity should be used. Thus in determining the increment of angle occurring during the first interval after the occurrence of fault at t = 0, Eq. (7.57) becomes –

$$\Delta\delta_i = P_{A^{\circ}+} /2M\ (\Delta t)^2$$

whereas $P_{A^{\circ}+}$ is the accelerating power immediately after the occurrence of the fault. Since immediately before the occurrence of the fault the system is in steady state, $P_{A^{\circ}-} = 0$ and δ_0 is of known value.

If the fault is cleared at the beginning of the nth interval, in calculation for this interval the value of $P_{A(n-1)}$ should be taken as

$$\frac{1}{2}\left[P_{A(n-1)^-} + P_{A(n-1)^+}\right]$$

where $P_{A\ (n-1)-}$ is the accelerating power immediately before clearing and $P_{A\ (n-1)+}$ is that immediately after clearing the fault.

In second case, i.e., when the discontinuity occurs at the middle of an interval, no special procedure is required. The increment of the angle during such an interval is computed, as usual, from the value of P_A at the beginning of the interval, i.e.,

$P_A = P_s$ – output during the fault

Where-as at the beginning of the interval following clearing of the fault, P_A is given as

$P_A = P_s$ – output after clearance of fault.

To compute accelerating power P_A in the third case, a weighted average value of P_A before and after the discontinuity may be used. It is found in practice that such a precise computation of accelerating power P_A is not required as the time interval used in computation is so short that it is sufficiently accurate to consider the discontinuity to occur at the beginning or at the middle of an interval and accelerating power P_A is computed as outlined above in the first two cases.

Digital Solution of Swing Equation:

There are number of numerical techniques that can be used for the solution of swing equation. These techniques can be used for digital computations. In such techniques a nth order differential equation is written as n first order equations. These techniques include– (1) Euler's method (2) Modified Euler's method and (3) Runge-Kutta method. The last two methods are most popular methods used for solution of swing equation.

1. Euler's Method:

This is a single step method. In case of a single step method for determining solution of x_{i+1}, we consider dependence at only one earlier point say x_i.

In this case algorithm will be of the following form:

$$y_{i+1} = y_1 \quad \underbrace{h\,\phi\,(x_i, y_i, h)}_{\text{Increment Function}} \tag{7.59}$$

In the case of initial value problem, we know dependence at the earlier point from initial condition, and therefore, no difficulty arises in determination of solution at x_i for the first order differential equation. There is another advantage of single step method and it is that the step size can be changed at any stage.

$$\text{Let } \frac{dy}{dx} = y' = f(x, y)$$

$$\text{with initial condition } y(x_0) = y_0 \tag{7.60}$$

$$\text{Algorithm is } y_{i+1} = y_i + h f(x_i, y_i) \tag{7.61}$$

Geometrical integration of the algorithm (7.61) is clear from Fig. 7.32.

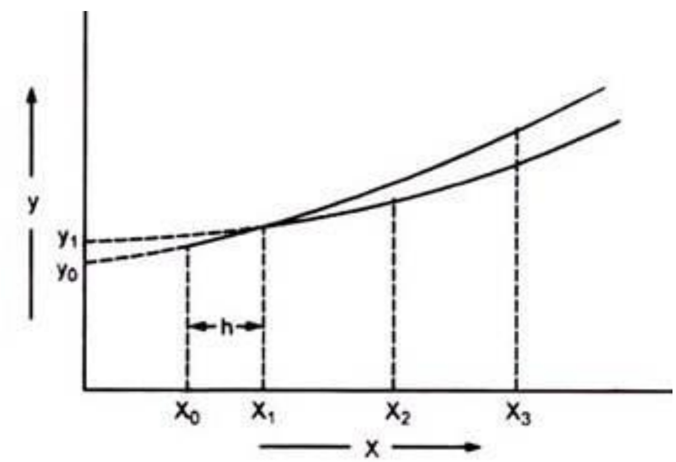

Fig. 7.32

Here the part of the curve in the interval h is being approximated by segment of the straight line whose slope is the same as that at the beginning of the subinterval. This method is only of theoretical importance.

