UNITIII

3.1 TORSION

3.1.1 INTRODUCTION:

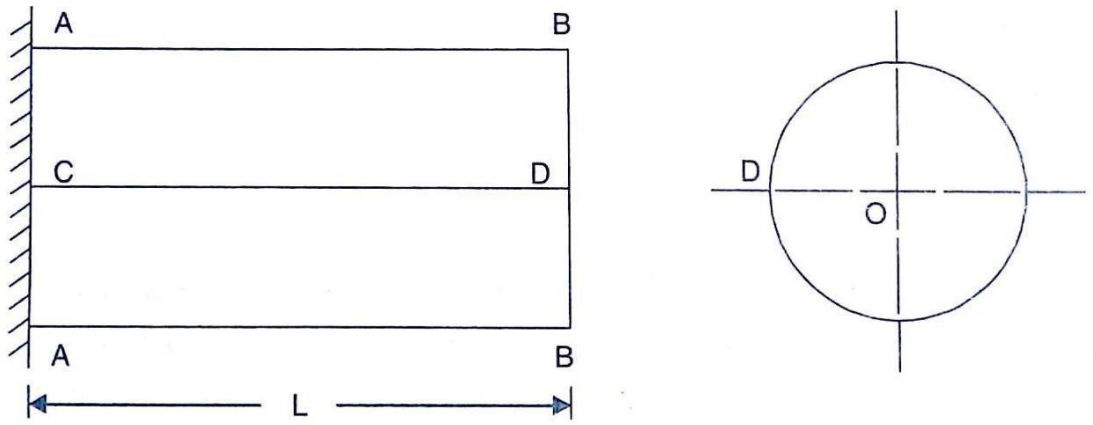
Ashaftissaidtobeintorsion,whenequalandoppositeforcesareappliedatthe two ends of the shaft. The torque is equal to the product of the force applied and radius of the shaft. Due to the application of the force at the ends the shaft is subjected to a twisting moment. This causes the shear stress and shear strains in the material of the shaft.

3.1.2DERIVATIONOFSHEARSTRESSPRODUCEDINACIRCULARSH SUBJECTEDTOTORSION:

Before the derivation of shear stress produced ina circular shaft the followingassumption are to be made as:

Assumption made in the Derivation of Shear Stress Produced in a Circular Shaft Subjected to Torsion:

1. Thematerialoftheshaftisuniformthroughout.
2. Thetwistalongtheshaftis uniform.
3. Theshaftisuniformcircularsectionthroughout.
4. Crosssectionoftheshaft,whichareplanebeforeandaftertwist.
5. Allradiiwhicharestraightbeforeandaftertwist.

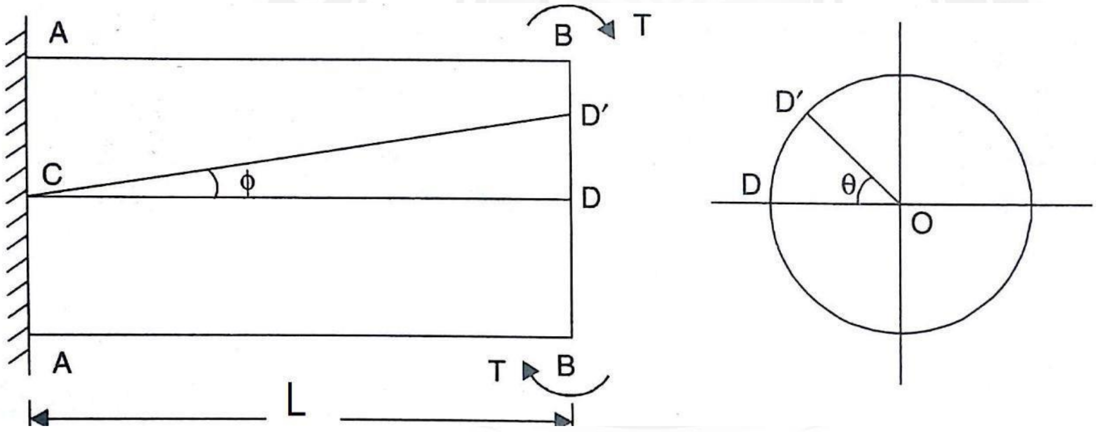


Considerashaftoflength$l$,radiusRfixedatoneendandfreeotherendisfreeis subjected to a torque T as shown in figure.

