5.5 CONCEPTS OF CONTROLLABILITY AND OBSERVABILLITY

Controllability

The controllability verifies the usefulness of state variable. In the controllability test we can find, whether the state variable can be controlled to achieve the desired output. If the state variable is not controllable then we have to go for another choice of state variable.

A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $X(t_0)$ to any other desired state $X(t_d)$ in specified finite time by a control vector U(t)

The controllability of state model can be tested by kalman's test KALMAN'S METHOD OF TESTING CONTROLLABILLITY Consider a system with state equation,

$$X(t) = A\,X(t) + B\,U(t).$$

For this system, a composite matrix, Q_c can be formed such that,

$$Q_c = [B\ A\ B\ A^2B \ldots\ldots\ldots\ldots\ldots\ldots\ A^{n\sim} {*}B\ ]$$

Where, n is the order of the system (n is also equal to number of state variables). In this case the system is completely state controllable if the rank of the composite matrix, Q_c is n.

The rank of the matrix is n, if the determinant of (n x n) composite matrix Q_C is nonzero. If $|\ Q_c\ |$ A 0, then rank of Q_c = n and the system is completely state controllable.

Observability

In observability test we can find whether the state variable is observable or measurable. The concept of observability is useful in solving the problem of reconstructing unmeasurable state variables from measurable ones in the minimum possible length of time.

A system is said to be completely observable if every state X(t) an be completely identified by measurements of the output Y(t) over a finite time interval.

KALMAN'S TEST FOR OBSERVABILITY Consider a system with state model,

$X(t) = A\,X(t)$ + B U(t) ;

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For this system, a composite matrix, Q_o can be formed such that,

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Where n is the order of the system.

In this case the system is completely observable if the rank of composite matrix Q_o is n.

The rank of the matrix is n, if the determinant of (n x n) composite matrix Q_O is nonzero.