UNIT I

## Mechanics

1.4 Moment of Inertia of a diatomic molecules

A diatomic molecule, in its stable equilibrium position consists two atoms that are at a distance 'R' apart. The distance 'R' is called the band length between the two atoms.

Presently we can consider that it consists of two tiny spheres at either end of a thin weightless rigid rod, as shown in fig. This kind of arrangement can be called as rigid rotor.
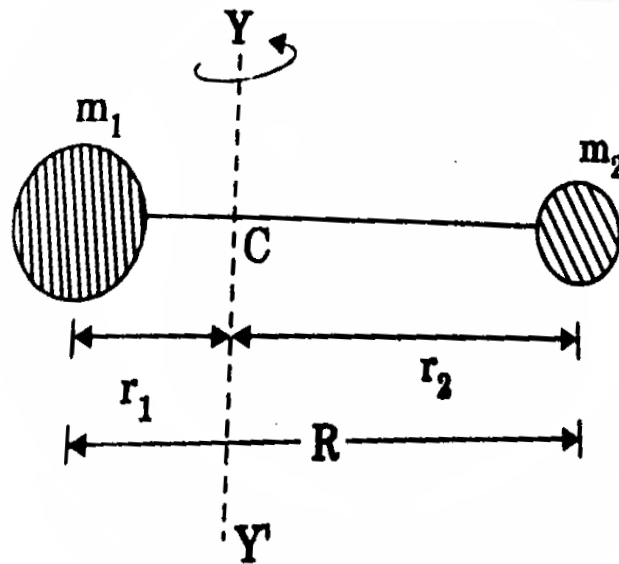
Let 'C' be the center of mass of the molecule and r_1 and r_2 respective distances of the two atoms from it.

Then

$$r_1 + r_2 = R \qquad \text{-------- (1)}$$

and

$$m_1 \, r_1 = m_2 r_2 \qquad \text{------- (2)}$$

where m_1 and m_2 are the masses of two atoms respectively.

From eqn. (1),

$$r_1 = R - r_2 \qquad \text{-------- (3)}$$

and from eqn. (2),

$$r_2 = \frac{m_1 r_1}{m_2} \qquad \text{-------- (4)}$$

so,

$$r_1 = R - \frac{m_1 r_1}{m_2}$$

$$R = r_1 + \frac{m_1 r_1}{m_2} = r_1 \left[1 + \frac{m_1}{m_2} \right] \text{-------- (5)}$$

$$R = \frac{R}{\left[1 + \frac{m_1}{m_2} \right]} \qquad \text{-------- (6)}$$

Now, the moment of inertia of the molecule (i.e., of the two atoms) about an axis passing through the centre of mass 'C' and perpendicular to the bond is given as

$$I = m_1 r_1^{\ 2} + m_2 r_2^{\ 2} \qquad \text{-------- (7)}$$

$$I = m_1 r_1 r_1 + m_1 r_1 r_2 \qquad \text{-------- (6)} \qquad [\therefore \text{ from eqn. (2)}]$$

$$I = m_1 r_1 (r_1 + r_2)$$

(or) by using eqn. (1),

$$I = m_1 r_1 R \qquad \text{-------- (8)}$$

Substituting eqn. (6) in (8) gives

$$I = m_1 R \left[\frac{R}{\left[1 + \frac{m_1}{m_2} \right]} \right]$$

$$I = \frac{m_1 R^2}{\left[1 + \frac{m_1}{m_2} \right]}$$

$$= \frac{m_1 R^2}{\left[\frac{m_2 + m_1}{m_2} \right]}$$
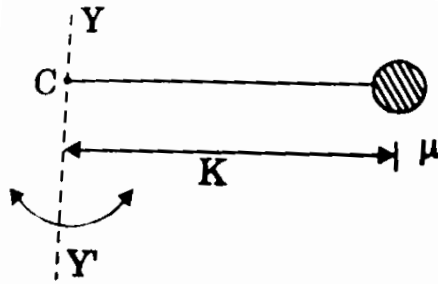
$$= \frac{m_1 m_2 R^2}{m_2 + m_1}$$

(or)

$$I = \left[\frac{m_1 m_2}{m_2 + m_1} \right] R^2$$

$$I = \mu R^2 \qquad \text{-------- (9)}$$

Where $\mu = \frac{m_1 m_2}{m_2 + m_1}$ is called as reduced mass of the molecule. Thus the figure can also be redrawn

as

In figure, K=R, which is called radius of gyration, so moment of inertia

$$I = \mu K^2 \qquad \text{-------- (10)}$$