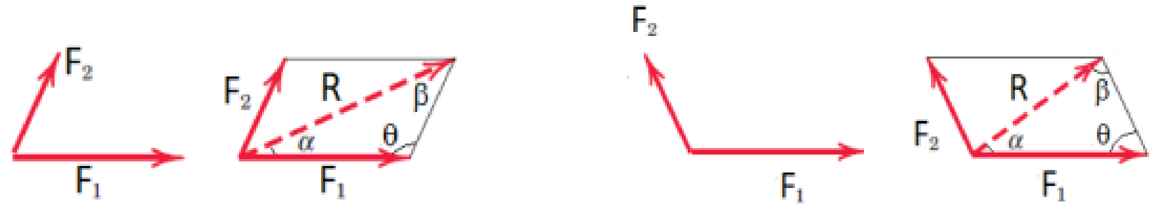
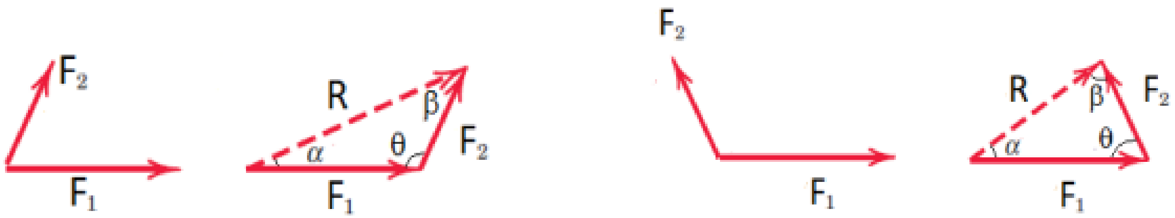
1.2 Composition & Resolution of Forces

Composition is the process of replacing a force system by its resultant.

a. Parallelogram Law



b. Triangle Law



The resultant of a pair of concurrent forces can be determined by:

$$R = \sqrt{{F_1}^2 + {F_2}^2 - 2F_1F_2\cos\theta}$$

Also, it can be found the direction of R or unknown one of forces by:

$$\frac{R}{\sin\theta} = \frac{F_1}{\sin\beta} = \frac{F_2}{\sin\alpha}$$

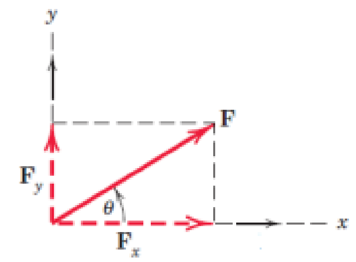
Resolution is the process of replacing a single force by its components.

If a force (F) lies in the $x - y$ plane. The force (F) may be resolved into two rectangular components. The component of a force parallel to the x-axis is called the Horizontal component (Fx), and parallel to y-axis the is called Vertical component (Fy).

For Example:

$$\cos\theta = \frac{Fx}{F} \quad \rightarrow \quad Fx = F\cos\theta \quad \rightarrow$$

$$\sin\theta = \frac{Fy}{F} \quad \rightarrow \quad Fy = F\sin\theta \quad \uparrow$$



$$F = \sqrt{Fx^2 + Fy^2}$$

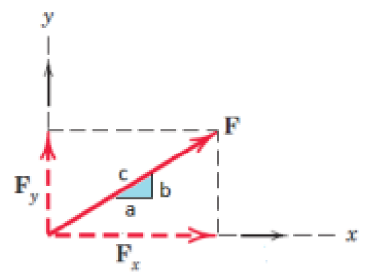
$$\theta_x = \tan^{-1}\left(\frac{Fy}{Fx}\right)$$

The direction of ***F*** can also be defined using a small "slope" triangle. Given the slope of the line of action of the force as

$$c = \sqrt{a^2 + b^2}$$

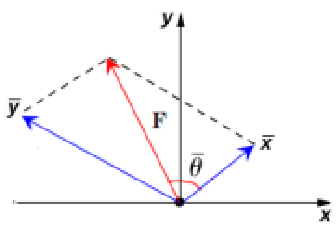
$$Fx = F\cos\theta \quad \rightarrow \quad Fx = F \cdot \frac{a}{c} \quad \rightarrow$$

$$Fy = F\sin\theta \quad \rightarrow \quad Fy = F \cdot \frac{b}{c} \quad \uparrow$$

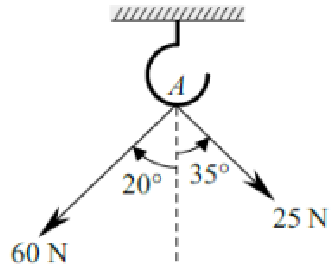


$$F\bar{x} = F\cos\theta \quad \nearrow$$

$$F\bar{y} = F\sin\bar{\theta} \quad \nwarrow$$

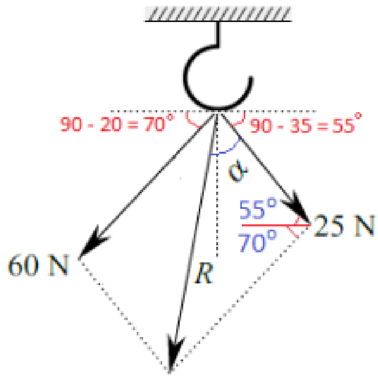


Example No. 1: Two forces are applied at the point A of a hook support as shown in Figure. Determine the magnitude and direction of the resultant force by using (i) parallelogram law, and (ii) triangle law.