Let C=Modulusofrigidityofthe material

τ=ShearstressinducedatthesurfaceoftheshaftduetotorqueT

Let 'O' be the centre of the shaft D a point on surface and AB be the line on the shaftparallel to the axis of the shaft.



When the shaft is subjected to torque T then D is moved to D^l. If 'ϕ' be the shear strain and 'θ' be the angle of twist in length l then

Shear strain ϕ =Distortionperunitlength= $\frac{\text{Distortion at the outer surface}}{\text{Length of the shaft}}$ ϕ

$= \frac{DD^1}{l}$

Then $DD^l = l\,\phi$

But,ArclengthDD^l $= R\theta = l\,\phi$ ....(3.1)

Shearstrainϕ $= \frac{\text{Shear stress}}{\text{Modulus of rigidity}} = \frac{\tau}{C}$ ……………...(3.2)

Substituteϕvalueineqn3.1

$$= R\,\theta = l\frac{\tau}{C}$$

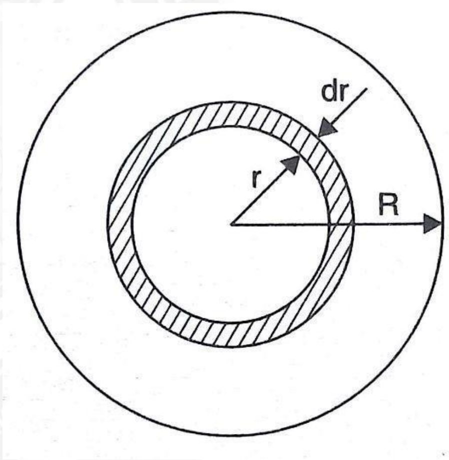
$$= \frac{C\theta}{l} = \frac{\tau}{R} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.3)$$

3.1.3STRENGTH(OR)MAXIMUMTORQUETRANSMITTEDBYA CIRCULAR SOLID SHAFT

Thestrengthof ashaft means themaximumtorqueormaximumpowertheshaft can transmit.

Themaximumtorquetransmittedbyacircularshaftisobtainedfromthemaximumshear stress induced at the outer surface of the solid shaft. ie., τ α R

ConsiderashaftsubjectedtoatorqueT.Alsoconsiderasmallelementarycircularring ofthicknessdratadistancerfromthecenteras showninfigure.



Letτ = Shear stress induced at the surface of theshaft due to torque T

R=Radius oftheshaft

q=shearstressattheradius’r’fromthe centre.

TheareaoftheringdA =2πrdr

Ifqistheshearstressinducedat aradius rfromthecentreoftheshaft then

$$\frac{q}{r} = \frac{\tau}{R}$$

∴Shearstressattheradiusr,

$$q = \frac{\tau}{R} r$$

Tuningforceontheelementarycircular ring

=Shearstressactingontheringxareaofring

= q xdA

$$= \frac{\tau}{R} x r x 2\pi r dr = \frac{\tau}{R} x\, 2\pi r^2 dr$$

Now tuning moment due to tuning force on the elementary circular ring

dT = Tuning force x distance of the ring from axis

$= \frac{\tau}{R} x 2\pi r^2 dr x r = \frac{\tau}{R} x 2\pi r^3 dr$

Nowthetotalturningmomentortorqueontheshaftisobtainedbyintegratingtheabove eqn. ( ) between the limit 0 to R

$$T = \int_o^R dT = \int_0^R \frac{\tau}{R} 2\pi r^3 dr$$

$$= \frac{\tau}{R} x\ 2\pi \int_0^R r^3 dr$$

$$= \frac{\tau}{R} x\ 2\pi \left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{\tau}{R} x\ 2\pi\ x\ \frac{R^4}{4}$$

$$= \frac{\tau}{2} x\ \pi R^3$$

$$= \frac{\tau}{2} x\ \pi\ x \left(\frac{D}{2}\right)^3 \qquad \left(R = \frac{D}{2}\right)$$

$$= \frac{\tau}{2} x\ \pi\ x\ \frac{D^3}{8}$$

$$T = \frac{\pi}{16} \tau D^3 \qquad \text{……………..(3.6)}$$

DERIVETHETORSIONALEQUTION:

