1.4 BLOCK DIAGRAMS

Block diagrams consist of a single block or a combination of blocks. These are used to represent the control systems in pictorial form.

Basic Elements of Block Diagram

The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following fig 1.4.1 to identify these elements.

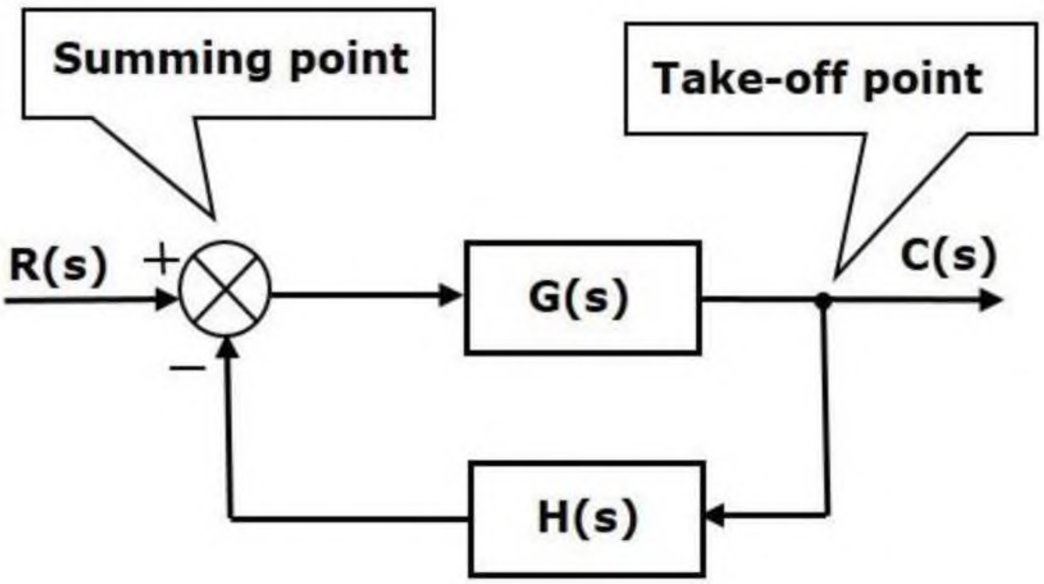


Figure 1.4.1: Basic elements of Block diagram

[Source: "Control System Engineering" by Nagoor Kani, page : 1.50]

The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals. Let us now discuss these elements one by one.

Block

The transfer function of a component is represented by a block. Block has single input and single output. The following fig 1.4.2 shows a block having input X(s), output Y(s) and the transfer function G(s).

Figure 1.4.2: Open loop transfer function

[Source: "Control System Engineering" by Nagoor Kani, page : 1.49]

Transfer Function, G(s)=Y(s) / X(s)

^Y(s)=G(s)X(s)

Output of the block is obtained by multiplying transfer function of the block with input.

Summing Point

The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one.

The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as **sum of A and B.**

i.e.,Y = A + B.

The following fig 1.4.3& fig 1.14 shows the summing point with two inputs (A, B) and one output (Y).

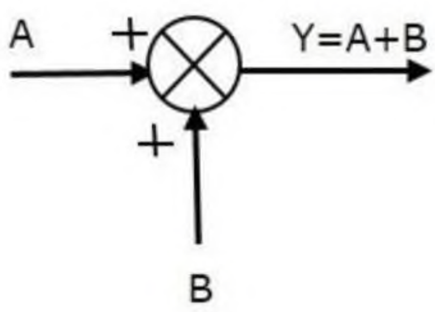


Figure 1.4.3: Summing point with two inputs

[Source: "Control System Engineering" by Nagoor Kani, page : 1.50]

Here, the inputs A and B are having opposite signs, i.e., A is having positive sign and B is having negative sign. So, the summing point produces the output **Y** as the **difference of A and B**.

$$Y = A + (-B) = A - B.$$

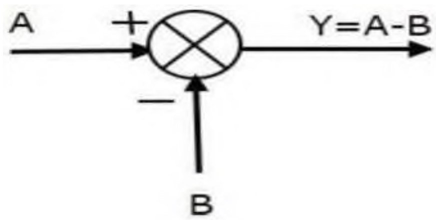


Figure 1.4.4: Summing point with two inputs (difference of A & B)

[Source: "Control System Engineering" by Nagoor Kani, page : 1.50]

The following fig 1.4.5 shows the summing point with three inputs (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output **Y** as

$$Y = A + B + (-C) = A + B - C.$$

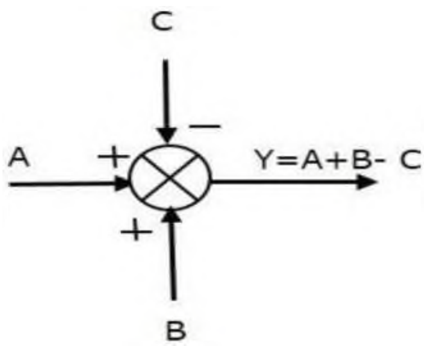


Figure 1.4.5: Summing point with three inputs

[Source: "Control System Engineering" by Nagoor Kani, page: 1.50]

Take-off Point

The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.

In the following fig 1.4.6 shows the take-off point is used to connect the same input, R(s) to two more blocks.

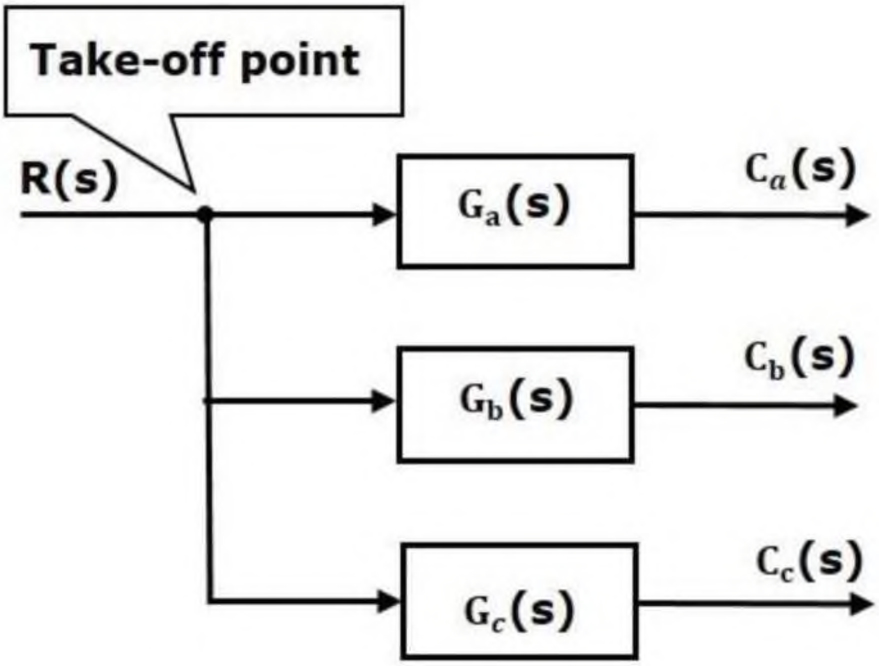


Figure 1.4.6: Take-off point from same input

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

In the following fig 1.4.7 , the take-off point is used to connect the output C(s), as one of the inputs to the summing point.

