2.2 INTENSITY OF AN EM WAVE IN VACUUM

The magnitude of the average value of $\vec{S}$ at a point is called the intensity of radiation at that point. The S.I unit of intensity is $\mathrm{W/m^2}$.

Let us consider the electric and magnetic field solutions

$$\vec{E}(x,t) = E_y \cos(\omega t - kx)$$

and

$$\vec{B}(x,t) = B_z \cos(\omega)t - hz\big)$$

From eqn. (6)

$$\vec{S} = \frac{1}{\mu_o}\vec{E} \times \vec{B} \text{ becomes}$$

$$\vec{S}(x,t) = \frac{1}{\mu_o}E_y \cos(\omega t - kx) \times B_z \cos(\omega t - kz)$$

The x-component (Direction of propagation) of the poynting vector is given as

$$S_y(x,t) = \frac{E_y B_z}{\mu_o}\cos^2(\omega t - hx)$$

$$= \frac{E_y B_z}{\mu_o}\left(\frac{1 + \cos 2(\omega t - hx)}{2}\right)$$

The time average value of $\cos 2(\omega t - hx)$ is zero. So the fle value of the poynting vector is

$$S_{\text{average}} = \overline{S_x}(x,t) = \frac{E_x B_y}{2\mu_o}$$

or simply

$$S_{av} = \frac{E_y B_z}{2\mu_0} = \frac{E_y \cdot E_y}{2\mu_0 c}$$

$$= \frac{E_y E_y}{2\mu_0 \times \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

$$S_{av} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_y^2$$

$$S_{av} = \frac{1}{2}\sqrt{\frac{\varepsilon_o \varepsilon_o}{\mu_o \varepsilon_o}} E_y^2$$

$$S_{av} = \frac{E_o}{\sqrt{\mu_o \varepsilon_o} E_y^2}$$

$$I = S_{av} = \frac{1}{2}\varepsilon_0 c E_y^2$$

(or)

This is the intensity of an EM wave in vacuum.

Also intensity is represented as for localized sources as $I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2}$

MOMENTUM AND RADIATION PRESSURE

It is important to note that as EM waves carry energy, they also carry momentum. Maxwell proved that wave energy U and momentum are related by

$$P = \frac{u}{c}$$

where v is energy density and c is the velocity by of light. As the electromagnetic waves carry momentum, they exert pressure when they are reflected or absorbed at the surface of a body. This is known as radiation pressure. From Newton's second law, the change in momentum is related to a force by

$$F = \frac{\Delta P}{\Delta t}$$

As intensity I $= \frac{\text{Power}}{\text{Area}} = \frac{\text{energy / time}}{\text{Area}}$

then for a flat surface of area A, which is perpendicular to the path of an EM wave radiation, the energy intercepted in a given time Δt is

$$\Delta U = I \cdot A \cdot \Delta t$$

So, from eqn. (1), the momentum is

$$\Delta P = \frac{\Delta u}{c} = \frac{I \cdot A \cdot \Delta t}{c}$$

and as

$$F = \frac{\Delta P}{\Delta t} = \frac{I \cdot A}{c}$$

This is the relation for the total absorption of EM radiation. This is due to ' ΔP ' is the momentum change and the direction of momentum change of the object is the direction of the incident EM radiation that the object absorbs.

If the radiation is completely reflected back by the object along the original path then

$$F = \frac{2IA}{c}$$

Thus if the radiation is partly absorbed or completely reflected by the object, the magnitude of the force on area A varies between the values $\frac{IA}{c}$ and $\frac{2IA}{c}$

Radiation pressure

The force per unit area on an object due to EM radiation is the radiation pressure P_r. Thus from eqns. (5) and (6) we obtain

Radiation pressure $P_r = \frac{F}{A}$

$$P_r = \frac{I}{c}$$

for total absorption of radiation and

$$P_r = \frac{2I}{c}$$

for total reflection back along the path