4.7 EMF EQUATION OF PRACTICAL BLPM SINE WAVE MOTOR

In a practical BLPM sine wave motor at the time of design it is taken care to have the flux density is sinusoidal distributed and rotor rotates with uniform angular velocity. However armature winding consists of short chorded coils properly distributed over a set of slot. These aspects reduce the magnitude of E_{ph} of an ideal winding by a factor K_{w1} which is known as the winding factor the fundamental component of flux.

$e = -N\, d\phi/dt$

$-d\phi/dt \quad \text{as } N=1$

$= -d\phi/dt \quad ((2 B\square\, lr/p) \cos p\theta\, \omega_{mt})$

$= (2 B\square\, lr/p)\, p\, \omega_m \sin p\, \omega_{mt}$

$$e = 2 B\square\, lr\, \omega_m \sin p\, \omega_{mt} \qquad \text{.......(5.2)}$$

$$K_{w1} = K_{s1} K_{p1} K_{b1} \qquad \text{.......(5.8)}$$

K_{s1} =slew factor

$K_{s1} = (\sin \sigma/2)/(\sigma/2)$

$K_{s1} = 1$ (slightly less than 1)

σ – Skew angle in elec. Radians.

K_{p1} = pitch factor (or) short chording factor

$= \sin m\pi/2$ or $\cos \rho/2$

Where m = coil span/pole pitch

= fraction < 1

$\pi(1 - m) = \rho$

[Coil span = τ

= π elec rad

= π/ρ mech. Rad]

K_{p1}=sin $\frac{m\pi}{2}$ or cos $\frac{\rho}{2}$

[mπ *is elec rad* $\frac{m\pi}{p}$ mech. Rad.]

K_{b1} = Distribution factor or width factor

$$K_{b1} = \frac{\sin q\frac{v}{2}}{q \sin\frac{v}{2}}$$

Where v = slot angle in elec. Radians

$= \frac{2\pi\rho}{n_s}$; n_s = no. of slots (total)

q = slots/pole/phase for 60° phase spread

= slots/pair of poles/phase

$K_{b1}< 1$; $K_{p1}< 1$; $K_{s1}< 1$

Therefore $K_{w1} = K_{p1} K_{b1} K_{s1}< 1$ (winding factor)

Thus rms value of the per phase emf is

$$\mathbf{E_{ph} = 4.44\ f\ \emptyset_m\ T_{ph}\ K_{w1}\ volts.}$$

4.8 TORQUE EQUATION OF PERMANENT MAGNET SYNCHRONOUS MOTOR

5.7.1 Torque equation of ideal PMSM

When a balanced three phase voltage is applied to the armature, a three phase current flows through the conductors. This current produces armature flux for deriving the torque equation, the concept of armature ampere conductor density is used. A sinusoidally distributed ampere conductor density is assumed as shown in figure 5.9

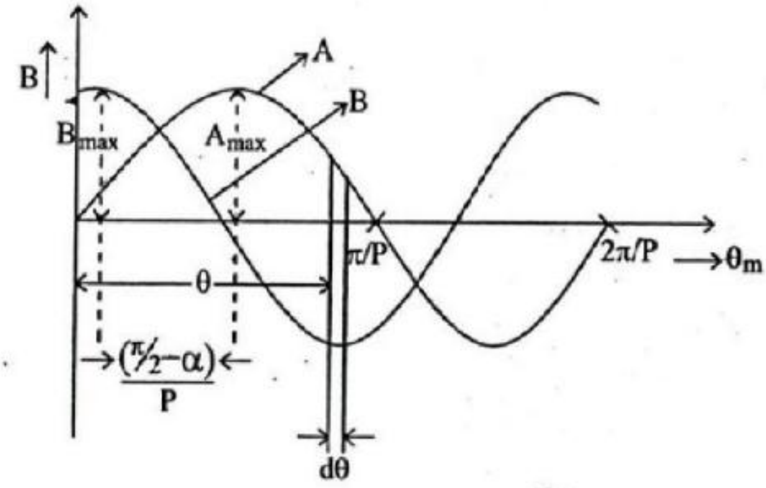

Figure 5.9 Ampere conductor and flux density distribution

Let the operation point of PMSM is such that the ampere conductor density and the flux density are as shown in figure5.9 In figure 5.9 the angle between the axes of ampere conductor and flux density is $\left(\frac{\pi}{2}-\alpha\right)$. A strip of width $d\theta$ is considered at θ