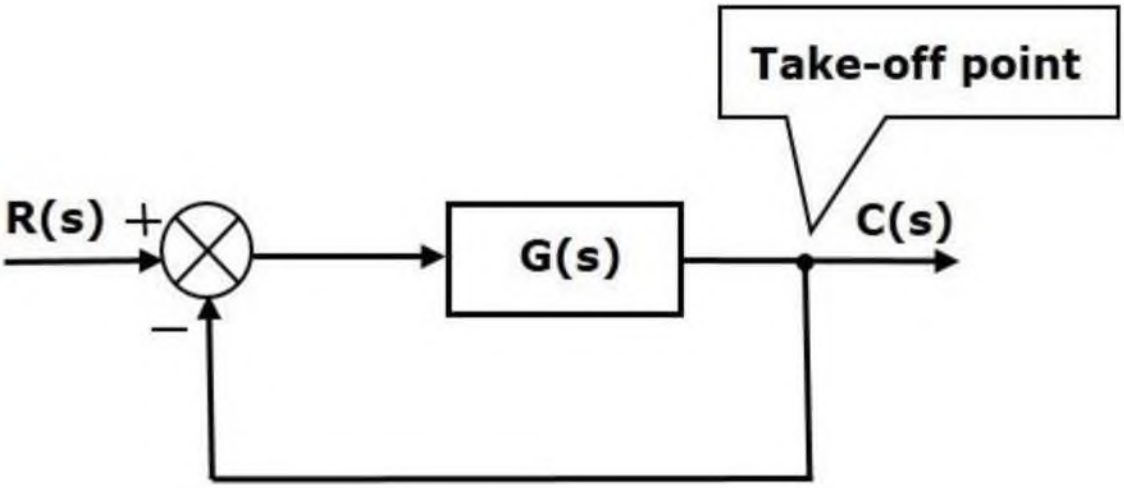


Figure 1.4.7: Take-off point from output

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

Block Diagram Representation of Electrical Systems

In this section, let us represent an electrical system with a block diagram. Electrical systems contain mainly three basic elements — **resistor, inductor and capacitor**.

Consider a series of RLC circuit as shown in the fig 1.4.8. Where, $V_i(t)$ and $V_o(t)$ are the input and output voltages. Let i(t) be the current passing through the circuit. This circuit is in time domain.

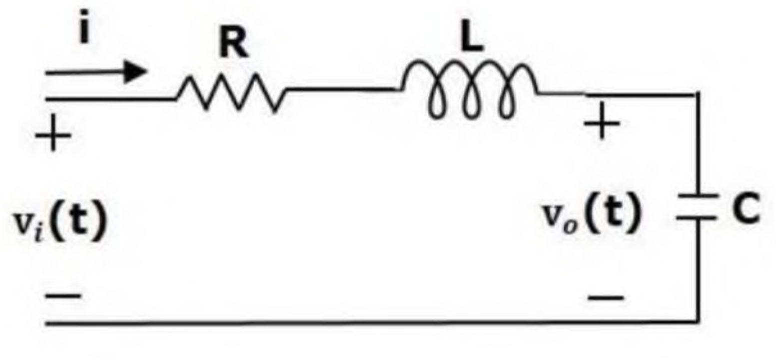


Figure 1.4.8: RLC Series Circuit

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

By applying the Laplace transform to this circuit, will get the circuit in s-domain. The circuit is as shown in the following fig 1.4.9.

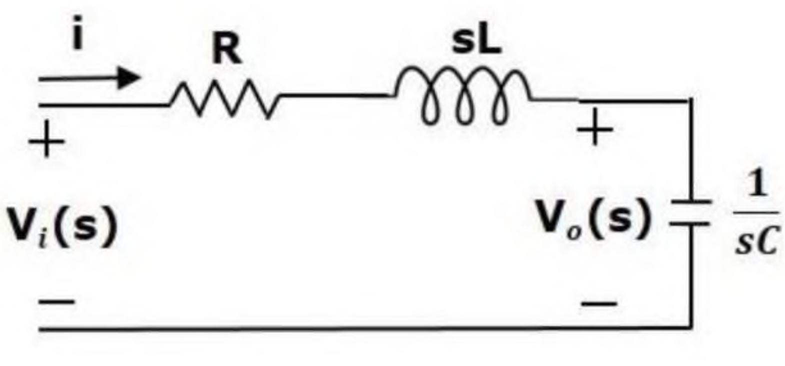


Figure 1.4.9: RLC Series Circuit with s-domain

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

From the above circuit, we can write

$$/(s) = \frac{Ws) - V_O(S)}{R + sL}$$

$$^\wedge I(s)=\{1/(R+sL)\}\ \{V_i(s)-V_o(s)\}$$ **(Equation 1)**

$$V_o(s)=(1/sC)I(s)$$ **(Equation 2)**

Let us now draw the block diagrams for these two equations individually. And then combine those block diagrams properly in order to get the overall block diagram of series of RLC Circuit (s-domain).

Equation 1 can be implemented with a block having the transfer function, 1/R+sL. The input and output of this block are $\{V_i(s)-V_o(s)\}$ and I(s). We require a summing point to get $(V_i(s)-V_o(s)\}$. The block diagram of Equation 1 is shown in fig 1.4.10.

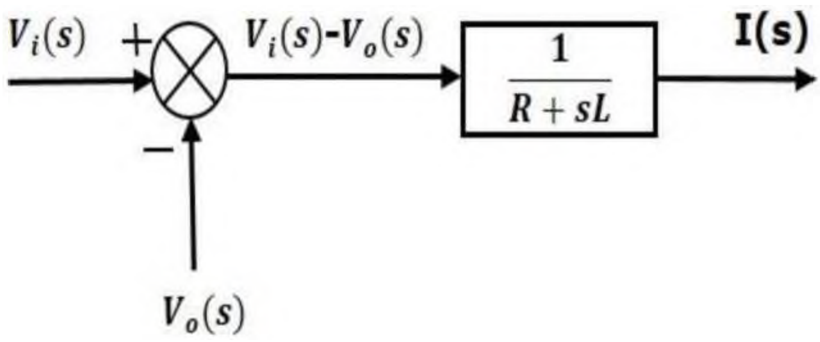


Figure 1.4.10: block diagram of equationl

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

Equation 2 can be implemented with a block having transfer function, 1/sC. The input and output of this block are I(s) and $V_o(s)$. The block diagram of Equation 2 is shown in the fig 1.4.11.