Fromtheeqn(3.3)weknow that

$$\frac{C\theta}{l} = \frac{\tau}{R}$$

Butfromtorquetransmissiononashaftofeqn.(3.6)

$$\tau = \frac{T\ x\ 16}{\pi\ x D^3}$$

Substitutetheτvalueintheeqn(3.3)

$$\frac{C\theta}{l} = \frac{T\ x\ 16}{\frac{D}{2} x\ \pi\ x\ D^3}$$

$$\frac{C\theta}{l} = \frac{T}{\frac{\pi}{32} D^4}$$

Where $\frac{\pi}{32} D^4$ isthe**polarmomentofinertia(J)** ofthesolidshaft.Thentheabove equation become $\frac{C\theta}{l} = \frac{T}{J}$ ……………….3.10

Similarly polarmomentofinertiaofhollow circularshaft = $\frac{\pi}{32}(D^4 - d^4)$

WhereD=outerdiameterandd=innerdiameterofhollowshaft From eqn. (3.3) and eqn. (3.10)

$$\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$$

TORQUETRANSMITTEDBYAHOLLOWCIRCULARSHAFT:

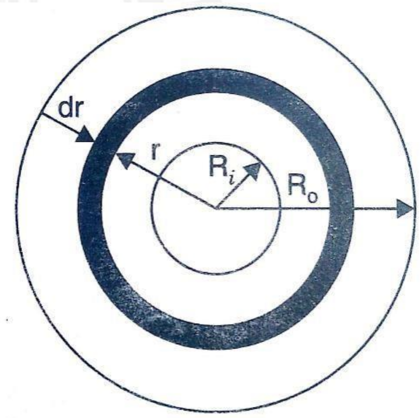
Consider a hollow circular shaft of outer and inner radius are Ro and Ri is subjected to a torque T. Take an elementary circular ring of thickness ‘dr’ at a distance r from the centre as shown in figure

Letq=shearstressinducedontheelementaryring.

dA=2πrdrareaoftheelementarycircularringshear stress at the elementary ring is obtained from shear stress ratio

$$\frac{q}{r} = \frac{\tau}{R_o}$$

$$q = \frac{\tau}{R_o} r$$



Tuningforceontheelementarycircular ring

=Shearstressactingontheringxareaofring

= q xdA

$= \frac{\tau}{R_o}$x r x 2πrdr $= \frac{\tau}{R_o}$x 2πr²dr

Nowtuningmomentduetotuningforceontheelementarycircularring dT = Tuning force x distance of the ring from axis

$= \frac{\tau}{R_o}$x2πr²drxr x

$= \frac{\tau}{R_o} 2\pi r^3 dr$

Nowthetotalturningmomentortorqueontheshaftisobtainedbyintegratingtheabove eqn. between the limit Rito Ro

$$T = \int_{R_i}^{R_o} dT = \int_{R_i}^{R_o} \frac{\tau}{R_o} \times 2\pi r^3 dr$$

$$= \frac{\tau}{R_o} \times 2\pi \int_{R_i}^{R_o} r^3 dr$$

$$= \frac{\tau}{R_o} \times 2\pi \left[\frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$= \frac{\tau}{R_o} \times 2\pi \text{ x } \left[\frac{{R_o}^4 - {R_i}^4}{4}\right]$$

$$= \frac{\tau}{2} \times \pi \times \left[\frac{{R_o}^4 - {R_i}^4}{R_o}\right]$$

$$= \frac{\tau}{2} \times \pi \times \left[\frac{\left(\frac{D_o}{2}\right)^4 - \left(\frac{D_i}{2}\right)^4}{\frac{D_o}{2}}\right] \qquad \left(R_o = \frac{D_o}{2} \ ; R_i = \frac{D_i}{2}\right)$$

$$= \frac{\tau}{2} \times \pi \times \left[\frac{{D_o}^4 - {D_i}^4}{16} \text{ x } \frac{2}{D_o}\right]$$

$$T = \frac{\pi}{16} \tau\left[\frac{{D_o}^4 - {D_i}^4}{D_o}\right]$$

TORSIONALRIGIDITY:

Theproductofmodulusofrigidityandpolarmomentofinertiaofacircularshaft is known as torsional rigidity. It is denoted by (K).

Torsionalrigidity(K)=CxJ

Torsionalrigidityisalsodefinedasthetorquerequiredtoproduceatwistofradian per unit length of the shaft.

