Symmetrical components

Introduction

Installment 3 of these notes dealt primarily with networks that are balanced, in which the three voltages (and three currents) are identical but for exact 120◦ phase shifts. Unbalanced conditions may arise from unequal voltage sources or loads. It is possible to analyze some simple types of unbalanced networks using straightforward solution techniques and wye-delta transformations. However, power networks can be come quite complex and many situations would be very difficult to handle using ordinary network analysis. For this reason, a technique which has come to be called symmetrical components has been developed.

Symmetrical components, in addition to being a powerful analytical tool, are also conceptually useful. The symmetrical components themselves, which are obtained from a transformation of the ordinary line voltages and currents, are useful in their own right. Symmetrical components have become accepted as one way of describing the properties of many types of network elements such as transmission lines, motors and generators

Symmetrical components of a 3 phase system

In a 3 phase system, the unbalanced vectors (either currents or voltage) can be resolved into three balanced system of vectors.

They are Positive sequence components

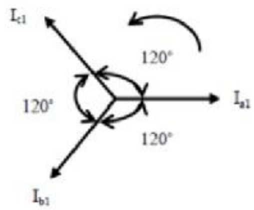
Negative sequence components

Zero sequence components

Unsymmetrical fault analysis can be done by using symmetrical components.

Positive sequence components

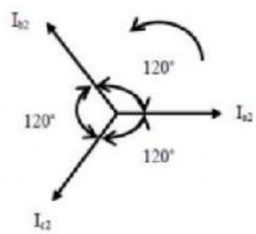
It consists of three components of equal magnitude, displaced each other by 120˚ in phase and having the phase sequence abc .



$$I_{a1} = I_{a1}\angle 0^\circ$$
$$I_{b1} = I_{a1}\angle 240^\circ = I_{a1}\angle -120^\circ$$
$$I_{c1} = I_{a1}\angle 120^\circ$$

Negative sequence components

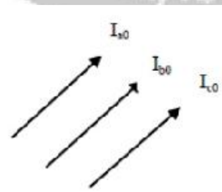
It consists of three components of equal magnitude, displaced each other by 120˚ in phase and having the phase sequence acb .



$$I_{a2} = I_{a2}\angle 0^\circ$$
$$I_{b2} = I_{a2}\angle 120^{\circ\circ}$$
$$I_{c1} = I_{a2}\angle 240^\circ = I_{a2}\angle -120^\circ$$

Zero sequence components

It consists of three phasors equal in magnitude and with zero phase displacement from each other.



$$I_{a0} = I_{b0} = I_{c0}$$

Sequence operator:

In unbalanced problem, to find the relationship between phase voltages and phase currents, we use sequence operator ‘a’.

a = 1∠120˚ == - 0.5+j0.86

$$a^2 = 1\angle 240^\circ = -0.5 - j0.866$$

$$1 + a + a^2 = 0$$

Unbalanced currents from symmetrical currents

Let, Ia, Ib, Ic be the unbalanced phase currents

Let, Ia0, Ia1, Ia2 be the symmetrical components of phase a

$$\begin{bmatrix} I_a \\ I_b \\ I_b \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Determination of symmetrical currents from unbalanced currents.

Let, Ia, Ib, Ic be the unbalanced phase currents

Let, Ia0, Ia1, Ia2 be the symmetrical components of phase a

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_b \end{bmatrix}$$

The Symmetrical Component Transformation

The basis for this analytical technique is a transformation of the three voltages and three currents

In to a second set of voltages and currents. This second set is known as the symmetrical components.

