1.1 Magnetic circuits

Introduction

In general, magnetic materials can be classified as magnetically "soft" and "hard" materials. Soft materials are normally used as the magnetic core materials for inductors, transformers, actuators and rotating machines, in which the magnetic fields vary frequently, whereas hard materials, or permanent magnets, are used to replace magnetization coils for generating static magnetic fields in devices such as electric motors and actuators. The B-H relationships and hysteresis loops have been discussed earlier. In this chapter, we are going to examine the power losses in a soft magnetic core under an alternating magnetization, and further develop an electrical circuit model of a magnetic core with a coil. For performance prediction of electromagnetic devices, magnetic field analysis is required.

Analytical magnetic field analysis by the Maxwell's equations, however, has been shown very difficult for engineering problems owing to the fact that most practical devices are of complicated structures. Powerful numerical methods, such as the finite difference and finite element methods, are out of the scope of this subject. In this chapter, we introduce a simple method of magnetic circuit analysis based on an analogy to dc electrical circuits.

Soft Magnetic Materials under Alternating Excitations

Core Losses

Core losses occur in magnetic cores of ferromagnetic materials under alternating magnetic field excitations. The Figure 1.1 on the right hand side plots the alternating core losses of M-36, 0.356 mm steel sheet against the excitation frequency. In this section, we will discuss the mechanisms and prediction of alternating core losses.

As the external magnetic field varies at a very low rate periodically, as mentioned earlier, due to the effects of magnetic domain wall motion the B- H. relationship is a hysteresis loop. The area enclosed by the loop is a power loss known as the hysteresis loss, and can be calculated by,

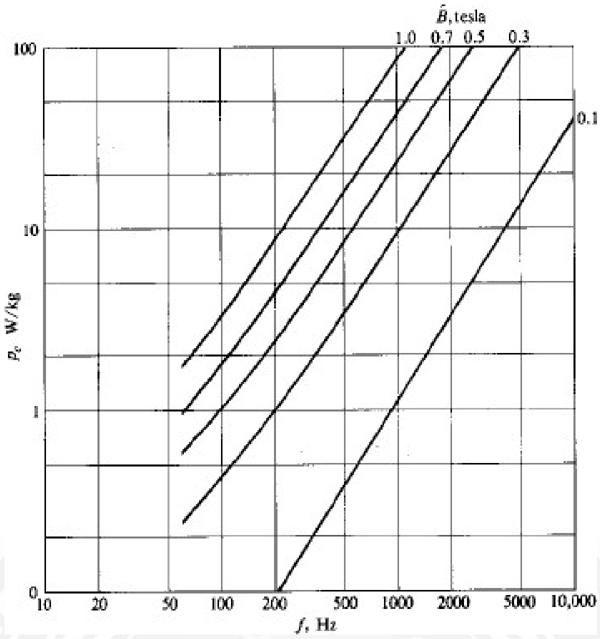


Figure 1.1.1 Alternating core loss of steel sheet at different excitation frequencies

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 45]*

For magnetic materials commonly used in the construction of electric machines an

$$P_{hyst} = \oint \mathbf{H} \bullet d\mathbf{B} \quad (\text{W/m}^3/\text{cycle}) \text{ or } (\text{J/m}^3)$$

approximate relation is

$$P_{hyst} = C_h f B_p^n \quad (1.5 < n < 2.5) \quad (\text{W/kg})$$

where

C_h is a constant determined by the nature of the ferromagnetic material,

f is the frequency of excitation, and B_p is the peak value of the flux density.

Example:

A B-H loop for a type of electric steel sheet is shown in the diagram below. Determine approximately the hysteresis loss per cycle in a torus of 300 mm mean diameter and a square cross section of 50*50 mm.

Solution:

The area of each square in the diagram represents

$$(0.1\ \text{T}) \times (25\ \text{A/m}) = 2.5\ (\text{Wb/m}^2) \times (\text{A/m}) = 2.5\ \text{VsA/m}^3 = 2.5\ \text{J/m}^3$$

If a square that is more than half within the loop is regarded as totally enclosed, and one that is more than half outside is disregarded, then the area of the loop is