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$ gives $CJ = \frac{Tl}{\theta}$

If l = 1metre and θ =1radianthenTorsional rigidity =Torque

SinceC,Jand$l$areconstantforagivenshaft,theangleoftwistθisdirectlyproportional to the torque (T). The term CJ is known as torsional rigidity.

POLARMODULUS:

Itistheratiobetweenthepolarmomentofinertiaandtheradiusoftheshaft.Itis denoted by (Z).Its unit is mm^3. It is also called torsional section modulus.

PolarModulus$(Z) = \frac{\text{Polar moment of inertia}}{Radius\ of\ shaft} = \frac{J}{R}$

For solid shaft $(J) = \frac{\pi}{32} D^4$ then $Z = \frac{\frac{\pi}{32} D^4}{\frac{D}{2}} = \frac{\pi}{16} D^3$

For hollow shaft $(J) = \frac{\pi}{32} (D^4 - d^4)$ then$Z = \frac{\frac{\pi}{32}(D^4 - d^4)}{\frac{D}{2}} = \frac{\pi}{16}\left[\frac{D^4 - d^4}{D}\right]$

2

POWERTRANSMITTEDBYSHAFT:

Once the torque (T) for a solid or a hollow shaft is obtained, power transmitted by the shaft can be determined,

Let N=Speedofshaftinrpm

T = mean torque transmitted in Nm Power P = $\frac{2\pi N T_{mean}}{60}$ Watts or P = T x ω where ω = $\frac{2\pi N}{60}$

SAVINGOFMATERIALANDWEIGHTOFSOLIDANDHOLLOW SHAFT:(i)PercentageofsavinginMaterial:

$$= \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}} \text{ x } 100$$

Let D=diameterofsolidshaft

l_S=lengthofsolidshaft

ρ_S=densityofsolidshaft

l_H=lengthofhollowshaft

D_H=Outerdiameterofhollowshaft

d_H=innerdiameterofhollowshaft

ρ_H=densityofHollowshaft

$$= \frac{\frac{\pi \times D^2}{4} - \frac{\pi \times (D_H{}^2 - d_H{}^2)}{4}}{\frac{\pi \times D^2}{4}} \times 100 \quad = \frac{D^2 - (D_H{}^2 - d_H{}^2)}{D^2} \times 100$$

(ii)Percentageofsavinginweight:

$$= \frac{\text{Weight of solid shaft} - \text{Weight of hollow shaft}}{\text{Weight of solid shaft}} \times 100$$

Weightofsolidshaft=densityxvolume=densityxareaxlength

$$= \rho_S \times \frac{\pi \times D^2}{4} \times l_S$$

Weightofhollowshaft=$\rho_H \times \frac{\pi \times (D_H{}^2 - d_H{}^2)}{4} \times l_H$Then

Percentage of saving in material

$$= \frac{\rho_S \times \frac{\pi \times D^2}{4} \times l_S - \rho_H \times \frac{\pi \times \left(D_H{}^2 - d_H{}^2\right)}{4} \times l_H}{\rho_S \times \frac{\pi \times D^2}{4} \times l_S} \text{ x100}$$

Forsamematerialandsamelengthρ_S=$\rho_H$andl_S=$l_H$then Percentage of saving in material = $\frac{D^2 - \left(D_H{}^2 - d_H{}^2\right)}{D^2}$ x 100

Problem4.1.1 Asolidshaftofistotransmitatorqueof25kNm.Iftheshearingstressis not to exceed 60 Mpa. Find the minimum diameter of the shaft.

Given Data:

TorquetransmittedT=25kNm=25 ×10^6Nmm

Shear stress τ = 60 Mpa = 60 N/mm^2**To find:**

DiameteroftheshaftD=?

Solution:

WktTorque transmitted T =$\frac{\pi}{16} D^3$thenD =$\sqrt[3]{\frac{T \times 16}{\pi}}$

$$= \sqrt[3]{\frac{25 \times 10^6 \ x \ 16}{\pi}} =$$ **128.5mm**

Result:

DiameteroftheshaftD=**128.5mm**

Problem 4.1.2. A hollow circular shaft of external diameter 50mm and internal diameter 40mm transmit a torque of 10 kNm. Find the maximum shear induced in the shaft.

Given Data: 6Nmm

ExternaldiameTorquetransmittedT=10kNm=10erD= 50mm×10 Internal diameterd = 40mm

Tofind:

Shearstressτ=?

