### 1.9 SIGNAL FLOW GRAPH

The diagrammatic or pictorial representation of a set of simultaneous linear algebraic equations of a more complicated system is known as signal flow graph (SFG). It shows the flow of signals in the system. It is important to note that the flow of signals in SFG is only in one direction. To represent the set of algebraic equations using SFG, it is necessary that those algebraic equations are to be represented in the s-domain. The transfer function of the system which is represented by SFG can be obtained by using Mason's gain formula. The dependent and independent variables in the set of algebraic equations are represented by the nodes in the SFG. The branches are used to connect different nodes present in SFG. The connection between the different nodes is based on the relationship given in the algebraic equation. The arrow and the multiplication factor indicated on the branch of SFG represent the signal direction. The SFG and the block diagram representation of a system yield the same transfer function; but when a system is represented by SFG, the transfer function is obtained easily and quickly without using the SFG reduction techniques. The terminologies used in SFG rae explained with the help of SFG of a system as shown in figure 1.9.1.


Figure 1.9.1 Signal flow graph of a system
[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 4.1]
Node: The variables present in the set of algebraic equations are represented by a point called node.


Figure 1.9.2 Node in signal flow graph
[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 4.2]

## Branch

The line segment joining the two nodes with a specific direction is known as a branch. The specific direction is indicated by an arrow in the branch.


## Figure 1.9.3 Branch in signal flow graph

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 4.2]

## MASON'S GAIN FORMULA

A technique to reduce a signal flow graph to a single transfer function requires the application of one formula. The transfer function of a system represented by a signal flow graph is

$$
T=\frac{C(s)}{R(s)}=\frac{\sum_{i=1}^{n} P_{i} \Delta_{i}}{\Delta}
$$

where, $k$ - number of forward path
$\mathrm{P}_{\mathrm{i}} \quad$ - ith forward path gain
$\Delta \quad-1$ - (sum of individual loop gains)+(sum of product of two non-touching loop gains)-(sum of product of three non-touching loop gains)+......
$\Delta_{\mathrm{i}} \quad-1$ - ( $\Delta$ of the loop non-touching the ith forward path)

## Steps to determine the transfer function of a system using SFG Method

Step 1: Identify the number of forward paths.
Step 2: Identify the individual loops and find their respective loop gains.
Step 3: Identify the two non-touching loops and find the product of their gains.
Step 4: Identify the three non-touching loops and find the gain product and so on...
Step 5: Calculate the $\Delta$ value.
Step 6: Calculate the $\Delta_{\mathrm{i}}$ value.
Step 7: Use Mason's gain formula to calculate the transfer function value, T.

| Characteristics | Block Diagram | Signal flow graph |
| :---: | :---: | :---: |
| Time Consumption | More since the diagrams have to be redrawn repeatedly | Less since there is no necessary to redraw the diagrams |
| Technique applied | Block Diagram reduction technique | Mason's gain formula |
| Representation of elements | Blocks are used to represent the element. | Nodes are need to represent the elements |
| Representation of transfer function of each element | Represented inside the block of each element | Represented along the branches above the arrow ahead |
| Feedback paths | Present and hence the formula, $(\mathrm{G} /(1 \pm \mathrm{GH}))$ is used to reduce the paths | Present, but there is no need for any formulae to reduce the paths |
| Self-loops | Absence of self-loops | Presence of self-loops |
| Summing points and takeoff points | Present in block diagram | Absence in SFG |