Solution:

i. Parallelogram law



$F_1 = 25\ N\ , \qquad F_2 = 60\ N$

$\theta = 70 + 55 = 125^{\circ}$

To find the value of resultant:

$$R = \sqrt{F_1{}^2 + F_2{}^2 - 2F_1F_2\cos\theta}$$

$$R = \sqrt{25^2 + 60^2 - 2 \times 25 \times 60 \times \cos 125}$$

$$= 77.11\ N$$

the direction of resultant:

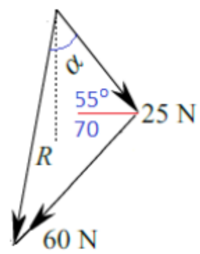
$$\frac{R}{\sin\theta} = \frac{F_2}{\sin\alpha}$$

$$\frac{77.11}{\sin 125} = \frac{60}{\sin\alpha} \quad \rightarrow \quad \sin\alpha = \frac{60 \times \sin 125}{77.11} = 0.637$$

$\alpha = \sin^{-1} 0.637 = 39.597^{\circ}$

The direction of R from the vertical axis $= 39.597 - 35$

$= 4.597^{\circ}$



ii. Triangle Law

by the same above equations to get:

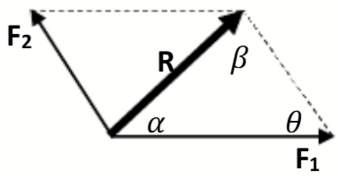
$R = 77.11\ N$ inclined 4.597° with vertical direction

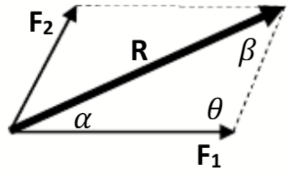
Composition and Resolution of Forces

Composition:

Composition is the process of replacing a force system by its resultant. For any **two concurrent forces** the resultant can be determined by the ***Parallelogram Law.***

Parallelogram Law: the resultant of two concurrent forces is the diagonal of the parallelogram formed on the vectors of these forces.





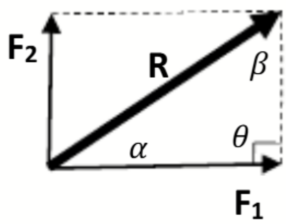
The magnitude of the resultant can be determined by **Cosine Law**:

$$\boldsymbol{R = \sqrt{(F_1)^2 + (F_2)^2 - 2F_1F_2\cos\theta}}$$

and the slope (or angle) can be determined by **Sine Law**:

$$\frac{\boldsymbol{F_1}}{\boldsymbol{\sin\beta}} = \frac{\boldsymbol{F_2}}{\boldsymbol{\sin\alpha}} = \frac{\boldsymbol{R}}{\boldsymbol{\sin\ \theta}}$$

Note: for the case of rectangular or perpendicular components ($\theta = 90°$), the parallelogram becomes rectangle:

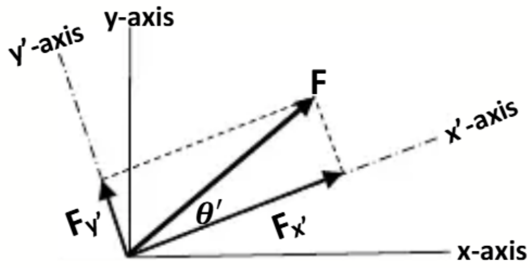


Therefore, the magnitude of the resultant (by cosine law) becomes:

$$\boldsymbol{R = \sqrt{(F_1)^2 + (F_2)^2}}$$

and the slope of resultant (by sine law) becomes:

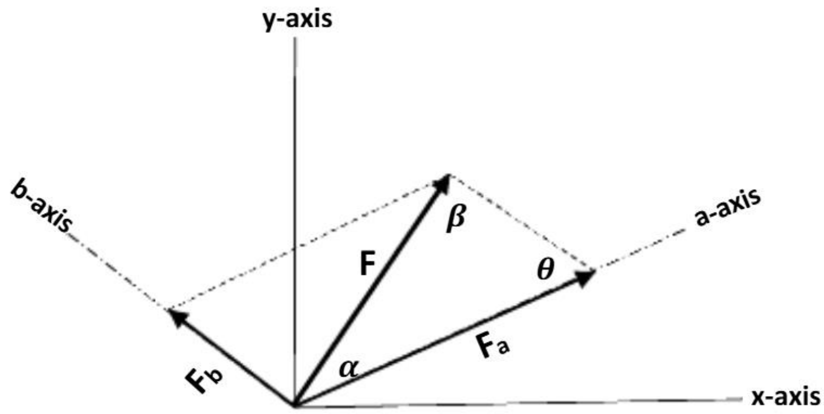
$$\frac{\boldsymbol{F_1}}{\boldsymbol{\sin\beta}} = \frac{\boldsymbol{F_2}}{\boldsymbol{\sin\alpha}} = \frac{\boldsymbol{R}}{\boldsymbol{\sin\ 90}}$$



$$F_{\acute{x}} = F\cos\acute{\theta}$$

$$F_{\acute{y}} = F\sin\acute{\theta}$$

b) Non-rectangular components:



By using the sine law find F_a and F_b :

$$\frac{F_a}{\sin\beta} = \frac{F_b}{\sin\alpha} = \frac{F}{\sin\ \theta}$$