Solution:

Wkt TorquetransmittedT $= \frac{\pi}{16}\ \tau\ \left[\frac{D^4-d^4}{D}\right]$

$10\times10^6 = \frac{\pi}{16}\ \tau\ \left[\frac{50^4-40^4}{50}\right]$

$\boldsymbol{\tau}$**= 690.1N/mm²**

Result:

Shearstress τ=**690.1N/mm²**

Problem 4.1.3. Find thepower that can betransmitted by ashaft of50mmdiameter at a speed of 120 rpm. If the shear stress is 60 N/mm² **Given Data:**

Diameter D=50mm

Speed N=120rpm

Shearstress τ=60N/mm² **To find:**

Power P =?

Solution:

Wkt Power transmittedP $= \frac{2\pi N T_{mean}}{60}$

But $T_{mean} = \frac{\pi}{16}\ \tau\ D^3 = \frac{\pi}{16}\ 60\ \times 50^3$

T_{mean} =1472621.5Nmm =1.472× 10³Nm

Then P $= \frac{2\times\pi\ \times 120\ \times 1.472\times10^3}{60} =$ 18476.5W

=**18.476kW**

Result:

PowerP=**18.476 kW**

Problem 4.1.4. A solid circular shaft transmits85kW power at 200 rpm. Find the shaft diameter if the shear stress is 50 MN/m².

Given Data:

Power P =85kW=85× 10³W

Speed N=200rpm

Shearstress τ=50MN/m^2=50N/mm^2**To find:**

ShaftdiameterD=?

Solution:

Wkt PowertransmittedP $= \frac{2\pi NT}{60}$

Then $85 \times 10^3 = \frac{2\times\pi\times200\times T}{60}$

T =4.05×10^{3}Nm =4.05×10^{6}Nmm

ButWkt, $T = \frac{\pi}{16}\tau D^3 = \frac{\pi}{16}\times 50 \times D^3$

D =**74.4mm**

Result:

ShaftdiameterD=**74.4mm**

Problem4.1.5. Ahollowshaftistotransmit200kWat80rpm.Iftheshearstressisnot toexceed 70MN/m^2.and internal diameteris0.5oftheexternal diameter.Find the external and internal diameters assuming that maximum torque is 1.6 times the mean. **Given Data:**

Power P=200kW=200×10^{3}W

Internal Diameter d = 0.5 D

Speed N=80rpm

Shear stress τ=70MN/m^2=70×10^{6}N/m^2=70N/mm^{2}Max.

Torque T_{max}= 1.6 T_{mean}**To find:**

Externalandinternaldiameter D,d=?

Solution:

Wkt PowertransmittedP $= \frac{2\pi NT}{60}$

$200 \times 10^3 = \frac{2\times\pi\times80\times T}{60}$

T=T_{mean}=23.87×10^{3}Nm=23.87×10^{6}Nmm

Ingivendata T_{max}=1.6T_{mean}

Then T_{max}=1.6×23.87×10^6=38.19×10^{6}Nmm

But $T_{max} = \frac{\pi}{16}\ \tau \left[\frac{D^4 - d^4}{D}\right]$

$$T_{max} = \frac{\pi}{16}\ \tau \left[\frac{D^4 - (0.5\ D)^4}{D}\right] \quad (\because d = 0.5D)$$

$$38.19 \times 10^6 = \frac{\pi}{16} \times 70 \times D^4 \left[\frac{1 - 0.5^4}{D}\right]$$

D=143.6mm

⇒ d=0.5×143.6=71.82mm

Result:

OuterdiameterD=**143.6mm**

Innerdiameterd=**71.82mm**

Problem4.1.6. Findthemaximumtorquethatcanbesafelyappliedtoashaftof120mm diameter. If the allowable twist is 3° in a length of 1.5m. Take C = 1 ×^{5} N/mm10 ^{2}

Given Data:

Diameter D=120mm

Angle of twist $\theta = 3^o = 3 \times \frac{\pi}{180} = 0.05$rad

Length l = 1.5m = 1.5× 10^3 mm

ModulusofrigidityC=1×10^5N/mm^2**Tofind:**

MaximumtorquetransmittedT=?