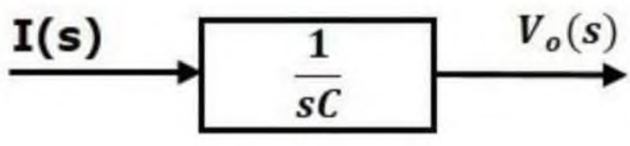


Figure 1.4.11: block diagram of equation2

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

The overall block diagram of the series of RLC Circuit (s-domain) is shown in fig 1.4.12.

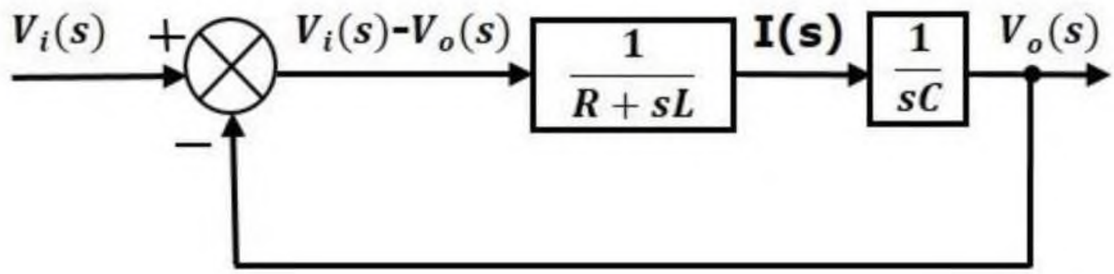


Figure 1.4.12: block diagram of RLC series circuit

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

Similarly, you can draw the **block diagram** of any electrical circuit or system just by following this simple procedure.

- Convert the time domain electrical circuit into an s-domain electrical circuit by applying Laplace transform.
- Write down the equations for the current passing through all series branch elements and voltage across all shunt branches
- Draw the block diagrams for all the above equations individually.
- Combine all these block diagrams properly in order to get the overall block diagram of the electrical circuit (s-domain).

Block diagram algebra

Block diagram algebra is nothing but the algebra involved with the basic elements of the block diagram. This algebra deals with the pictorial representation of algebraic equations.

Basic Connections for Blocks

There are three basic types of connections between two blocks.

Series Connection

Series connection is also called **cascade connection**. In the following fig 1.4.13, two blocks having transfer functions $G_1(s)$ and $G_2(s)$ are connected in series.

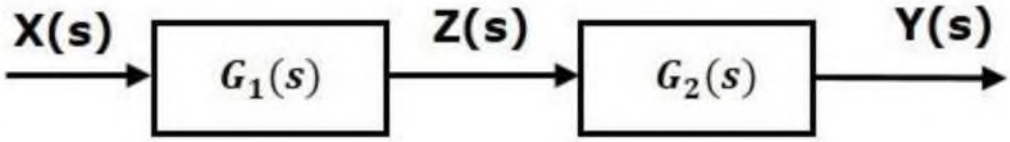


Figure 1.4.13: block diagram of Series connection of two blocks

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

For this combination, we will get the output Y(s)Y(s) as

$$Y(s)=G_2(s)Z(s)$$

Where, $Z(s)=G_1(s)X(s)$

$$\wedge Y(s)=G_2(s)[G_1(s)X(s)]=G_1(s)G_2(s)X(s)$$

$$\wedge Y(s)=\{G_1(s)G_2(s)\}X(s)$$

Compare this equation with the standard form of the output equation, $Y(s)=G(s)X(s)$. Where, $G(s)=G_1(s)G_2(s)$.

That means we can represent the **series connection** of two blocks with a single block. The transfer function of this single block is the **product of the transfer functions** of those two blocks. The equivalent block diagram is shown in fig 1.4.14.

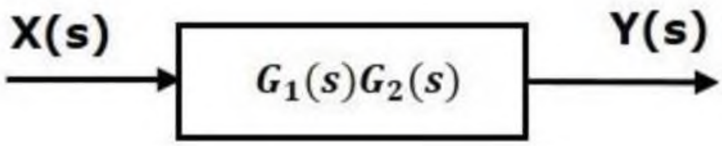


Figure 1.4.14: block diagram of Series connection of two block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

Similarly, you can represent series connection of 'n' blocks with a single block. The transfer function of this single block is the product of the transfer functions of all those 'n' blocks.

Parallel Connection

The blocks which are connected in **parallel** will have the **same input**. In the following fig 1.25, two blocks having transfer functions G1(s) and G2(s) are connected in parallel. The outputs of these two blocks are connected to the summing point.

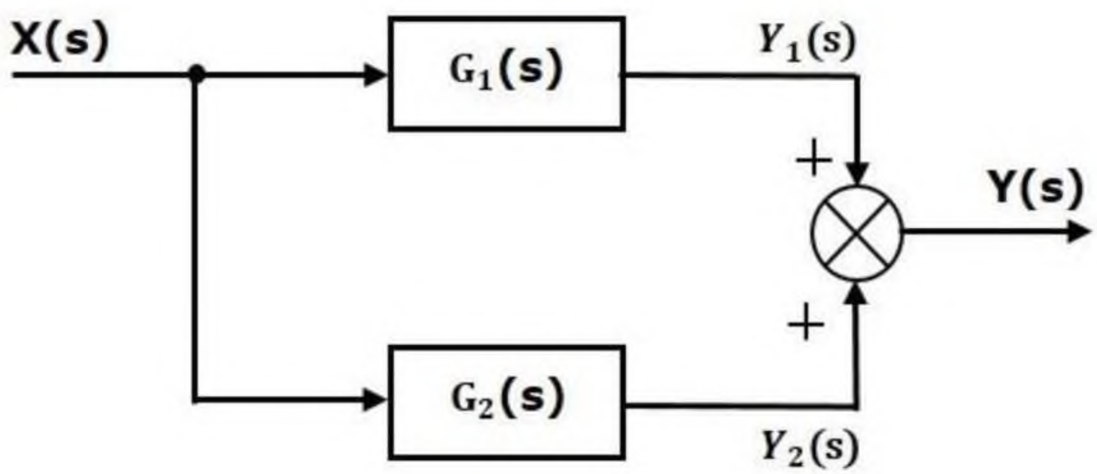


Figure 1.4.15: block diagram of parallel connection of two blocks

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

For this combination, we will get the output Y(s)Y(s) as

Y(s)=Y1(s)+Y2(s)

Where, Y1(s)=G1(s)X(s) and Y2(s)=G2(s)X(s)

^Y(s)=G1(s)X(s)+G2(s)X(s)={G1(s)+G2(s)}

Compare this equation with the standard form of the output equation, Y(s)=G(s)X(s). Where, G(s)=G1(s)+G2(s).

