Series Resonance

The basic series-resonant circuit is shown in fig. 1. Of interest here in how the steady state amplitude and the phase angle of the current vary with the frequency of the sinusoidal voltage source. As the frequency of the source changes, the maximum amplitude of the source voltage (Vm) is held constant

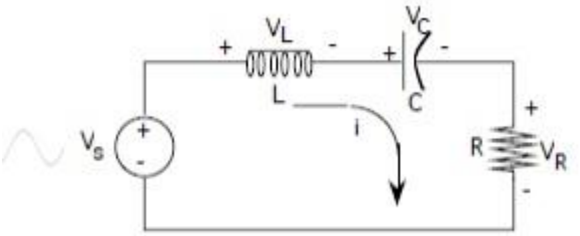

Fig. 1 The Series Resonant Circuit

$V_s = V_m \cos(\omega t)$ $\qquad$ $i = I_m \cos(\omega t + \theta)$

The frequency at which the reactance of the inductance and the capacitance cancel each other is the resonant frequency (or the unity power factor frequency) of this circuit. This occurs at

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad (1)$$

Since i = VR /R, then the current i can be studied by studying the voltage across the resistor. The current i has the expression

$$i = I_m \cos(\omega t + \theta)$$

$$I_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad (2A)$$

$$\theta = -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \qquad (2B)$$

The bandwidth of the series circuit is defined as the range of frequencies in which the amplitude of the current is equal to or greater than (1 / 2 = 2 / 2) times its maximum amplitude, as shown in fig. 2. This yields the bandwidth B = !2-!1= R/L

$$\omega_{2,1} = \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \pm \frac{R}{2L}$$

are called the half power frequencies or the 3 dB frequencies, i.e the frequencies at which the value of Im equals the maximum possible value divided by = 1.414 . The quality factor

$$Q = \frac{\omega_o}{B} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad (4)$$

Then the maximum value of :

- VR occurs at $\omega =$
- VL occurs at $\dfrac{\omega_o}{\sqrt{1 - \dfrac{R^2 C}{2L}}}$ (5B)
- VC occurs at $\omega_o\sqrt{1 - \dfrac{R^2 C}{2L}}$ (5C)

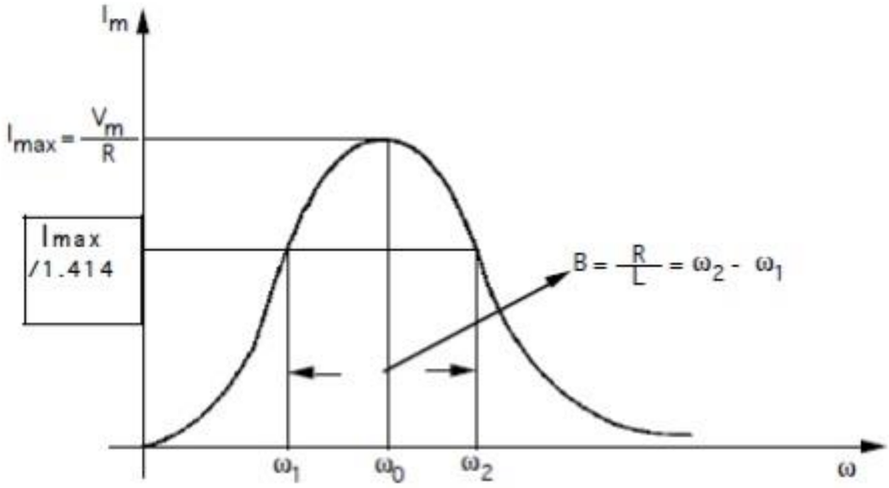

Fig. 2 Frequency Response of a Series - Resonant Circuit