Solution:

Fromthetorsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J =polar moment ofinertia $= \frac{\pi}{32} \times D^4$

$$J = \frac{\pi}{32} \times 120^4 \quad = 20.3 \times 10^6 mm^4$$

SubstituteJvalueinthetorsionequation,then

$$T = \frac{1 \times 10^5 \times 0.05}{1.5 \times 10^3} \times 20.3 \times 10^6\ T$$

$$= 67.6 \times 10^6 \text{ Nmm}$$

Result:

TorquetransmittedT=**67.6×10⁶Nmm**

Asolidshaftofdiameter100mmisrequiredtotransmit150kWat 120 rpm.Ifthelength oftheshaft is4m and modulus ofrigidity forshaft is 75 Gpa,find the angle of twist.

Given Data:

Diameter D =100mm

Power P=150kW=150×10^3W

Speed N =120rpm

Length *l*= 4m =4×10^3mm

ModulusofrigidityC=75Gpa=75×10^9pa=75×10^9N/m^2

= 75×10^3N/mm^2

Tofind:

Angleoftwist θ =?

Solution:

Fromthetorsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, *J* =polar moment ofinertia =$\frac{\pi}{32} \times D^4$

$$J = \frac{\pi}{32} \times 100^4 = 9.81\times 10^6 \text{ mm}^4$$

Wkt, PowertransmittedP $= \frac{2\pi NT}{60}$

$$150 \times 10^3 = \frac{2\times\pi\times100\times T}{60}$$

≫ T=11.93×10^3Nm=11.93×10^6Nmm

SubstituteJandTvalueinthetorsionequation,then

$$\frac{11.93\times10^6}{9.81\times10^6} = \frac{75\times10^6\times\theta}{4\times10^3}$$

≫ θ=0.06rad

$$\theta=0.06\times \frac{180}{\pi} = 3.7^o$$

Result:

Angleoftwistθ=**3.7º**

A hollow shaft of 120mm external diameter and 80mm internal diameter is required to transmit 200kWat 120 rpm. Ifthe angleof twist is not to exceed 3^o find the length of the shaft. Take modulus of rigidity for shaft is 80 Gpa.

Given Data:

ExternalDiameterD=120mm

Internal diameter   d = 80 mm

Power   P=200kW=200×10^3W

Speed   N =120rpm

Angleoftwist   θ= $3^o = 3 \times \frac{\pi}{180} =$ 0.05rad

Modulusofrigidity C=80Gpa=80×10^9pa=80×10^9N/m^2

=80×10^3N/mm^2**To**

find:

Lengthofshaft l = ?

Solution:

Fromthetorsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J =polar moment ofinertia $=\frac{\pi}{32} \times [D^4 - d^4]$

$$J = \frac{\pi}{32} \times [120^4 - 80^4]$$

$$=16.3 \times 10^6 \quad mm^4$$

Wkt,   PowertransmittedP $= \frac{2\pi NT}{60}$

$$200 \times 10^3 = \frac{2\times\pi\times120\times T}{60}$$

≫   T=15.93×10^3Nm=11.93×10^6Nmm

SubstituteJandTvalueinthetorsionequation,then

$$\frac{15.93\times10^6}{16.3\times10^6} = \frac{80\times10^3\times0.05}{l}$$

≫ l=4264.6 mm

Result: Length ofshaft$l$=**4264.6 mm**

:Findthemaximumtorquethatcanbesafelyappliedtoashaftof120mmdiameter. Thepermissibleshearstressandallowabletwistare200N/mm^{2}and3^orespectively.Take C = 75Gpa and length of shaft = 4m.

Given data:

Diameter   D =120mm

Shearstress   τ=200N/mm^2

Angleoftwist   θ= $3^o = 3 \times \frac{\pi}{180} = 0.052$ rad

Modulusofrigidity C=75Gpa=75×10^9pa=75×10^9N/m^2

= 75×10^3N/mm^2

Lengthofshaft l=4m =4 $\times10^3$mm

Tofind:

MaximumtorqueT_{max}=?

Solution:

Consider based on shear stress

Torque $T = \frac{\pi}{16} \tau D^3$

$= \frac{\pi}{16} \times 200 \times 120^3$=$67.8\times10^6$Nmm

Consideringangleoftwist

Fromthetorsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J = polar moment of inertia =$\frac{\pi}{32} \times D^4 = \frac{\pi}{32} \times 120^4 = 20.35 \times 10^6$ mm^4

$$\frac{T}{20.35 \times10^6} = \frac{75 \times10^3 \times0.052}{4 \times10^3}$$

Then torsion equation become

Then T =19.8×10^6Nmm

Fromtheabovetwotorquevaluewehaveto findthemaximumvaluethatcanbesafely applied on the shaft is take the minimum value as 19.8×10^6 Nmm.