From figure 5.9,

$$B = B_{max}\sin\left(P\theta + \left(\frac{\pi}{2}-\alpha\right)\right)$$

$$= B_{max}\sin\left(\frac{\pi}{2} + (P\theta - \alpha)\right)$$

$$\therefore \quad B = B_{max}\cos(P\theta - \alpha) \quad \text{... (5.21)}$$

$$A = A_{max}\sin P\theta$$

Force experinced by the armature conductors in $d\theta$ is

$$dF = B l A\, d\theta$$

$$= A_{max}\, B_{max}\, l \sin P\theta \cos(P\theta - \alpha)\, d\theta$$

Torque experienced by the armature conductors in $d\theta$ is

$$dT = A_{max}\, B_{max}\, rl \sin P\theta \cos(P\theta - \alpha)\, d\theta$$

Torque experienced by armature conductors/pole $= \int_{\theta=0}^{\theta=\pi/P} dT$

$$= A_{max}\, B_{max}\, rl \int_{o}^{\pi/P} \sin P\theta \cos(P\theta - \alpha)\, d\theta$$

$$= \frac{A_{max}\, B_{max}}{2} rl \int_{o}^{\pi/P} [\sin(P\theta + P\theta - \alpha) + \sin\alpha]\, d\theta$$

$$= \frac{A_{max}\, B_{max}}{2} rl \left[\frac{-\cos(2P\theta - \alpha)}{2P} + \theta \sin\alpha\right]_{o}^{\pi/P}$$

$$= \frac{A_{max} B_{max} rl}{2}\left[\frac{-\cos\left(2P \times \frac{\pi}{P} - \alpha\right)}{2P} + \frac{\pi}{P}\sin\alpha + \frac{\cos(-\alpha)}{2P}\right]$$

$$= \frac{A_{max} B_{max} rl}{2}\left[\frac{-\cos\alpha}{2P} + \frac{\cos\alpha}{2P} + \frac{\pi}{P}\sin\alpha\right]$$

$$= \frac{A_{max} B_{max} rl}{2} \frac{\pi}{P}\sin\alpha \qquad \text{... (5.22)}$$

Total electromagnetic torque developed by all the armature conductors = 2P × Torque per pole

$$= 2P \frac{\pi}{P} \frac{A_{max} B_{max} rl \sin\alpha}{2}$$

$$= \pi A_{max} B_{max} rl \sin\alpha \qquad \text{... (5.23)}$$

As armature is stationary, this torque is experienced by the rotor and rotor rotates

$$T = -\pi A_{max} B_{max} rl \sin\alpha \qquad \text{... (5.24)}$$

Since $\beta = -\alpha$

$$\Rightarrow T = \pi A_{max} B_{max} rl \sin\beta \qquad \text{... (5.25)}$$

Where β is the torque angle or power angle

5.7.2 Ampere conductor density distribution

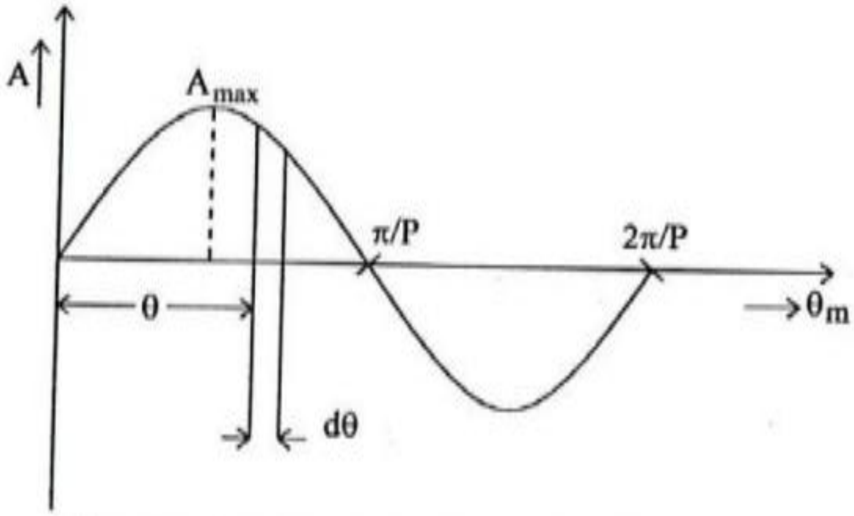

Figure: 5.10 Ampere conductor density

The above figure shown the ampere conductor density distribution in the air gap due to current carrying armature winding

$$A = A_{max} \sin P\theta$$

Where A → ampere conductor density

consider a strip of width $d\theta$ at angle θ from the reference axis.