That means we can represent the **parallel connection** of two blocks with a single block. The transfer function of this single block is the **sum of the transfer functions** of those two blocks. The equivalent block diagram is shown below.

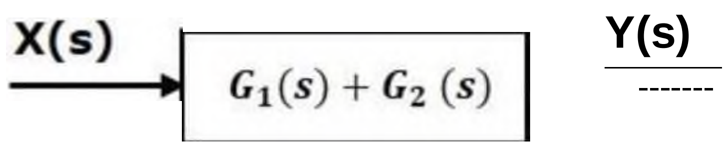


Figure 1.4.16: block diagram of parallel connection of two block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.5] Similarly,

you can represent parallel connection of 'n' blocks with a single block. The transfer function of this single block is the algebraic sum of the transfer functions of all those 'n' blocks.

Feedback Connection

As we discussed in previous chapters, there are two types of **feedback** — positive feedback and negative feedback. The following figure 1.4.17 shows negative feedback control system. Here, two blocks having transfer functions G(s) and H(s) form a closed loop.

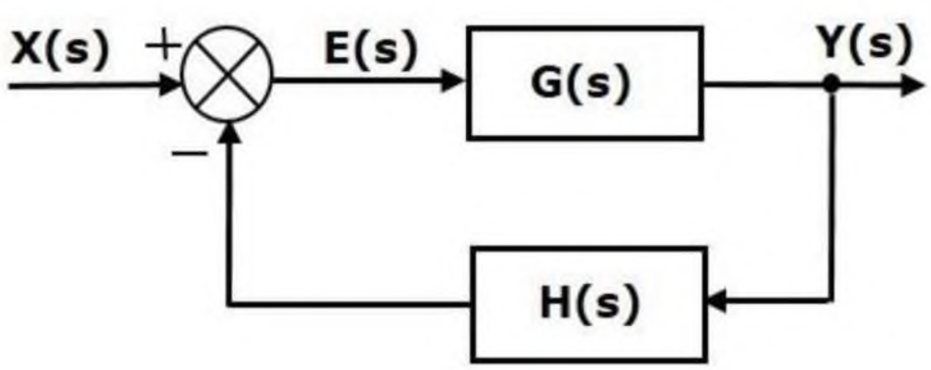


Figure 1.4.17: block diagram of negative feedback system

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

The output of the summing point is -

$$E(s)=X(s)-H(s)Y(s)$$

The output Y(s)Y(s) is -

$$Y(s)=E(s)G(s)$$

Substitute E(s) value in the above equation.

$$Y(s)=\{X(s)-H(s)Y(s)\}G(s)\}$$

$$Y(s)\{1+G(s)H(s)\}=X(s)G(s)\}$$

$$\wedge Y(s)/X(s) =G(s)/(1+G(s)H(s))$$

Therefore, the negative feedback closed loop transfer function is G(s) /(1+G(s)H(s)) This means we can represent the negative feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the negative feedback. The equivalent block diagram is shown in figure 1.4.18.

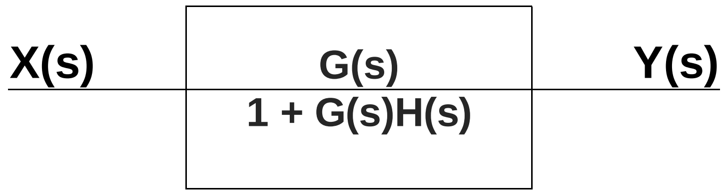


Figure 1.4.18: equivalent block diagram of negative feedback system

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

Similarly, you can represent the positive feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the positive feedback, i.e., G(s)/(1-G(s)H(s))

Block Diagram Algebra for Summing Points

There are two possibilities of shifting summing points with respect to blocks -

- Shifting summing point after the block
- Shifting summing point before the block

Let us now see what kind of arrangements need to be done in the above two cases one by one.

Shifting Summing Point After the Block

Consider the block diagram shown in the following figure 1.29. Here, the summing point is present before the block.

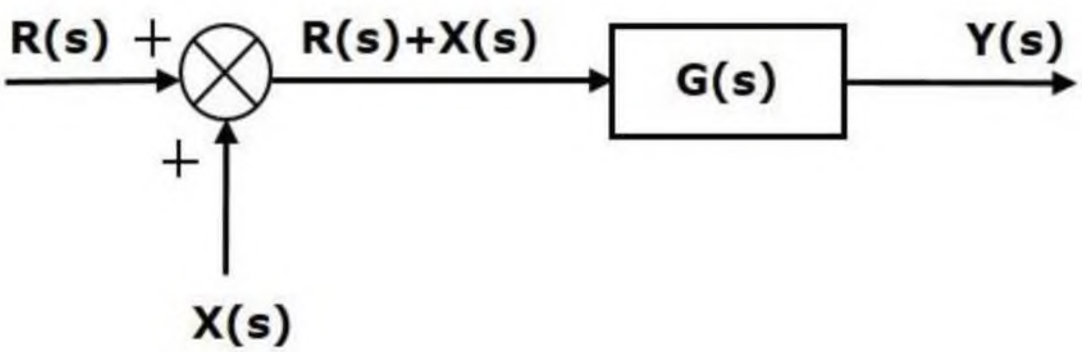


Figure 1.4.19: shifting summing point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

Summing point has two inputs R(s) and X(s). The output of it is {R(s)+X(s)}.

So, the input to the block G(s) is {R(s)+X(s)} and the output of it is -

Y(s)=G(s){R(s)+X(s)}

^Y(s)==G(s)R(s)+G(s)X(s) **(Equation 1)**

Now, shift the summing point after the block. This block diagram is shown in the following figure 1.4.20.