Working in complex amplitudes:

$$
\begin{aligned}
v_a &= Re\left(\underline{V}_a e^{j\omega t}\right) \\
v_b &= Re\left(\underline{V}_b e^{j(\omega t - \frac{2\pi}{3})}\right) \\
v_c &= Re\left(\underline{V}_c e^{j(\omega t + \frac{2\pi}{3})}\right)
\end{aligned}
$$

The transformation is defined as:

$$
\begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix}
$$

where the complex number a is:

$$
\begin{aligned}
\underline{a} &= e^{j\frac{2\pi}{3}} && = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\
\underline{a}^2 &= e^{j\frac{4\pi}{3}} = e^{\ j\frac{2\pi}{3}} && = -\frac{1}{2} - j\frac{\sqrt{3}}{2} \\
\underline{a}^3 & && = 1
\end{aligned}
$$

This transformation may be used for both voltage and current, and works for variables in

ordinary form as well as variables that have been normalized and are in per-unit form. The inverse

of this transformation is:

$$
\begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \underline{a}^2 & \underline{a} & 1 \\ \underline{a} & \underline{a}^2 & 1 \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_0 \end{bmatrix}
$$

The three component variables V 1, V 2, V 0 are called, respectively, positive sequence, negative

sequence and zero sequence. They are called symmetrical components because, taken separately,

they transform into symmetrical sets of voltages. The properties of these components can be

demonstrated by tranforming each one back into phase variables.

Consider first the positive sequence component taken by itself:

$$\underline{V}_1 = V$$
$$\underline{V}_2 = 0$$
$$\underline{V}_0 = 0$$

yields:

$$\underline{V}_a = V \quad \text{or} \quad v_a = V\cos\omega t$$
$$\underline{V}_b = \underline{a}^2 V \quad \text{or} \quad v_b = V\cos(\omega t - \frac{2\pi}{3})$$
$$\underline{V}_c = \underline{a} V \quad \text{or} \quad v_c = V\cos(\omega t + \frac{2\pi}{3})$$

This is the familiar balanced set of voltages: Phase b lags phase a by 120◦, phase c lags phase

b and phase a lags phase c.

The same transformation carried out on a negative sequence voltage:

$$\underline{V}_1 = 0$$
$$\underline{V}_2 = V$$
$$\underline{V}_0 = 0$$

yields:

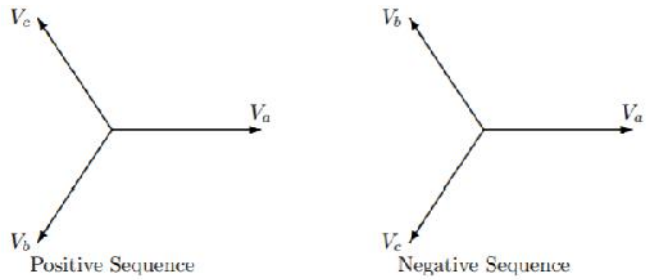
$$\underline{V}_a = V \quad \text{or} \quad v_a = V\cos\omega t$$
$$\underline{V}_b = \underline{a} V \quad \text{or} \quad v_b = V\cos(\omega t + \frac{2\pi}{3})$$
$$\underline{V}_c = \underline{a}^2 V \quad \text{or} \quad v_c = V\cos(\omega t - \frac{2\pi}{3})$$

This is called negative sequence because the sequence of voltages is reversed: phase b now leads

phase a rather than lagging. Note that the negative sequence set is still balanced in the sense

that the phase components still have the same magnitude and are separated by 120◦ . The only

difference between positive and negative sequence is the phase rotation. This is shown in Figure



The third symmetrical component is zero sequence. If:

$$\underline{V}_1 = 0$$
$$\underline{V}_2 = 0$$
$$\underline{V}_0 = V$$

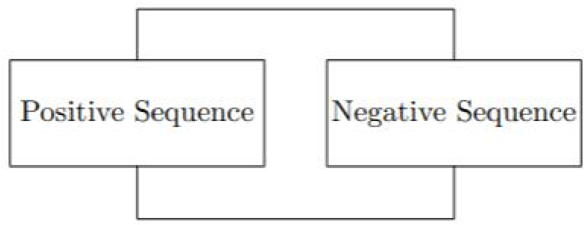
Then:

$$\underline{V}_a = V \quad \text{or} \quad v_a = V\cos\omega t$$
$$\underline{V}_b = V \quad \text{or} \quad v_b = V\cos\omega t$$
$$\underline{V}_c = V \quad \text{or} \quad v_c = V\cos\omega t$$