2. RungeKutta Method:

This is the most powerful method of solving swing equation on digital computer. Let us consider two first order differential equations in two variables x and y such that –

$$\frac{dx}{dt} = f(x, y) \tag{7.64}$$

$$\frac{dy}{dt} = g(x, y) \tag{7.65}$$

Let us start with known initial conditions x° and y° and a time step Δt.

We compute the following eight constants –

$$h_1^0 = f(x^0, y^0)\,\Delta t \qquad \text{...(7.66 a)}$$

$$k_1^0 = g(x^0, y^0)\,\Delta t \qquad \text{...(7.66 b)}$$

$$h_2^0 = f\left(x^0 + 0.5h_1^0, y^0 + 0.5k_1^0\right)\Delta t \qquad \text{...(7.66 c)}$$

$$k_2^0 = g\left(x^0 + 0.5h_1^0, y^0 + 0.5k_1^0\right)\Delta t \qquad \text{...(7.66 d)}$$

$$h_3^0 = f\left(x^0 + 0.5h_2^0, y^0 + 0.5k_2^0\right)\Delta t \qquad \text{...(7.66 e)}$$

$$k_3^0 = g\left(x^0 + 0.5h_2^0, y^0 + 0.5k_2^0\right)\Delta t \qquad \text{...(7.66 f)}$$

$$h_4^0 = f\left(x^0 + h_3^0, y^0 + k_3^0\right)\Delta t \qquad \text{...(7.66 g)}$$

$$k_4^0 = g\,(x^0 + h_3^0, y^0 + k_3^0)\,\Delta t \qquad \text{...(7.66 h)}$$

We use the above eight constants to estimate the changes in x and y as follows –

$$\Delta x^0 = \frac{1}{6}\left(h_1^0 + 2h_2^0 + 2h_3^0 + h_4^0\right) \qquad \text{...(7.67 a)}$$

$$\Delta y^0 = \frac{1}{6}\left(k_1^0 + 2k_2^0 + 2k_3^0 + k_4^0\right) \qquad \text{...(7.67 b)}$$

The values of x, y and t are updated as

$$x^1 = x^0 + \Delta x^0 \qquad \text{...(7.68 a)}$$
$$y^1 = y^0 + \Delta y^0 \qquad \text{...(7.68 b)}$$
$$t^1 = 0 + \Delta t \qquad \text{...(7.68 c)}$$

Then we replace x^0 and y^0 by x^1 and y^1 and recalculate h's, k's, Δ x and Δ y. In general, we may write

$$x^{n+1} = x^n + \Delta x^n \qquad \text{...(7.69 a)}$$
$$y^{n+1} = y^n + \Delta y^n \qquad \text{...(7.69 b)}$$
$$t^{n+1} = (n+1)\,\Delta t \qquad \text{...(7.69 c)}$$

Application to Solution of Swing Equation:

For using the Runge-Kutta method for solving the swing equation of one machine connected to infinite bus, let us substitute –

$$x = \delta \qquad ...(7.70\ a)$$

$$y = \alpha = \frac{d\delta}{dt} \qquad ...(7.70\ b)$$

Then $\frac{d\delta}{dt} = f(\delta, \alpha) = \alpha \qquad ...(7.71\ a)$

and $\frac{d\alpha}{dt} = g(\delta, \alpha) = \frac{1}{M}(P_S - P_E) \qquad ...(7.71\ b)$

where $M = \frac{GH}{\pi f}$ MJ-s/electrical radian

The initial value of load angle δ_0 is determined using the pre-fault values.

$$P_S = P_E = P_{max} \sin \delta_0 \qquad ...(7.72\ a)$$

$$\delta_0 = \text{Sin}^{-1} \frac{P_S}{P_{max}} \qquad ...(7.72\ b)$$

The values of P_E used in Eq. (7.71*b*) are

For $0 < t < t_c$ $\qquad P_e = \gamma_1 P_{max} \sin \delta \qquad ...(7.73)$

and for $t > t_c$ $\qquad P_e = \gamma_2 P_{max} \sin \delta \qquad ...(7.74)$

where t_c is the fault clearing time.