HISTOGRAM MODELING

Histogram of an image **represents the relative frequency of occurrence of the various gray levels in the image.**

Histogram modeling techniques modify an image so that its histogram has a desired shape. This is useful in stretching the low contrast levels of images with narrow histograms. Thus histogram is **used to either increase or decrease**, the intensity level of the pixels by any kind of transformation.

The histogram of a digital image with gray levels in the image [0,L-1] is a discrete function given as,

$h(r_k) = n_k$, where r_k is the kth grey level, $k = 0, 1, \ldots, L-1$

n_k is the number of pixels in the image with grey level r_k

Normalized histogram is given as,

$$p(r_k) = \frac{n_k}{n}$$

Where , n is the total number of pixels in the image and n ≥ n_k

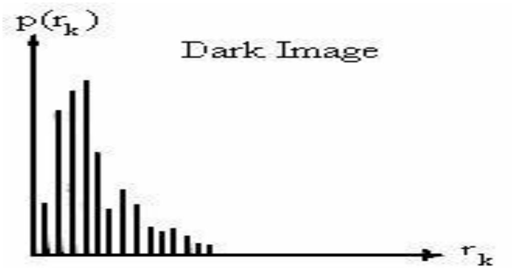
The **Normalized histogram** is the histogram **divided by the total number of pixels in the source image.**

1. The sum of all values in the normalized histogram is 1.
2. The value given by the normalized histogram for a certain gray value can be read as the probability of randomly picking a pixel having that gray value
3. The horizontal axis of each histogram plot corresponds to grey level values r_k in the image and the vertical axis corresponds to $h(r_k) = n_k$ or $p(r_k) = \frac{n_k}{n}$, if the values are normalized.

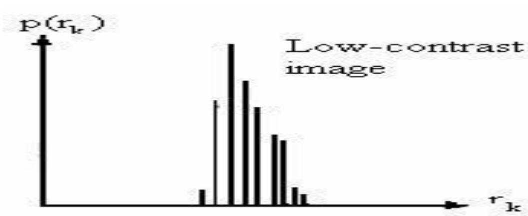
Histogram of **four basic gray level characteristics:**

- Dark image: (under exposed image): Here components of the histogram are concentrated on the low or dark side of the gray scale.
- Bright image: (over exposed image): Here components of the histogram are

concentrated on the high or bright side of the gray scale.



- Low-contrast image: Here components of the histogram at the middle of the gray scale and are narrow.
- High-contrast image: Here components of the histogram cover a broad range of the gray scale and they are almost uniformly distributed.



HISTOGRAM EQUALIZATION:

A technique which is used to obtain uniform histogram is known as <u>histogram equalization or histogram linearization</u>**.**

Let r represent the grey levels in the image to be enhanced. Assume r to be normalized in the interval [0, 1], with r = 0 representing black and r = 1 representing white.

For any value r in the interval [0,1], the image transformation is given as,

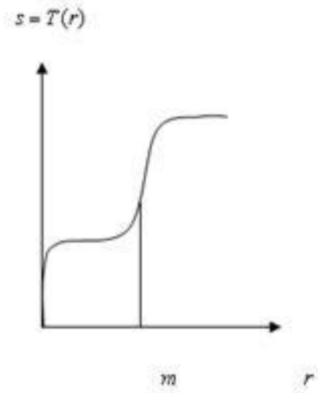
$$S = T(r),\ 0 \leq r \leq 1$$

This transformation produces a level **s** for every pixel value **r** in the original image. The transformation function T(r) satisfies the following conditions,

(a) T(r) is single –valued and monotonically increases in the interval $0 \leq r \leq 1$ (ie., preserves the order from black to white in the output image) and

(b) $0 \leq T(r)) \leq 1$ for $0 \leq r \leq 1$ (ie., guarantees that the output gray levels will be in the same range as the input levels)

The following shows the transformation function satisfying the above two conditions.



The Inverse transformation is given as,

$$r = T^{-1}(s),\ 0 \le s \le 1$$

If the gray levels in an image can be viewed as random variables in the interval [0,1], then $P_r(r)$ and $P_s(s)$ denote the pdfs of the random variables r and s.

If $P_r(r)$ and T(r) are known and $T^{-1}(s)$ satisfies the condition (a) , then

$$P_s(s) = \left[P_r(r) \left| \frac{dr}{ds} \right| \right]_{r = T^{-1}(s)} \quad \text{-------------} \quad (1)$$

Thus , the pdf of the transformed variables is determined by the gray level pdf of the input image and by the chosen transformation function.

