Bus Impedance matrix building algorithm

Bus impedance matrix Zbus of a power network can be obtained byinverting the bus admittance matrix Ybus , which is easy to construct. However, when the order of matrix is large, direct inversion requires more core storage and enormous computer time. Therefore inversion of Ybus is prohibited for large size network. Bus impedance matrix can be constructed by adding the network elements one after the other. Using impedance parameters, performance equations in bus frame of reference can be written as

$$E_{bus} = Z_{bus} I_{bus}$$

In the expanded form the above becomes

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

From this we can write

$E_p = Z_{p1} I_1 + Z_{p2} I_2 + ... + Z_{pq} I_q + ... + Z_{pN}$

From the above, it can be noted that with Iq = 1 p.u. other bus currents set to

zero, Ep = Zpq . Thus Zpq can be obtained by measuring Ep when 1 p.u. current is injected at bus q and leaving the other bus currents as zero. In fact p and q can be varied from 1 to N.While making measurements all the buses except one, are opencircuited. Hence, the bus impedance parameters are called open circuitimpedances. The diagonal elements in Zbus are known as driving pointimpedances, while the off-diagonal elements are called transferimpedances.

Symmetrical fault analysis through bus impedance matrix. Once the bus impedance matrix is constructed, symmetrical fault analysiscan be carried out with a very few calculations. Bus voltages andcurrents in various elements can be computed quickly. When faults areto be simulated at different buses, this method is proved to be good.Symmetrical short circuit analysis essentially consists of determining thesteady state solution of linear network with balanced sources.Since the short circuit currents are much larger compared to prefault currentsthe following assumptions are made while conducting short circuit study.

1. all the shunt parameters like loads, line charging admittances

   etc. are neglected.

2. all the transformer taps are at nominal position.

3. prior to the fault, all the generators are assumed to operate at

   rated voltage of 1.0 p.u. with their emf's in phase. With these assumptions, in the prefault condition, there will not be anycurrent flow in the network and all the bus voltages will be equal to1.0 p.u.

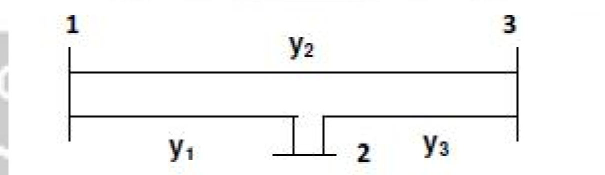
The linear network that has to be solved comprises of

i) Transmission network
ii) ii) Generation system and iii) Fault

By properly combining the representations of the above three components,

we can solve the short circuit problem.

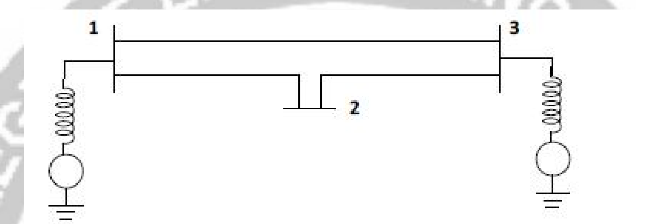
Consider the transmission network shown in Fig



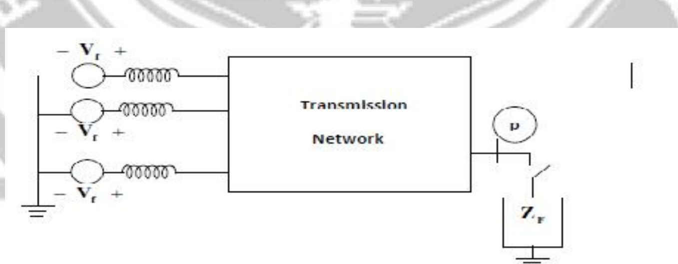
Taking the ground as the reference bus, the bus admittance matrix is obtained as

$$Y_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} y_1 + y_2 & -y_1 & -y_2 \\ -y_1 & y_1 + y_3 & -y_3 \\ -y_2 & -y_3 & y_2 + y_3 \end{bmatrix} \end{array}$$

If we add all the columns (or rows) we get a column (or row) of allzero elements. Hence this Ybus matrix is singular and hence correspondingZbus matrix of this transmission network does not exist. Thus, when all the shunt parameters are neglected, Zbus matrix will not exist for thetransmission network. However, connection to ground is established at the generator buses, representing the generator as a constant voltage source behind appropriatereactance as shown in Fig.



If the generator reactance are included with the transmission network, Zbus matrix of the combined network can be obtained. As stated earlier, there is no current flow in the network in the pre-fault condition and all the bus voltages will be 1.0 p.u. Consider the network shown in Fig. Symmetrical fault occurring at bus 2can be simulated by closing the switch shown in Fig.HereZf is the fault impedance



In the faulted system there are two types of sources:

1. Current injection at the faulted bus

2. Generated voltage sources.

The bus voltages in the faulted system can be obtained using Superposition

Theorem.