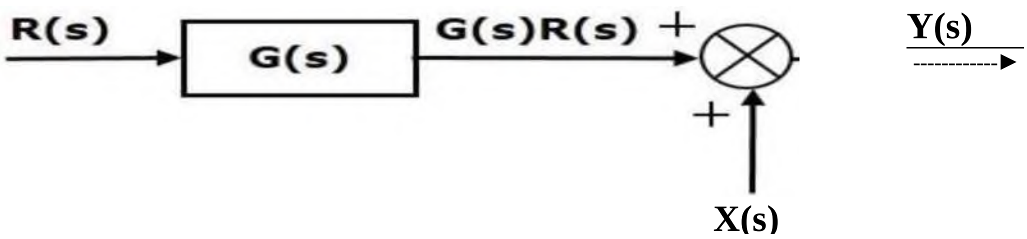


Figure 1.4.20: shifting summing point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

Output of the block G(s) is G(s)R(s).

The output of the summing point is
Y(s)=G(s)R(s)+X(s)**(Equation 2)**

Compare Equation 1 and Equation 2.

The first term 'G(s)R(s)' is same in both the equations. But, there is difference in the second term. In order to get the second term also same, we require one more block G(s). It is having the input X(s) and the output of this block is given as input to summing point instead of X(s). This block diagram is shown in the following figure 1.4.21.

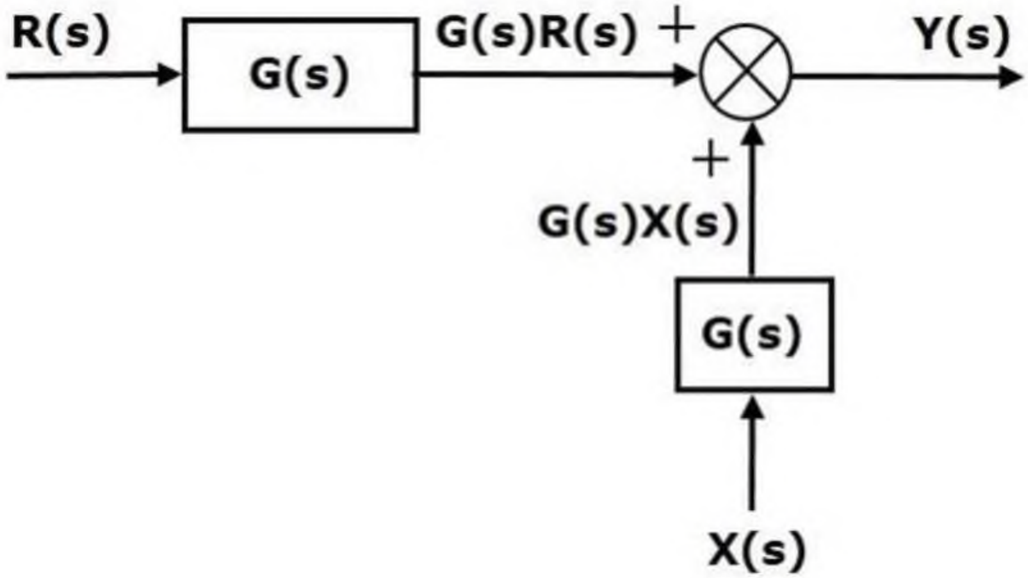


Figure 1.4.21: shifting summing point after the block(equation1 and equation2)

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Shifting Summing Point before the Block

Consider the block diagram shown in the following figure 1.4.22. Here, the summing point is present after the block.

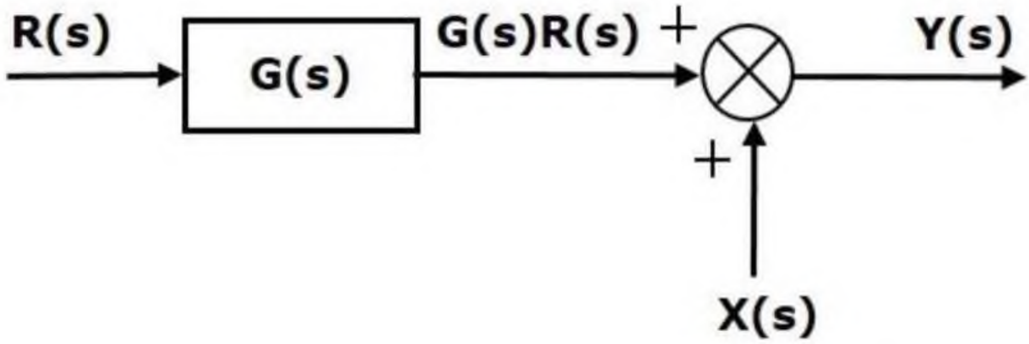


Figure 1.4.22: shifting summing point before the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Output of this block diagram is -

Y(s)=G(s)R(s)+X(s) (**Equation 3)**

Now, shift the summing point before the block. This block diagram is shown in the following figure 1.4.23.

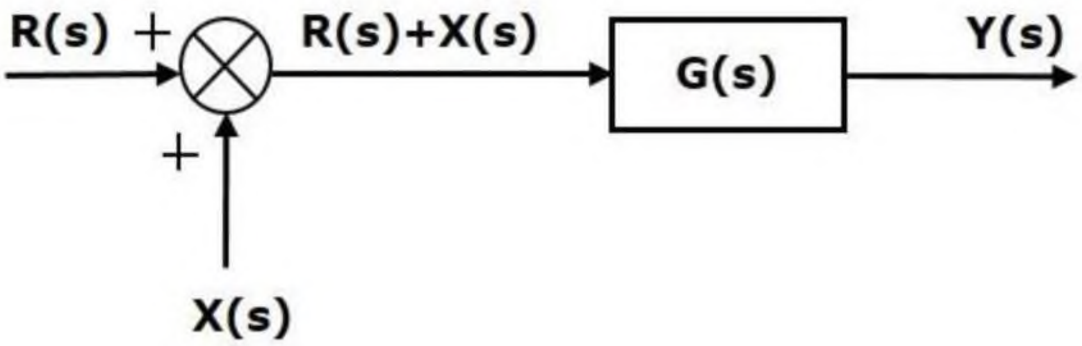


Figure 1.4.23: shifting summing point after the block (equation 3)

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Output of this block diagram is -

Y(S)=G(s)R(s)+G(s)X(s) **(Equation 4)**

Compare Equation 3 and Equation 4,

The first term 'G(s)R(s)' is same in both equations. But, there is difference in the second term. In order to get the second term also same, we require one more block 1/G(s). It is having the input X(s) and the output of this block is given as input to summing point instead of X(s). This block diagram is shown in the following figure 1.4.24.

