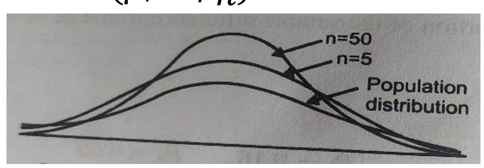
Central Limit Theorem

We know that the mean of the sampling distribution of the mean will equal to the population mean. As the sample size increases, the sampling distribution of the mean will approach normality. The relationship between the shape of the population distribution and the shape of the population distribution of the mean is called the central limit Theorem.

Definition

When sampling is done from a population with mean μ and finite standard deviation σ, the sampling distribution of the sample mean $\bar{X}$ will tend to a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$ as the sample size n becomes large.

For "large enough" n $\bar{X} \sim N(\mu, \sigma^2/n)$



The significance of the central limit theorem is that it permits us to use sample statistics (mean, variance etc) to make inferences about population without knowing anything about the shape of the frequency distribution of that population other than what we can get from the sample.

1. The life time of a certain brand of an electric bulb may be considered as a randon variable with mean 1200 hours and standard deviation 250 hours. Find the probability using central limit theorem hat the average life time of 60 bulbs exceed 1250 hours.

Solution

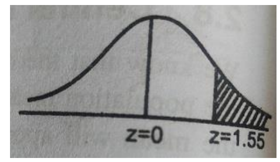
We are given n = 60, $Mean = \mu = 1200$ and $S.D = \sigma$=250

Let X_i be the life time if an electric bulb. $\bar{X} = \frac{X_1+X_2+\cdots.+X_{60}}{60}$

By Central limit theorem

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

$$p(\bar{X} > 1250) = p\left[\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{1250-1200}{250/\sqrt{60}}\right]$$



$$= p[Z > 1.55] = 0.5 - 0.4394 = 0.0606$$

2. A random sample of size 100 is taken from a population whose mean is 60 variance 400. Using central limit theorem find what probability that we can as That the mean of the sample will not differ from μ more than 4.

   Solution:

   Given, $\mu = 60, n = 100, \sigma = \sqrt{400} = 20$

   $\bar{X}$ =sample mean.

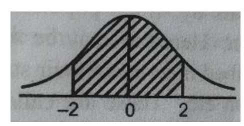
   $The\ required\ probability$ is $p[-4 < \bar{X} - \mu < 4]$

$$= p[-4 < \bar{X} - 60 < 4]$$

$$= p[56 < \bar{X} < 64]$$

$$= p\left[\frac{56-60}{2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{64-60}{2}\right]$$

$$= p[-2 < Z < 2]$$



$$= 2p[0 < Z < 2] = 2 * 0.4772 = 0.9544.$$

3. An economist wishes to estimate the average family income in a certain population. The population standard deviation is known to be $ 4.500, and the economist uses a random sample of size n=225. What is the probability that the sample mean will fall within $ 800 of the population mean?

   Solution:

Given μ = 800, n = 225, and σ = 4,500.

Let X = sample mean.

By central limit theorem

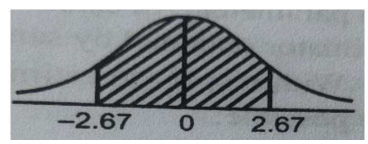
$$\bar{X} \sim N(\mu, \sigma^2/n)$$

$\textit{The required probability}$ is $p[-800 < \bar{X} - \mu < 800]$

$$= p\left[\frac{-800}{4500/\sqrt{225}} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{800}{4500/\sqrt{225}}\right]$$

$$= p\left[\frac{-800}{300} < Z < \frac{800}{300}\right]$$

$$= p[-2.67 < Z < 2.67]$$



$$= 2p[0 < Z < 2.67] = 2 * 0.4962 = 0.9924$$