Bus voltages due to current injection:

Make all the generator voltages to zero. Then we have Generator-Transmission system without voltage sources. Such network has transmission parameters and generator reactances between generator buses and the ground. LetZbus be the bus impedance matrix of such Generator-Transmission network. Then the bus voltages due to the current injection will be given by

$$Vbus = Z_{bus} I_{bus}\ (F)$$

where I_{bus} (F) is the bus current vector having only one non-zero element.

Thus when the fault is at the p[th] bus

$$I_{bus\,(F)} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I_{p\,(F)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Here Ip (F) is the faulted bus current

$$V_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2N} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pN} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Np} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_{p\,(F)} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{1p} \\ Z_{2p} \\ \vdots \\ Z_{pp} \\ \vdots \\ Z_{Np} \end{bmatrix} I_{p\,(F)}$$

Bus voltages due to generator voltages

Make the fault current to be zero. Since there is no shunt element, there will be no current flow and all the bus voltages are equal to V0 , the pre-fault voltage which will be normally equal to 1.0 p.u. Thus, bus voltages due to generator voltages will be

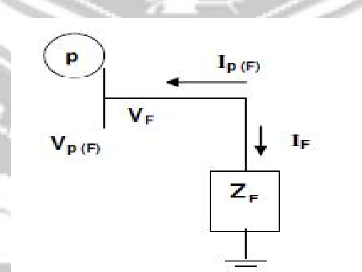
$$V_{bus} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} V_0$$

Thus for the faulted system, wherein both the current injection and generator sources are simultaneously present, the bus voltages can be obtained by adding the voltages. Therefore, for the faulted system the bus voltages are

$$V_{bus\,(F)} = \begin{bmatrix} V_{1(F)} \\ V_{2(F)} \\ \vdots \\ V_{p(F)} \\ \vdots \\ V_{N(F)} \end{bmatrix} = \begin{bmatrix} Z_{1p} \\ Z_{2p} \\ \vdots \\ Z_{pp} \\ \vdots \\ Z_{Np} \end{bmatrix} I_{p\,(F)} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} V_0$$

To calculate V_{bus} (F) we need the faulted bus current Ip (F) which can be

determined as discussed below. The fault can be described as shown in Fig.



It is clear that $V_F = Z_F I_F$,Vp (F) = V_F and Ip (F) = - I_F

Therefore Vp (F) = - Z_F Ip (F) .

The pth equation extracted from eqngivesVp (F) = $Z_{pp} I_p$ (F) + V0

Vp (F) = - $Z_F I_p$ (F)

The pth equation extracted from above equation gives

V_p (F) = $Z_{pp} I_p$ (F) + V0

Substituting eqn. in the above, we get

$- Z_F \, Ip\,(F) = Z_{pp} I_p\,(F) + V0$

Thus the faulted bus current I_p (F) is given by

$$I_{p\,(F)} = - \frac{V_0}{Z_{pp} + Z_F}$$

Substituting the above in eqn. (3.41), the faulted bus voltage Vp (F) is

$$V_{p\,(F)} = \frac{Z_F}{Z_{pp} + Z_F} V_0$$

Finally voltages at other buses at faulted condition are to be obtained. The ithequation extracted from equation gives

$$V_i\,(F) = Z_{ip}\, I_p\,(F) + V0$$

Substituting in the above, we get

$$V_{i\,(F)} = V_0 - \frac{Z_{ip}}{Z_{pp} + Z_F} V_0 \qquad \begin{matrix} i = 1,2,\ldots\ldots\ldots,N \\ i \neq p \end{matrix}$$

Knowing all the bus voltages, current flowing through the various networkelements can be computed asi k_m (F) = (V_k (F) – V_m (F)) yk_m where yk_m is the admittance of element k-m.

When the fault is direct, ZF = 0 and hence

$$I_{p\,(F)} = - \frac{V_0}{Z_{pp}}$$

$$V_{p\,(F)} = 0 \text{ and}$$

$$V_{i\,(F)} = V_0 - \frac{Z_{ip}}{Z_{pp}} V_0 \qquad \begin{matrix} i = 1,2,\ldots\ldots\ldots,N \\ i \neq p \end{matrix}$$

It is to be noted that when the fault occurs at the p_{th} bus, only the p^{th} column of Zbus matrix (and not the entire Zbus matrix) is required for further calculations.

The following are the various steps for conducting symmetrical short circuit analysis.

Step 1 **Read**
- i) **Transmission line data**
- ii) **Generator reactances data**
- iii) **Faulted bus number p and**
- iv) **Fault impedance Z_F.**

Step 2 **Construct the bus impedance matrix of the transmission network including the generator reactances.**

Step 3 **Compute** $I_{p(F)} = -\dfrac{V_0}{Z_{pp} + Z_F}$

Step 4 **Compute** $V_{p(F)} = \dfrac{Z_F}{Z_{pp} + Z_F} V_0$

Step 5 **Compute** $V_{i(F)} = V_0 - \dfrac{Z_{ip}}{Z_{pp} + Z_F} V_0 \qquad i = 1,2,\ldots\ldots\ldots,N;\ i \neq p$

Step 6 **Calculate the element currents from** $i_{km(F)} = (V_{k(F)} - V_{m(F)})\, y_{km}$