

4.2 ANALYSIS OF RIGID BODIES IN EQUILIBRIUM

The analysis of rigid bodies in equilibrium is a fundamental topic in mechanics that involves understanding how forces and moments act on objects to maintain their state of rest or uniform motion.

Key Concepts

1. Equilibrium Conditions

For a rigid body to be in equilibrium, it must satisfy two essential conditions:

1. Translational Equilibrium:

The net force acting on the body must be zero:

This ensures there is no linear acceleration.

2. Rotational Equilibrium:

The net moment (torque) about any axis must be zero:

This ensures there is no angular acceleration.

2. Types of Forces

Rigid bodies in equilibrium can be subjected to various types of forces:

- **Weight (gravitational force):** Acts vertically downward through the center of gravity.
- **Normal force:** Acts perpendicular to the surface of contact.
- **Frictional force:** Acts parallel to the surface to resist motion.
- **Tension force:** Acts along the length of a cable, rope, or string.
- **Applied force:** Any external force applied to the body.
- **Reactions at supports or joints:** Forces and moments exerted by supports to maintain equilibrium.

3. Free-Body Diagrams (FBDs)

A free-body diagram is a critical tool for analyzing rigid bodies in equilibrium. It involves:

1. Representing the body as a simplified shape (e.g., a rectangle or point).
2. Drawing all the forces and moments acting on the body, with their directions and points of application.

3. Including dimensions, angles, and relevant distances for torque calculations.

4. Steps for Solving Problems

1. **Draw the Free-Body Diagram (FBD):**
Clearly label all forces, moments, and dimensions.
2. **Apply Equilibrium Equations:**
 - Use and for forces in the horizontal and vertical directions.
 - Use for moments about a convenient point.
3. **Solve Simultaneous Equations:**
Depending on the complexity, solve for unknown forces, moments, or reactions.
4. **Check Results:**
Verify that all equilibrium conditions are satisfied.

5. Example Applications

- **Statics of Beams:** Calculating reactions at supports and internal forces in beams under various loads.
- **Trusses and Frames:** Analyzing member forces in trusses or frames using the method of joints or sections.
- **Rigid Body Rotation:** Studying cases where forces produce rotational equilibrium, such as seesaws or levers.

Practical Considerations

- **Distributed Loads:** For distributed forces (e.g., weight of a beam), replace them with an equivalent point load acting at the center of gravity.
- **Choosing the Moment Point:** Select the point of moments wisely to eliminate unknown forces from the equations and simplify calculations.
- **Units:** Ensure consistent units throughout calculations.

FREE BODY DIAGRAMS

A **free-body diagram** (FBD) is a graphical representation used to visualize the forces acting on an object. It simplifies complex problems in physics and engineering, making it easier to analyze forces, torques, and motion. Here's how to create and interpret a free-body diagram:

Steps to Draw a Free-Body Diagram

1. Identify the Object:

- Choose the object of interest and isolate it from its surroundings.

2. Simplify the Object:

- Represent the object as a point or a simple shape.

3. Show All Forces Acting on the Object:

- **Gravity:** Usually acts downward .
- **Normal Force:** Acts perpendicular to the surface in contact.
- **Friction:** Opposes the direction of motion (if applicable).
- **Tension:** Acts along a rope, string, or cable.
- **Applied Force:** Any external force pushing or pulling.
- **Air Resistance:** Acts opposite to motion in fluids.

4. Label the Forces:

- Use arrows to indicate the direction and approximate magnitude of each force.
- Label forces with symbols .



5. Define a Coordinate System:

- Establish the direction of positive - and -axes for consistency.

Key Rules

- The object should only show forces **acting on it**, not forces it exerts on others.
- Draw all forces from the **center** or the point of application on the object.