

2.5 DIELECTRIC POLARIZATION

Dipole described by its dipole moment \mathbf{P} . If Q is the charge and \mathbf{l} is the vector (distance) from the negative to the positive charge, the dipole moment is given by

$$\mathbf{P} = Q \cdot \mathbf{l}$$

If there are n dipole per unit volume ΔV , then there are $n\Delta V$ dipoles and the total dipole moment is given by

$$\mathbf{P}_{total} = \sum_{i=1}^{n\Delta V} \mathbf{P}_i$$

Polarization is defined as dipole moment per unit volume

$$\mathbf{P} = \lim_{\Delta V \rightarrow 0} \frac{1}{\Delta V} \sum_{i=1}^{n\Delta V} \mathbf{P}_i \quad C/n^2$$

If the dipole is signed in random orientation, polarization \mathbf{P} has zero values, whereas if dipoles are aligned in same direction polarization \mathbf{P} has a significant value.

Consider a bound charge Q_b across a small element surface ds in dielectric containing non polar molecules. Then

$$Q_b = - \oint_s \mathbf{P} \cdot d\mathbf{s}$$

If Q is the free charge enclosed by the surface S , the total enclosed charge is

$$Q_T = Q_b + Q$$

By Gauss's law

$$Q_T = \oint_s \epsilon_0 \mathbf{E} \cdot d\mathbf{s}$$

The free charge enclosed

$$Q = Q_T - Q_b$$

Substitute Q_T and Q_b in above equation

$$Q = Q_T - Q_b$$

$$Q = \oint_s \epsilon_0 \mathbf{E} \cdot d\mathbf{s} - \left(- \oint_s \mathbf{P} \cdot d\mathbf{s} \right)$$

$$Q = \oint_s \epsilon_0 \mathbf{E} \cdot d\mathbf{s} + \oint_s \mathbf{P} \cdot d\mathbf{s}$$

$$Q = \oint_s (\epsilon_0 \mathbf{E} + \mathbf{P}) \cdot d\mathbf{s}$$

But

$$Q = \oint_s \mathbf{D} \cdot d\mathbf{s}$$

Equate both the Q equation

$$\oint_s \mathbf{D} \cdot d\mathbf{s} = \oint_s (\epsilon_0 \mathbf{E} + \mathbf{P}) \cdot d\mathbf{s}$$

$$\mathbf{D} = (\epsilon_0 \mathbf{E} + \mathbf{P})$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

Polarization \mathbf{P} can be written as

$$\mathbf{P} = \chi \epsilon_0 \mathbf{E}$$

Where χ is electric susceptibility

Substitute \mathbf{P} expression in above equation

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \chi \epsilon_0 \mathbf{E}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} (1 + \chi)$$

Substitute

$$\epsilon_R = 1 + \chi$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} \epsilon_R$$

$$\mathbf{D} = \epsilon_0 \epsilon_R \mathbf{E}$$