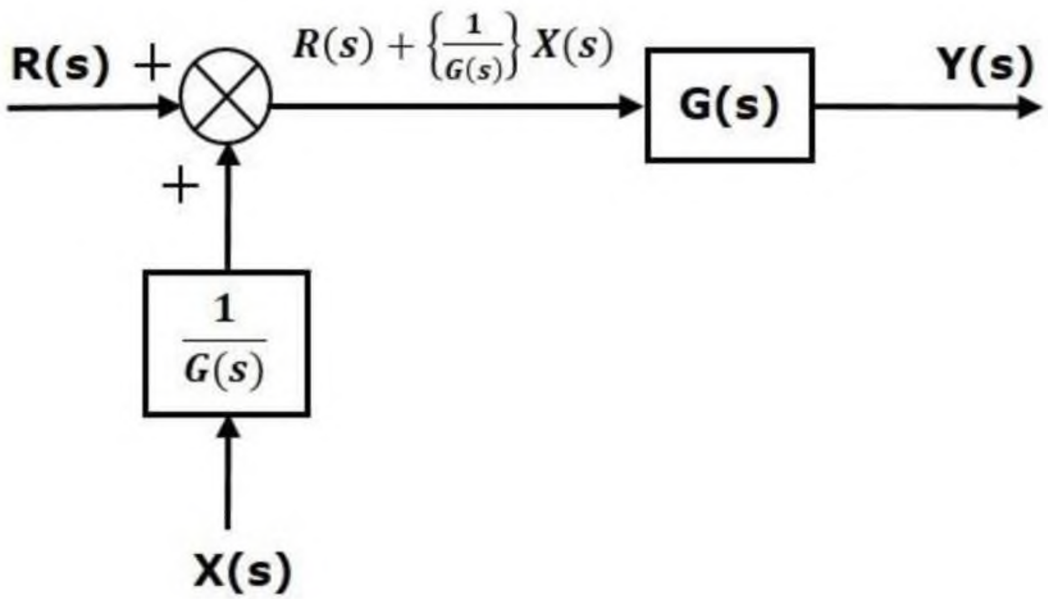


Figure 1.4.24: output of shifting summing point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Block Diagram Algebra for Take-off Points

There are two possibilities of shifting the take-off points with respect to blocks -

- Shifting take-off point after the block
- Shifting take-off point before the block

Let us now see what kind of arrangements is to be done in the above two cases, one by one.

Shifting Take-off Point after the Block

Consider the block diagram shown in the following figure 1.4.25. In this case, the takeoff point is present before the block.

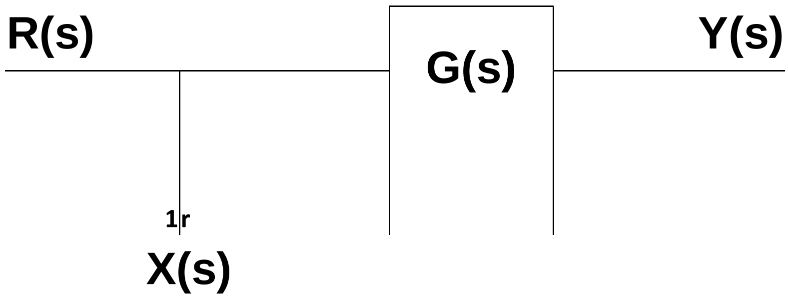


Figure 1.4.25: shifting take off point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Here, X(s)=R(s) and Y(s)=G(s)R(s)

When you shift the take-off point after the block, the output Y(s)Y(s) will be same. But, there is difference in X(s) value. So, in order to get the same X(s) value, we require one more block 1/G(s). It is having the input Y(s) and the output is X(s). This block diagram is shown in the following figure 1.4.26.

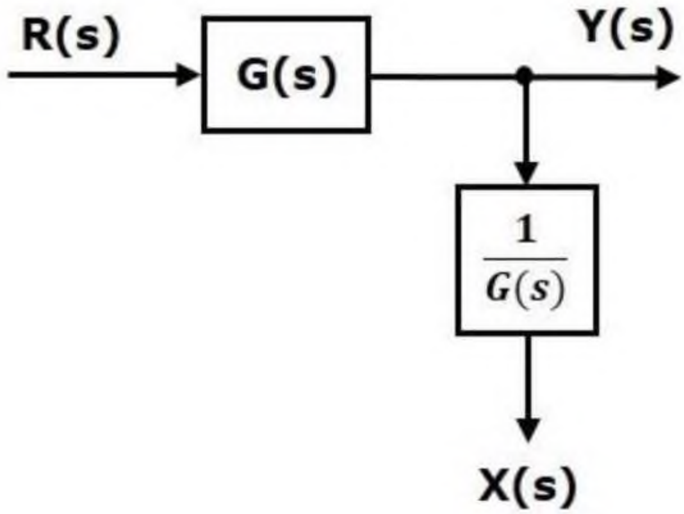


Figure 1.4.26: shifting take off point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Shifting Take-off Point Before the Block

Consider the block diagram shown in the following figure 1.4.27. Here, the take-off point is present after the block.

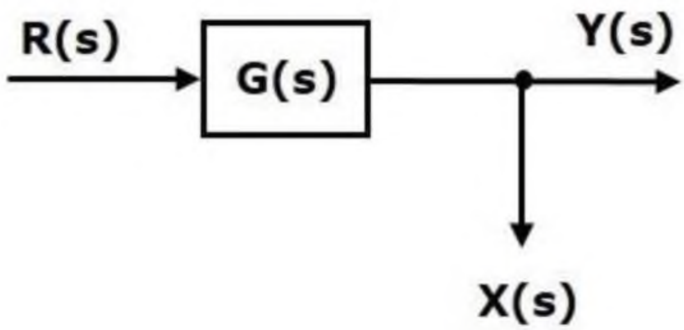


Figure 1.4.27: shifting take off point before the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Here, X(s)=Y(s)=G(s)R(s)

When you shift the take-off point before the block, the output Y(s) will be same. But, there is difference in X(s)X(s) value. So, in order to get same X(s) value, we require one more block G(s). It is having the input R(s)R(s) and the output is X(s)X(s). This block diagram is shown in the following figure 1.4.28.