Ampere conductors in the strip $d\theta$ is $A\,d\theta$.

$$A d\theta = A_{max} \sin P\theta \, d\theta \qquad \text{... (5.26)}$$

Ampere conductors per pole $= \int_{o}^{\pi/P} A d\theta$

$$= \int_{o}^{\pi/P} A_{max} \sin P\theta \, d\theta$$

$$= A_{max} \left[\frac{\cos P\theta}{P} \right]_{o}^{\pi/P}$$

$$= \frac{A_{max}}{P} (-1-1)$$

$$= \frac{2A_{max}}{P} \qquad \text{... (5.27)}$$

Let T_{ph} be the number of full pitched turns per phase

i be the current

Total ampere conductiors $= 2iT_{ph}$

Sinusoidally distributed ampere conductors /pole $= \frac{2iT_{ph}}{2P}$

$$= \frac{iT_{ph}}{P} \qquad \text{... (5.28)}$$

equating eqations (5.27) & (5.28)

$$= \frac{2A_{max}}{P} = \frac{iT_{ph}}{P}$$

$$A_{max} = \frac{iT_{ph}}{2} \qquad \text{...(5.29)}$$

For a PMSM supplied by balanced three phase sinusoidal voltage, the phase currents are given by

$$i_R = I_{max} \cos \omega t$$

$$i_y = I_{max} \cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$i_B = I_{max} \cos\left(\omega t - \frac{4\pi}{3}\right)$$

The turns are given by

$$T_{phR} = T_{ph} \cos\theta$$

$$T_{phy} = T_{ph} \cos\left(\theta - \frac{2\pi}{3}\right)$$

$$T_{phB} = T_{ph} \cos\left(\theta - \frac{4\pi}{3}\right)$$

Ampere turns at any instant is given by

$$iT_{ph} = i_R T_{phR} + i_y T_{phy} + i_B T_{phB}$$

$$= I_{max} T_{ph} \cos\omega t \cos\theta + I_{max} T_{ph} \cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$\cos\left(\theta - \frac{2\pi}{3}\right) + I_{max} T_{ph} \cos\left(\omega t - \frac{4\pi}{3}\right) \cos\left(\theta - \frac{4\pi}{3}\right)$$

By simplifying the above equation

$$iT_{ph} = \frac{3}{2} I_{max} T_{ph} \cos(\omega t - \theta)$$

$$= \frac{3}{2} \sqrt{2}\, I_{ph} T_{ph} \cos(\omega t - \theta)$$

$$A_{max} = \frac{iT_{ph}}{2}$$

$$A_{max} = \frac{3\sqrt{2}}{2} I_{ph} T_{ph} \qquad \text{... (5.30)}$$

In practical motor, the armature turns are short pitched and distributed further they may and be accommodated in skewed slots in such case for getting slots fundamental component of ampere turn distribution, the turns per phase is modified as $K_{\omega 1} T_{ph}$

Where $K_{\omega 1} = K_{s1} K_{c1} K_{d1}$

$K_{s1} \rightarrow$ skew factor

$$K_{s1} = \frac{\sin \sigma / 2}{\sigma / 2} \qquad \text{where } \sigma \rightarrow \text{skew angle}$$

$$K_{c1} = \frac{\cos \delta}{2}; \; K_{c1} \rightarrow \text{chording factor}$$

$K_{d1} \rightarrow$ distribution factor

$$K_{d1} = \frac{\sin q \, V/2}{q \sin V/2}$$

Fundamental component of ampere turns per phase of a practical motor

$$= \frac{4}{\pi} i T_{ph} K_{\omega 1} \qquad \text{...(5.31)}$$

When a balanced sinusoidally varying three phase ac current pass through a balanced three phase winding, it can be shown that total sinussoidally distributed ampere turns is equal to

$$= \frac{3}{2} \frac{4}{\pi} I_{max} K_{\omega 1} T_{ph}$$

$$= \frac{3 \cdot 2\sqrt{2}}{\pi} I_{ph} K_{\omega 1} T_{ph} \qquad \text{...(5.32)}$$

The amplitude of ampere conductor density distribution is equal to the total sinusoidally distributed ampere turns divided by 2

$\therefore A_{max}$ in practical 3ϕ motor

$$= \frac{3 \cdot 2\sqrt{2}}{\pi} \frac{I_{ph} K_{\omega 1}}{2} T_{ph}$$

$$= \frac{3\sqrt{2}}{\pi} I_{ph} K_{\omega 1} T_{ph} \qquad \text{... (5.33)}$$

electromagnetic torque developed in practical PMSM is

$$= \pi A_{max} B_{max} rl \sin\beta$$

$$= \pi \left[\frac{3\sqrt{2}}{\pi} I_{ph} K_{\omega 1} T_{ph} \right] B_{max} rl \sin\beta$$

$$= 3\sqrt{2} K_{\omega 1} I_{ph} T_{ph} B_{max} rl \sin\beta$$

$$= (3\sqrt{2} K_{\omega 1} T_{ph} B_{max} rl) I_{ph} \sin\beta$$

$$T = 3 \frac{E_{ph}}{\omega_m} I_{ph} \sin\beta \qquad \text{... (5.34)}$$