$$2 \times 43 \times 2.5 = 215 \ \text{J/m}^3$$

The volume of the torus is

$$0.05^2 \times 0.3\pi = 2.36 \times 10^{-3} \ \text{m}^3$$

Energy loss in the torus per cycle is thus

$$2.36 \times 10^{-3} \times 215 = 0.507 \ \text{J}$$

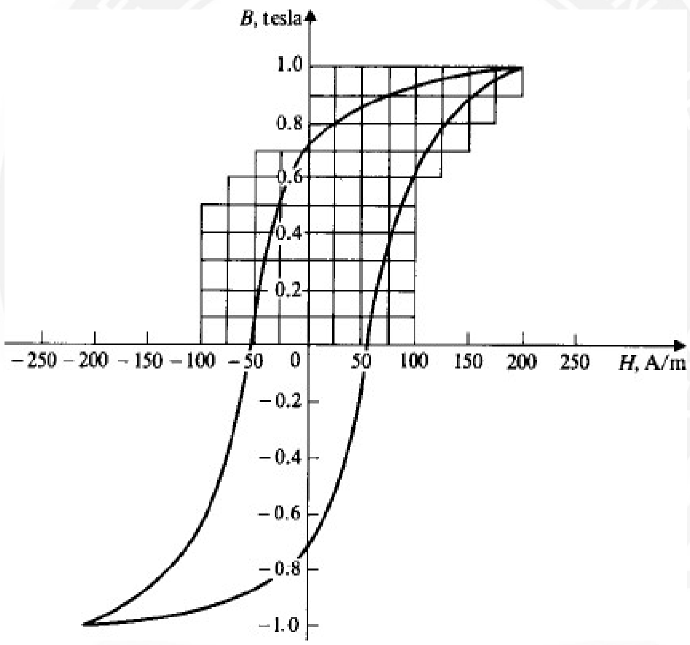


Figure 1.1.2 Hysteresis loop of M-36 steel sheet

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 46]*

A Simple Magnetic Circuit

Consider a simple structure consisting of a current carrying coil of N turns and a magnetic core of mean length lc and a cross sectional area Ac as shown in the diagram below. The permeability of the core material is μ_c. Assume that the size of the device and the operation frequency are such that the displacement current in Maxwell's equations are negligible, and that the permeability of the core material is very high so that all magnetic flux will be confined within the core. By Ampere's law, we can write

$$\oint_C \mathbf{H} \bullet d\mathbf{l} = \oint_S \mathbf{J} \bullet d\mathbf{a}$$

$$H_c l_c = Ni$$

where Hc is the magnetic field strength in the core, and Ni the magneto motive force. The magnetic flux through the cross section of the core can expressed as

$$\phi_c = B_c A_c$$

where ϕ_c is the flux in the core and *B*c the flux density in the core. The constitutive equation of the core material is

$$B_c = \mu H_c$$

Therefore, we obtain

$$\phi_c = \frac{Ni}{l_c/(\mu_c A_c)} = \frac{F}{R_c}$$

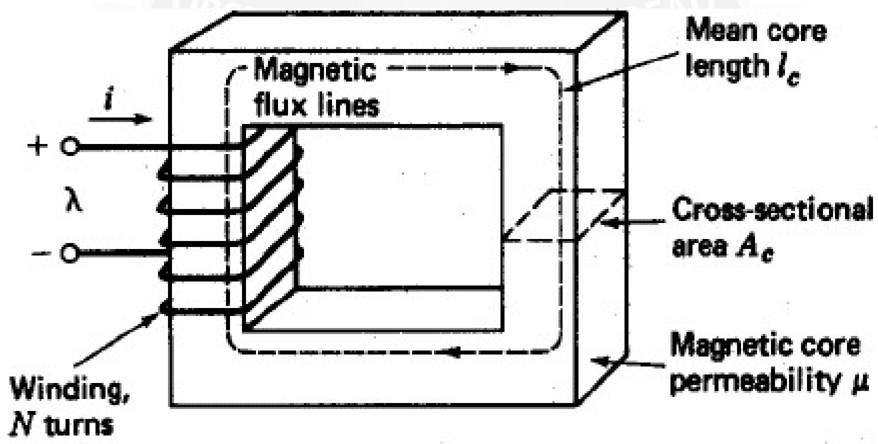
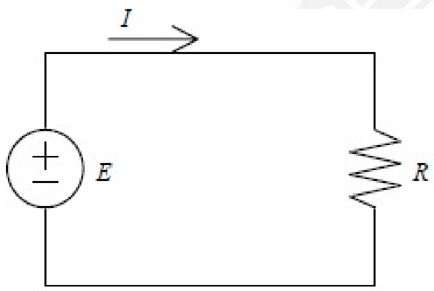


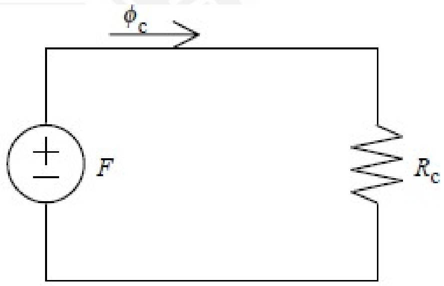
Figure 1.1.3 A Simple Magnetic Circuit

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 49]*

If we take the magnetic flux ϕ_c as the "current", the magneto motive force F=Ni as the "emf of a voltage source", and $R_c = l_c/(\phi_c A_c)$ (known as the magnetic reluctance) as the "resistance" in the magnetic circuit, we have an analog of Ohm's law in electrical circuit theory.