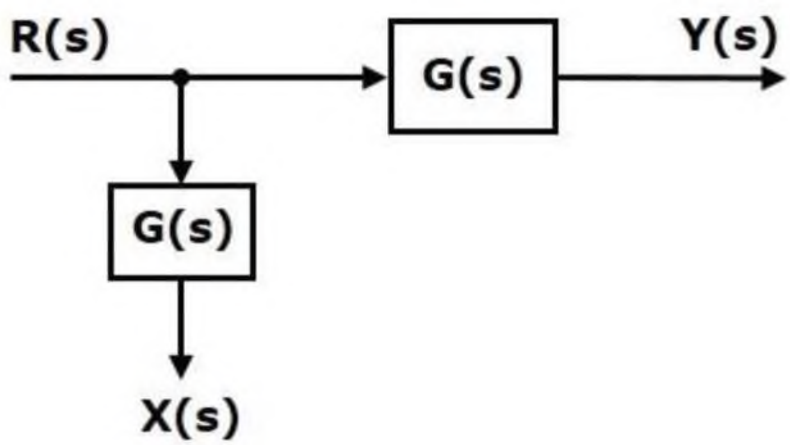


Figure 1.4.28: shifting take off point before the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Block Diagram Reduction

Block Diagram Reduction Rules

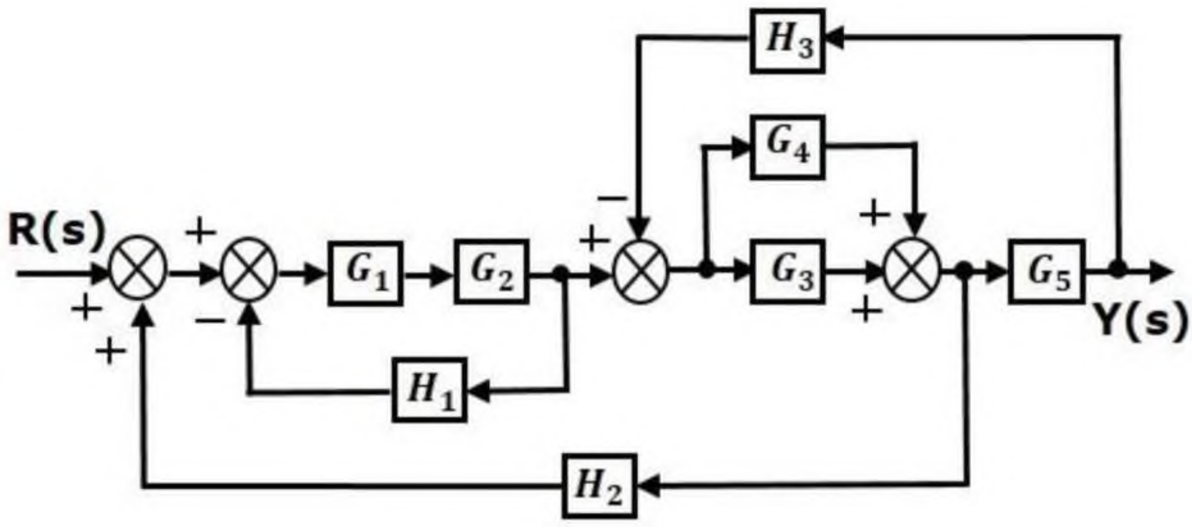
Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

- **Rule 1** - Check for the blocks connected in series and simplify.
- **Rule 2** - Check for the blocks connected in parallel and simplify.
- **Rule 3** - Check for the blocks connected in feedback loop and simplify.
- **Rule 4** - If there is difficulty with take-off point while simplifying, shift it towards right.
- **Rule 5** - If there is difficulty with summing point while simplifying, shift it towards left.
- **Rule 6** - Repeat the above steps till you get the simplified form, i.e., single block.

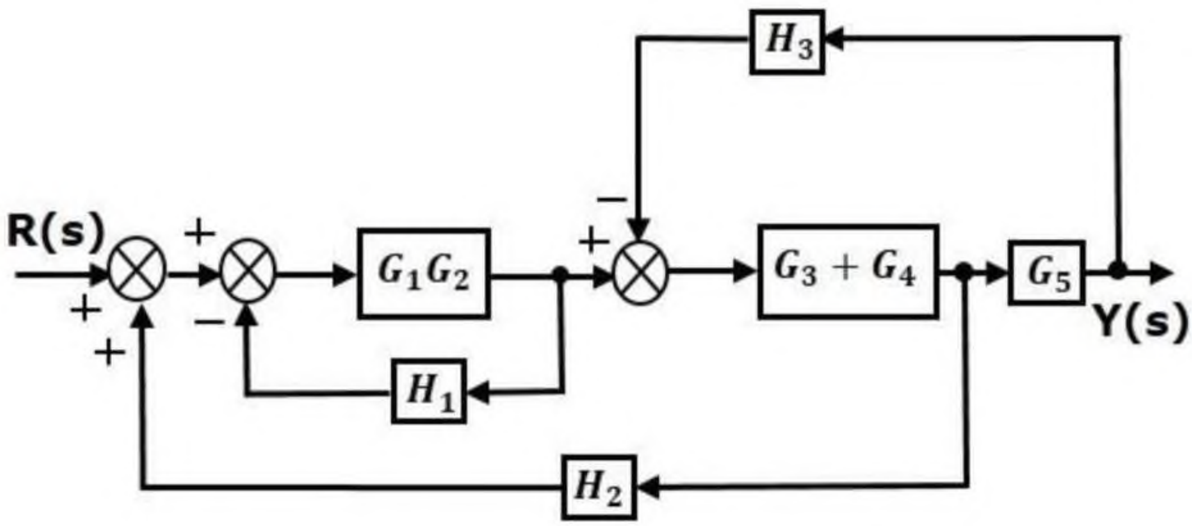
Note - the transfer function present in this single block is the transfer function of the overall block diagram.

Example

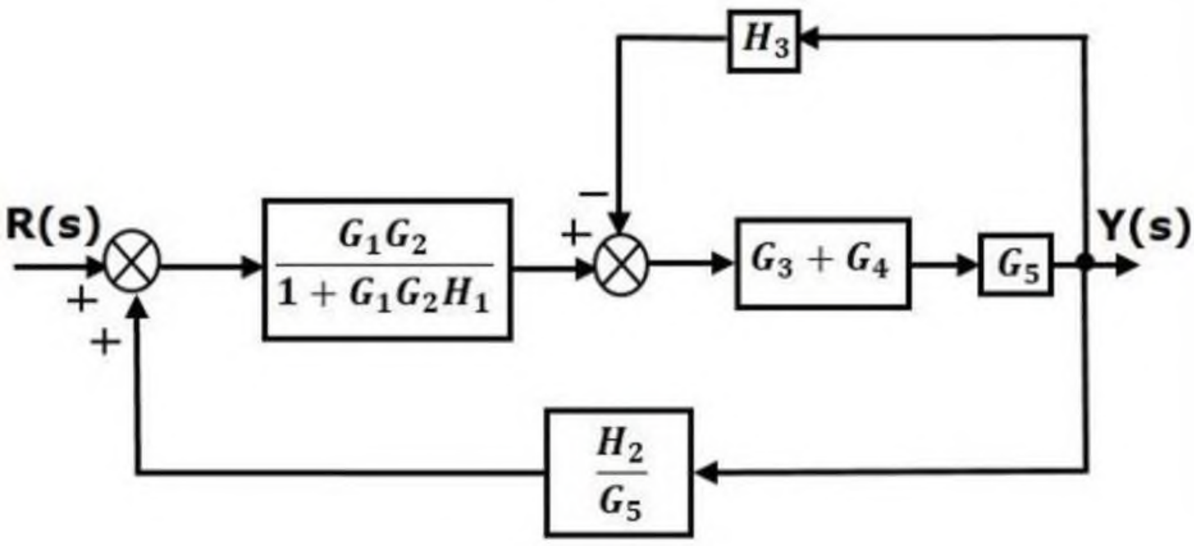
Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