Result:

MaximumtorqueT_{max}=**19.8×10^6Nmm**

:Asolidcircularshafttransmit70kWpowerat175rpm.Calculatethe shaft diameter if the twistin the shaft is not to exceed 2^o in 2 meter length of shaft andshear stress is limited to $50^3\times$kN/m10 ^{2}. Take C = 100×10^3MN/m^2.

Givendata:

Power P=70kW=70×10^3W

Speed N =175rpm

Angle of twist $\theta=2^o= 2\times \frac{\pi}{180} = 0.034$rad

Length of shaft l = 2m = 2×10^3 mm

Shearstress $\tau=50\times10^3$kN/m^2=50N/mm^2

ModulusofrigidityC=100 $\times10^3$MN/m^2=100$\times 10^9$N/m^2

$=100 \times 10^3 N/mm^2$

Tofind:

Diameterofshaft D=?

Solution:

Wkt,Power $P = \frac{2\pi NT}{60}$

$$70 \times 10^3 = \frac{2 \times \pi \times 175 \times T}{60}$$

≫ Torque $T = 3.81 \times 10^3 Nm = 3.81 \times 10^6 Nmm$

Considerbasedonshearstress

Torque $T = \frac{\pi}{16} \tau D^3$

$$3.81 \times 10^6 = \frac{\pi}{16} \times 200 \times D^3$$

≫ D=72.9 mm

Consideringangleoftwist

Fromthetorsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where,J=polarmomentofinertia=$\frac{\pi}{32} \times D^4 = 0.098 D^4$Then torsion equation become

$$\frac{3.81 \times 10^6}{0.098 D^4} = \frac{100 \times 10^3 \times 0.034}{2 \times 10^3}$$

Then D=69.12 mm

Fromtheabovetwocases,wehavetofindthesuitablediameterfortheshaftistakethe greatest value as 72.9 = 73 mm.

Result:

Shaftdiameter**D=73mm**

Problem4.1.11: Ahollowshaftistotransmit300kWpowerat80rpm.Iftheshearstress isnottoexceed50MN/m^2anddiameterratiois3/7.Findtheexternalandinternaldiameter if the twist of shaft is 1.2^o in 2 meter length. Assuming maximum torque is 20% greater than mean. Take C = 80 GN/m^2.

Given data:

Power P = 300kW=300×10^3W

Speed N=80rpm

Shear stress τ=50MN/m^2=50N/mm^2

Diameter ratio d/D = 3/7≫d = 0.428D

Angle of twist θ=1.2° = 1.2× $\frac{\pi}{180}$ = 0.020 rad

Length of shaft l = 2m = 2 × 10^3 mm

MaximumtorqueT$_{max}$=20%greaterthanT_{mean}=(100% +20%)T_{mean}

=1.2T_{mean}

ModulusofrigidityC=ModulusofrigidityC=80GN/m^2=80× 10^9N/m^2

= 80× 10^3N/mm^2

Tofind:

Externalandinternaldiameterofshaft D,d =?

Solution:

Wkt,Power P = $\frac{2\pi NT}{60}$

$$300 \times 10^3 = \frac{2\times\pi\times80\times T}{60}$$

≫ TorqueT=T_{mean}=35.8×10^3Nm=35.8×10^6Nmm

Inourdata,T$_{max}$=1.2T_{mean}=1.2×35.8×10^6=42.96×10^6Nmm Consider based on shear stress

Torque $T_{max} = \frac{\pi}{16}\,\tau\,\left[\frac{D^4-d^4}{D}\right]$

$$42.96\times 10^6 = \frac{\pi}{16}\times 50\times\left[\frac{D^4-(0.428D)^4}{D}\right]$$

$$42.96\times 10^6 = \frac{\pi}{16}\times 50\times D^3[1-(0.428)^4]$$

≫ D=165.4mm,thend=0.428×165.4=70.8mm

Consideringangleoftwist

Fromthetorsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where,J=polarmomentofinertia=$\frac{\pi}{32}\times[D^4-d^4] = \frac{\pi}{32}\times[D^4-(0.428D)^4]$Then