Electric Circuit **Magnetic Circuit**





$$I = \frac{E}{R}$$

$$\phi_c = \frac{F}{R_c}$$

LAWS GOVERNING MAGNETIC CIRCUITS

Consider the magnetic circuit in the last section with an air gap of length lg cut in the middle of a leg as shown in figure (a) in the diagram below. As they cross the air gap, the magnetic flux lines bulge outward somewhat as illustrate in figure (b). The effect of the fringing field is to increase the effective cross sectional area A_g of the air

$$F = Ni = H_c l_c + H_g l_g$$

where
$$H_c l_c = \frac{B_c}{\mu_c} l_c = \frac{\phi_c}{\mu_c A_c} l_c = \phi_c R_c$$

and
$$H_g l_g = \frac{B_g}{\mu_o} l_g = \frac{\phi_g}{\mu_o A_g} l_g = \phi_g R_g$$

gap. By Ampere's law, we can write

According to Gauss' law in magnetics,

$$\oint_S \mathbf{B} \bullet d\mathbf{a} = 0$$

we know

$$\phi_c = \phi_g = \phi$$

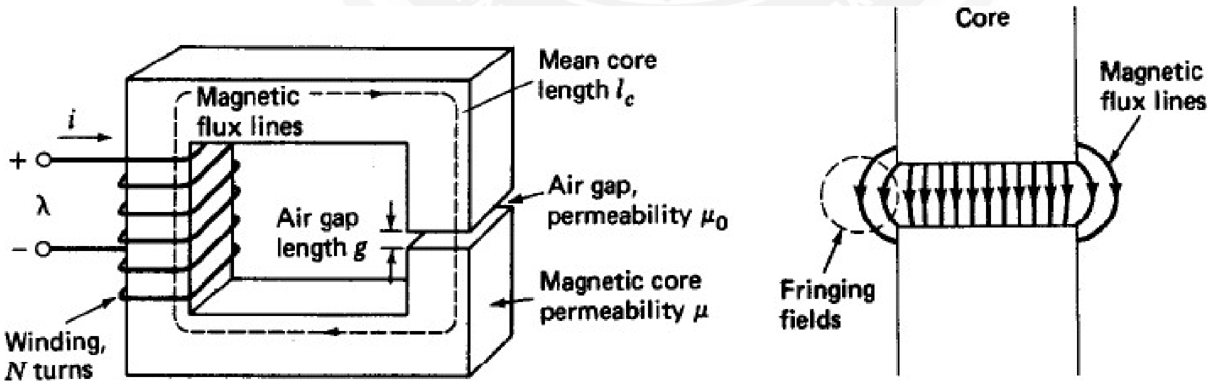


Figure 1.1.4 A Simple Magnetic Circuit with an air gap

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 54]*

Therefore,

$$F = \left(R_c + R_g\right)\phi$$

That is, the above magnetic circuit with an air gap is analogous to a series electric circuit. Further, if we regard $H_c l_c$ and $H_g l_g$ as the "voltage drops" across the reluctance of the core and airgap respectively, the above equation from Ampere's law can be

$$\sum R_k \phi_k = \sum F_k$$

interpreted as an analog to the Kirchhoff's voltage law (KVL) in electric circuit theory, or

The Kirchhoff's current law (KCL) can be derived from the Gauss' law in magnetics. Consider a magnetic circuit as shown below. When the Gauss' law is applied to the T joint in the circuit, we have