That is, all three phases are varying together. Positive and negative sequence sets contain those parts of the three-phase excitation that represent balanced normal and reverse phase sequence. Zero sequence is required to make up the difference between the total phase variables and the two rotating components. The great utility of symmetrical components is that, for most types of network elements, the symmetrical components are independent of each other. In particular, balanced impedances and rotating machines will draw only positive sequence currents in response to positive sequence voltages. It is thus possible to describe a network in terms of sub-networks, one for each of the symmetrical components. These are called sequence networks. A completely balanced network will have three entirely separate sequence networks. If a network is unbalanced at a particular spot, the sequence networks will be interconnected at that spot. The key to use of symmetrical components in handling unbalanced situations is in learning how to formulate those interconnections.

Sequence Impedances:

Many different types of network elements exhibit different behavior to the different symmetrical components. For example, as we will see shortly, transmission lines have one impedance for positive and negative sequence, but an entirely different impedance to zero sequence. Rotating machines have different impedances to all three sequences.



To illustrate the independence of symmetrical components in balanced networks, consider the transmission line illustrated back in Figure 20 of Installment 3 of these notes. The expressions for voltage drop in the lines may be written as a single vector expression:

$$\underline{V}_{ph1} - \underline{V}_{ph2} = j\omega \underline{\underline{L}}_{ph} \underline{I}_{ph}$$

Where

$$\underline{V}_{ph} = \begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix}$$

$$\underline{I}_{ph} = \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix}$$

$$\underline{\underline{L}}_{ph} = \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix}$$

Note that the symmetrical component transformation (4) may be written in compact form:

$$\underline{V}_s = \underline{\underline{T}} \underline{V}_p$$

Where

$$\underline{\underline{T}} = \frac{1}{3}\begin{bmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{bmatrix}$$

and V s is the vector of sequence voltages:

$$\underline{V}_s = \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_0 \end{bmatrix}$$

$$\underline{\underline{T}}^{-1}\underline{V}_{s1} - \underline{\underline{T}}^{-1}\underline{V}_{s2} = j\omega\underline{\underline{L}}_{ph}\underline{\underline{T}}^{-1}\underline{I}_s$$

Then transforming to get sequence voltages:

$$\underline{V}_{s1} - \underline{V}_{s2} = j\omega\underline{\underline{T}}\underline{\underline{L}}_{ph}\underline{\underline{T}}^{-1}\underline{I}_s$$

The sequence inductance matrix is defined by carrying out the operation indicated:

$$\underline{\underline{L}}_s = \underline{\underline{T}}\underline{\underline{L}}_{ph}\underline{\underline{T}}^{-1}$$

which is:

$$\underline{\underline{L}}_s = \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L+2M \end{bmatrix}$$

Thus the coupled set of expressions which described the transmission line in phase variables becomes an uncoupled set of expressions in the symmetrical components:

$$\begin{aligned} \underline{V}_{11} - \underline{V}_{12} &= j\omega(L-M)\underline{I}_1 \\ \underline{V}_{21} - \underline{V}_{22} &= j\omega(L-M)\underline{I}_2 \\ \underline{V}_{01} - \underline{V}_{02} &= j\omega(L+2M)\underline{I}_0 \end{aligned}$$

The positive, negative and zero sequence impedances of the balanced transmission line are then:

$$\begin{aligned} \underline{Z}_1 = \underline{Z}_2 &= j\omega(L-M) \\ \underline{Z}_0 &= j\omega(L+2M) \end{aligned}$$

So, in analysis of networks with transmission lines, it is now possible to replace the lines with three independent, single- phase networks. Consider next a balanced three-phase load with its neutral connected to ground through an impedance as shown in Figure. The symmetrical component voltage-current relationship for this network is found simply, byassuming positive, negative and zero sequence currents and finding the corresponding voltages. Ifthis is done, it is found that the symmetrical components are independent, and that the voltage current relationships are:

$$\underline{V}_1 = \underline{Z}\underline{I}_1$$
$$\underline{V}_2 = \underline{Z}\underline{I}_2$$
$$\underline{V}_0 = (\underline{Z} + 3\underline{Z}_g)\underline{I}_0$$