$= \frac{\pi}{32}\times D^4[1-(0.428)^4] = 0.095D^4$

torsion equation become

$$\frac{42.96\times10^6}{0.095D^4}=\frac{80\times10^3\times0.02}{2\times10^3}$$

≫ D=154.21mm,thend=0.428×154.21=66mm

Fromtheabovetwocases,wehavetofindthesuitablediameterfortheshaftistakethe greatest value as

ExternaldiameterD=165.4mmandInternaldiameterd=70.8mm**Result:**

ExternaldiameterofshaftD=**165.4mm**

Internaldiameterofshaftd=**70.8mm**

4.10REPLACINGSHAFTPROBLEMS:

Problem4.1.12: Asolidshaftof50mmdiameteristobereplacedbyahollowsteelshaft whoseinternaldiameteris0.5timesouterdiameter.Findthediameterofthehollowshaft andpercentageofsavinginweightandmaterial,themaximumshearingstressbeingthe same.

Given data:

SolidshaftdiameterD=50mm

Hollowshaftinternaldiameterd=0.5D_H

Shearstress τ=sameforsolidandhollowshaft**To find:**

Externalandinternaldiameterofshaft D_H,d =?

Solution:

Wkt,thetorquetransmittedbythesolidshaftshouldbeequaltothathollowshaftwhen solid shaft is replaced by hollow shaft.

Torquetransmittedbythesolid shaft

$$T=\frac{\pi}{16}\ \tau\ D^3=\frac{\pi}{16}\times\tau\times50^3=24543.6\ \tau\ \text{Nmm}$$

Torquetransmittedbythesolid shaft

$$T=\frac{\pi}{16}\ \tau\left[\frac{{D_H}^4-d^4}{D_H}\right]$$

$$=\frac{\pi}{16}\times\tau\times\left[\frac{{D_H}^4-(0.5D_H)^4}{D_H}\right]$$

$$=\frac{\pi}{16}\times\tau\times{D_H}^3[1-(0.5)^4]$$

≫ T=0.184 $\tau {D_H}^3$Nmm,

Toquetransmittedinbothshaftsaresame,

≫ $24543.6\tau = 0.184\tau D_H^3$

Basedongivendatashearstressaresame(notgivenassumesame), then

≫ $D_H = \sqrt[3]{\frac{24543.6\ \tau}{0.184\ \tau}} =$ 51.09 mm

Nowinternaldiameterofhollowshaftd=0.5D_H=0.5×51.09=25.54mm Percentage of saving in weight =

$$= \frac{\text{Weight of solid shaft} - \text{Weight of hollow shaft}}{\text{Weight of solid shaft}} \times 100$$

Weightofsolidshaft=density ×Area×length

$$= \rho_S \times \frac{\pi \times D^2}{4} \times l_S = \rho_S \times \frac{\pi \times 50^2}{4} \times l_S$$

$= 1963.4 \rho_S . l_S$ Weightofhollowshaft= $\rho_H \times \frac{\pi \times (D_H^2 - d^2)}{4} \times l_H$

$$= \rho_H \times \frac{\pi \times (51.09^2 - 25.54^2)}{4} \times l_H = 1537.5\ \rho_H . l_H$$

≫% of saving in weight $= \frac{1963.4\ \rho_S . l_S - 1537.5\ \rho_H . l_H}{1963.4\ \rho_S . l_S} \times 100$

For same material and same length $\rho_S = \rho_H . l_H = l_S$ then

≫% of saving in weight $= \frac{1963.4 - 1537.5}{1963.4} \times 100 =$ 21.7%

Percentage of saving in material =

$$= \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}} \times 100$$

Area of solid shaft= $\frac{\pi \times D^2}{4} = \frac{\pi \times 50^2}{4} = 1963.4\text{mm}^2$

Area of hollow shaft = $\frac{\pi \times (D_H^2 - d^2)}{4} = \frac{\pi \times (51.09^2 - 25.54^2)}{4} = 1537.5\text{mm}^2$

≫%ofsavinginmaterial $= \frac{1963.4 - 1537.5}{1963.4} \times 100 =$ 21.7%

Result:

ExternaldiameterofhollowshaftD_H=**51.09mm**

Internaldiameterofhollowshaftd=**25.54mm**

%ofsavinginweight =**21.7%**

%ofsavinginmaterial =**21.7%**