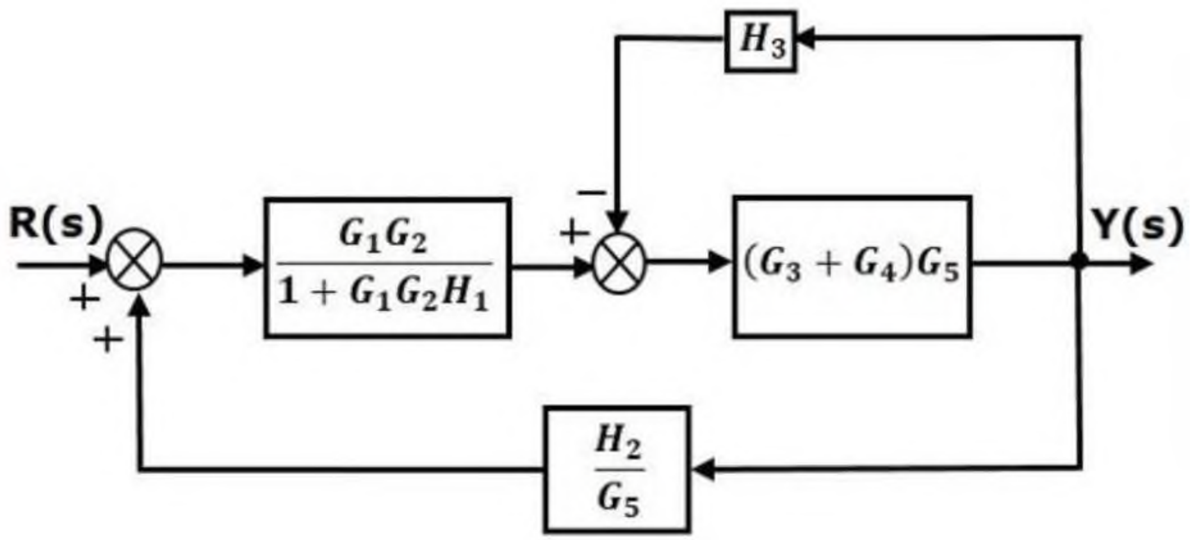
Step 1 - Use Rule 1 for blocks G1 and G2. Use Rule 2 for blocks G3 and G4. The modified block diagram is shown in the following figure.



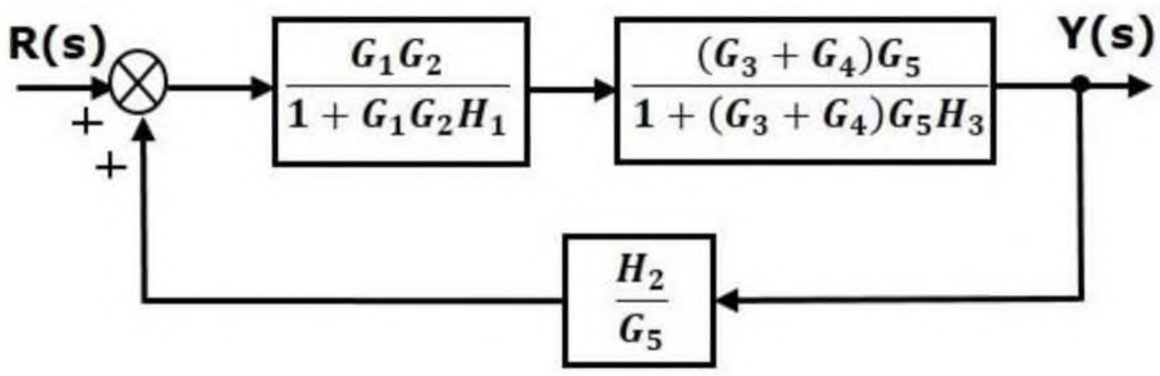
Step 2 - Use Rule 3 for blocks G1G2 and H1. Use Rule 4 for shifting take-off point after the block G5. The modified block diagram is shown in the following figure.



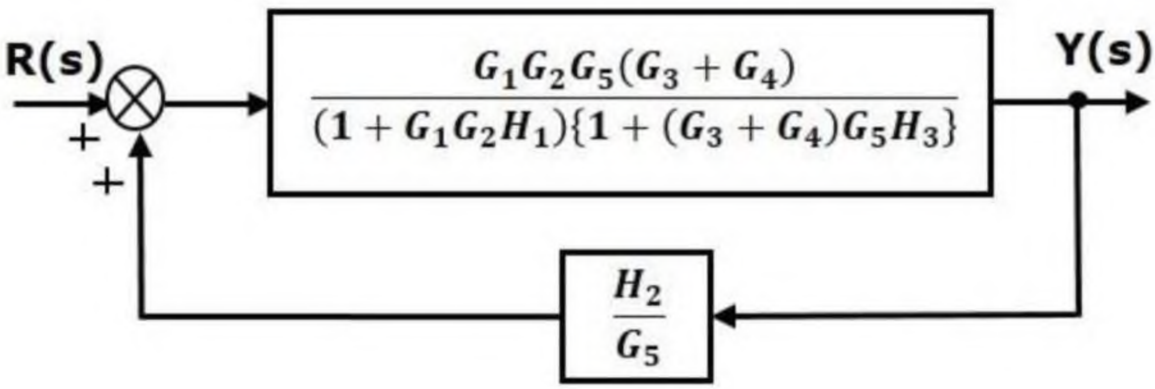
Step 3 - Use Rule 1 for blocks (G3+G4) and G5. The modified block diagram is shown in the following figure.



Step 4 - Use Rule 3 for blocks (G3+G4)G5 and H3. The modified block diagram is shown in the following figure.



Step 5 - Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



Step 6 - Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.

R(s) Y(s)

^1^2 ^5²(^3 + ^4)

(1 + + (63 + G4)G5H3]G5 — 616265(63 + 64)//2

Therefore, the transfer function of the system is

$Y(s) / R(s) = G_1G_2G_5^2(G_3+G_4) / «1+G_1G_2H_1)\{1+(G_3+G_„)G_5H_3\}G_5-G_1G_2G_5(G_3+G_4)H_2\}$