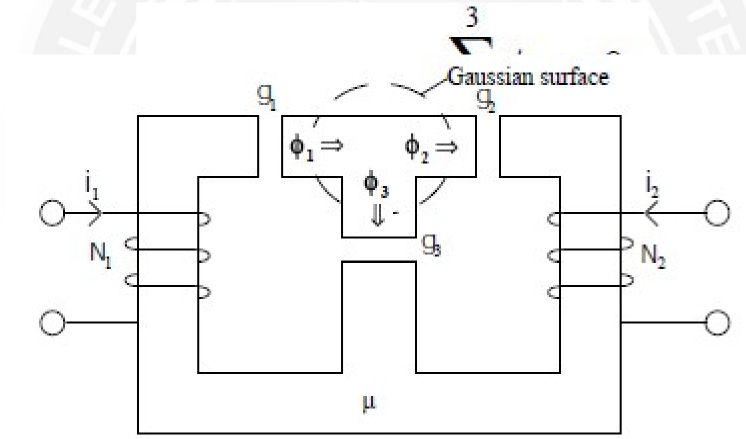


Figure 1.1.5 Magnetic circuit of T joints

[Source: ""*Electric Machinery Fundamentals" by* Stephen J. Chapman*, Page: 56]*

Having derived the Ohm's law, KVL and KCL in magnetic circuits, we can solve very complex magnetic circuits by applying these basic laws. All electrical dc circuit analysis techniques, such as mesh analysis and nodal analysis, can also be applied in magnetic circuit analysis. For nonlinear magnetic circuits where the nonlinear magnetization curves need to be considered, the magnetic reluctance is a function of magnetic flux since the permeability is a function of the magnetic field strength or flux density. Numerical or graphical methods are required to solve nonlinear problems.

1.2 FLUX LINKAGE, INDUCTANCE AND ENERGY

Inductance

Consider a two coil magnetic system as shown below. The magnetic flux linkage of the two coils can be express as

$$\lambda_1 = \lambda_{11} + \lambda_{12} \qquad \text{and} \qquad \lambda_2 = \lambda_{21} + \lambda_{22}$$

where the first subscript indicates the coil of flux linkage and the second the coil

$$L_{jk} = \frac{\lambda_{jk}}{i_k} \qquad (j=1,2 \text{ and } k=1,2)$$

carrying current. By defining the self and mutual inductances of the two coils as

where L_{jk} is the self inductance of the jth coil when $j=k$, the mutual inductance between the j^{th} coil and the k^{th} coil when $j \neq k$, and $L_{jk} = L_{kj}$, the flux linkages can be expressed as

$$\lambda_1 = L_{11}i_1 + L_{12}i_2 \qquad \text{and} \qquad \lambda_2 = L_{21}i_1 + L_{22}i_2$$

The above definition is also valid for a n coil system. For a linear magnetic system, the above calculation can be performed by switching on one coil while all other coils are switched off such that the magnetic circuit analysis can be simplified.

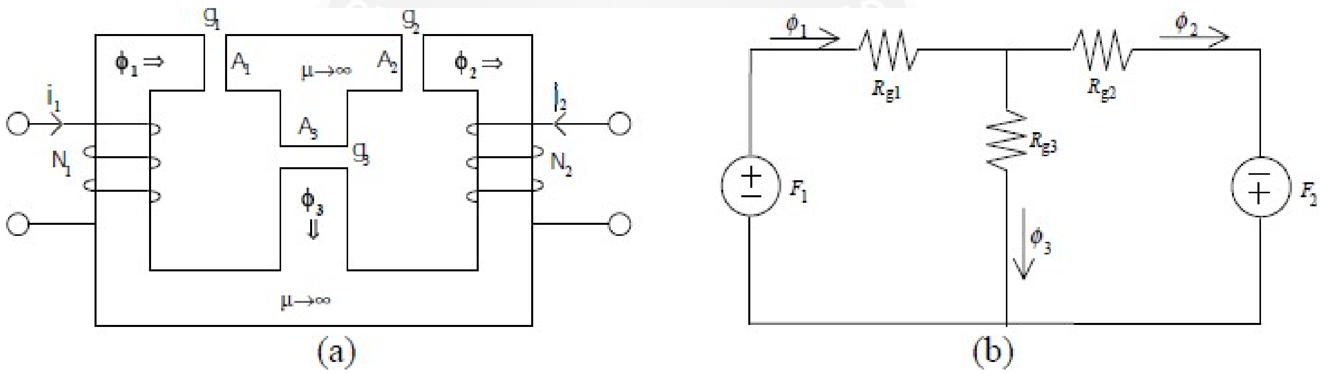


Figure 1.2.1 Magnetic circuit of a two coil system

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 59]*

This is especially significant for a complex magnetic circuit. For a nonlinear magnetic system, however, the inductances can only be calculated by the above definition with all coils switched on.

Electromotive Force

When a conductor of length *l* moves in a magnetic field of flux density **B** at a speed **v**,

$$\mathbf{e} = l\mathbf{v} \times \mathbf{B}$$

the induced electromotive force (*emf*) can be calculated by

For a coil linking a time varying magnetic field, the induced *emf* can be calculated from the flux linkage of the coil by

$$e_k = \frac{d\lambda_k}{dt} = \sum_{j=1}^{n} \frac{d\lambda_{kj}}{dt} = \sum_{j=1}^{n} L_{kj} \frac{di_j}{dt} \qquad (k=1,2,\ldots n)$$

Magnetic Energy

In terms of inductance, the magnetic energy stored in an *n* coil system can be expressed as

$$W_f = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_{jk} i_j = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \frac{\lambda_{jk}\lambda_{kj}}{L_{jk}} = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} L_{jk} i_j i_k$$