For Continuous gray level values:

Consider the transformation,

$$s = T(r) = \int_0^r p_r(w)dw \quad \text{--(2)}$$

where w is a dummy variable of integration and

$\int_0^r p_r(w)dw$ = cummulative distribution function [CDF] of random variable 'r' condition (a) and (b) are satisfied by this transformation as CDF increases monotonically from 0 to 1 as a function of r.

When T(r) is given, we can find $P_s(s)$ using equation (1),

We know that,

$$s = T(r)$$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = \frac{d}{dr}\int_0^r p_r(w)\,dw = p_r(r)$$

$$\frac{dr}{ds} = \frac{1}{\Pr(r)} \text{--}(3)$$

Substituting (3) in (1), we get,

$$p_s(s) = \left[p_r(r) \left| \frac{dr}{ds} \right| \right], \quad 0 \le s \le 1$$

$$= \left[p_r(r) \left| \frac{1}{p_r(r)} \right| \right]$$

$= 1$, $0 \le s \le 1$

This indicates that $P_s(s)$ is a uniform pdf.

For Discrete gray level values:

The probability of occurrence of gray level r_k in an image is given as,

$$p_r(r_k) = \frac{n_k}{n}, k = 0,1,.....L-1$$

Where, n = total no.of pixels in the image

n_k = No.of pixels that have gray level r_k

L = total no.of possible gray levels in the image

The discrete form of the transformation function is ,

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}, \; 0 \le r_k \le 1, \; k = 0,1,\ldots,L-1$$

Thus the processed output image is obtained by mapping each pixel with level r_k in the input image into a corresponding pixel with level s_k in the output image.

- The inverse transformation is given as ,

  $r_k = T^{-1}(s_k), k = 0,1,2,...L-1$

Histogram Modificatin: A technique which is used to modify the histogram is known as histogram modification.

Here the input gray level r is first transformed non-linearly by T(r) and the output is uniformly quantized.

In histogram equalization , the function $T(r) = \sum_{j=0}^{k} p_r(r_j), \quad k = 0,1,\ldots, L-1$

The above function performs a compression of the input variable.

Other choices of T(r) that have similar behavior are,

$T(r) = \log(1+r)$, $r \geq 0$

$T(r) = r^{1/n}$, $r \geq 0$, n =2,3…..

Histogram Specification:

It is defined as a method which is **used to generate a processed image that has a specified histogram.**Thus, it is used to specify the shape of the histogram, for the processed image & the shape is defined by the user.Also called **histogram matching**

For Continuous case:

Let r be an input image, Z be an output image

$p_r(r)$ is the original probability density function

$p_z(z)$ is the desired probability density function

Let S be a random variable and if the histogram equalization is first applied on the original image r, then

$$s = T(r) = \int_0^r p_r(w)dw$$

Suppose that the desired image z is available and histogram equalization is given as,

$$s = G(z) = \int_0^z p_z(z)dz$$

Where G is the transformation of z = s .

From equation (1) and (2),

G(z) = s = T(r)

The inverse process $z = G^{-1}(s) = G^{-1}[T(r)]$

Therefore, the process of histogram specification can be summarised in the following steps.

i) Obtain the transformation function T(r) using equation (1)

ii) Obtain the transformation function G(z) using equation (2)

iii) Obtain the Inverse transformation function G^{-1}

iv) Obtain the output image by applying using equation (3) to all pixels in the input image

This procedure yields an image whose gray levels z have the specified pdf $p_z(z)$

For Discrete gray level values:

The probability of occurrence of gray level r_k in an image is given as,

$$p_r(r_k) = \frac{n_k}{n}, k = 0,1,.....L-1$$

Where, n = total no.of pixels in the image

n_k = No.of pixels that have gray level r_k

L = total no.of possible gray levels in the image

The discrete form of the transformation function is ,

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}, \ 0 \le r_k \le 1, \ k = 0,1,\ldots,L-1$$

Also, the discrete form of the desired transformation function is ,

$$s_k = G(z_k) = \sum_{i=0}^{k} p_z(z_i),$$

From (1) and (2), the inverse transformation is given as ,

$$G(z_k) = s_k = T(r_k)$$

$$z_k = G^{-1}(s_k) = G^{-1}[T(r_k)]$$

The principal noise sources in the digital image arise during image acquisition or digitization or transmission. The performance of imaging sensors is affected by a variety of factors such as environmental conditions during image acquisition and by the quality of the sensing elements.

Assuming the noise is independent of spatial coordinates and it is uncorrelated with respect to the image itself.

The various Noise probability density functions are as follows,