UNIT-V

GasPowerCycles

Introduction

Forthepurposeofthermodynamicanalysisoftheinternalcombustionengines,the following approximations are made:

- Theengineisassumedtooperateonaclosedcyclewithafixedmassofairwhich does not undergo any chemical change.
- Thecombustionprocessisreplacedbyanequivalentenergyadditionprocessfrom an external source.
- The exhaust process is replaced by an equivalent energy rejection process to externalsurroundingsbymeansofwhichtheworkingfluidisrestoredtotheinitial state.
- Theairisassumedtobehavelikeanidealgaswithconstantspecificheat.These cycles are usually referred to as air standard cycle.

OttoCycle

The Air Standard Otto cycle is named after its inventor **Nikolaus A. Otto** . Figures 5.1 (a), (b) and (c) illustrate the working principles of an Otto cycle. The Otto cycle consists of the following processes.

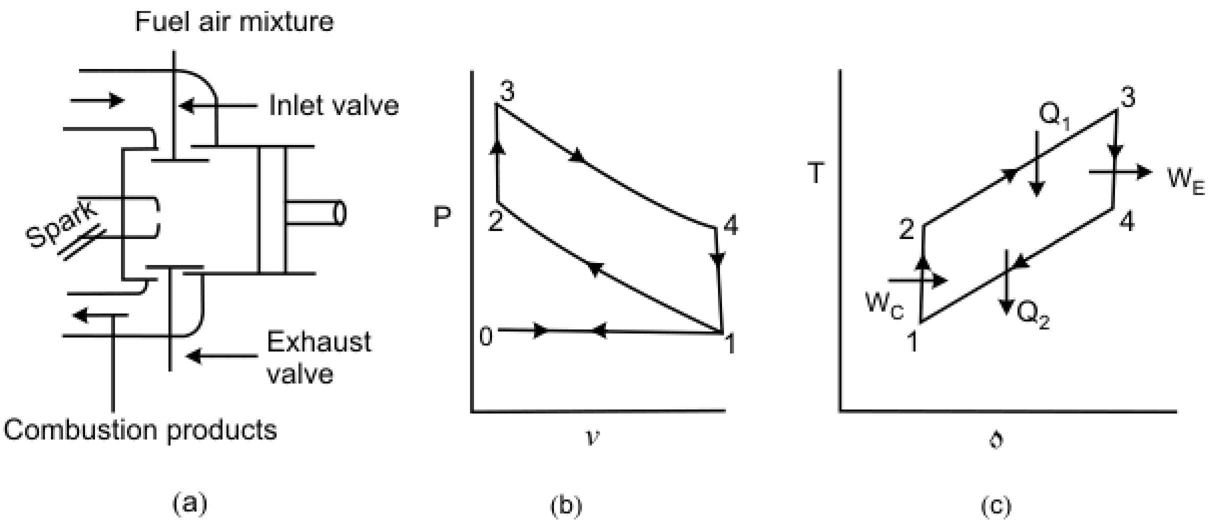


Figure 5.1

0-1: Constant pressure suction during which a mixture of fuel vapour and air is drawn into the cylinder as the piston executes an outward stroke.

1-2:Themixtureiscompressedisentropicallyduetotheinwardmotionofthepiston. Because of the **isentropic compression** , the temperature of the gas increases.

2-3: The hot fuel vapour-air mixture is ignited by means of an electric spark. Since the combustion is instantaneous, there is not enough time for the piston o move outward. This process is approximated as a **constant volume energy addition process** .

3-4: The hot combustion products undergo **isentropic expansion** and thepiston executesan outward motion.

4-1: The exhaust port opens and the combustion products are exhausted into the atmosphere. The process is conveniently approximated as a **constant-volume energy rejection process**.

1-0:Theremainingcombustionproductsareexhaustedbyaninwardmotionof thepistonat constant pressure.

Effectively there are four strokes in the cycle. These are suction, compression, expression, and exhaust strokes, respectively. From the P-V diagram it can be observed that the work done during the process 0-1 is exactly balanced by the work done during 1-0. Hence for the purpose of thermodynamic analysis we need to consider only the cycle 1-2-3-4, which is air-standard Otto Cycle.

$$\eta = \frac{W_{net}}{Q_1} = \frac{(Q_1 - Q_2)}{Q_1} \qquad (5.1)$$

Where Q_1 and Q_2 denotetheenergyabsorbedandenergyrejectedintheformofheat.
Application of the first law of thermodynamics to process 2-3 and 4-1gives:

$$Q_1 = U_3 - U_2 = m(u_3 - u_2) = mc_v(T_3 - T_2) \qquad (5.2)$$

$$Q_2 = U_4 - U_1 = m(u_4 - u_1) = mc_v(T_4 - T_1) \qquad (5.3)$$

Therefore,

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} \qquad (5.4)$$

1-2 and 3-4 are isentropic processes for which $TV^{\gamma-1} =$ constant

Therefore $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ or $\frac{T_2}{T_!} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

(5.5)

and

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \tag{5.6}$$

But

$$V_1 = V_4 \quad \text{and} \quad V_2 = V_3 \tag{5.7}$$

Hence

$$\frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \tag{5.8}$$

So,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_3}{T_2} = \frac{T_4}{T_1} \tag{5.9}$$

$$\frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1 \tag{5.10}$$

or

$$\frac{T_3 - T_2}{T_2} = \frac{T_4 - T_1}{T_1} \quad \text{or} \quad \frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} \tag{5.11}$$

and

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1}{r_0}\right)^{\gamma-1} \tag{5.12}$$

$$\eta = 1 - \left(\frac{1}{r_0}\right)^{\gamma-1} \tag{5.13}$$

Where

$$r_0 = \frac{V_1}{V_2} = \textbf{Compression ratio} \qquad (5.14)$$

Since $\gamma > 1$, the efficiency of the Otto cycle increases with increasing compression ratio.However, in an actual engine, the compression ratio can not be increased indefinitely since highercompressionratiosgivehighervaluesof T_2 andthisleads to **spontaneous** and **uncontrolled** combustion of the gasoline-air mixture in the cylinder. Such a condition is usually called knocking.

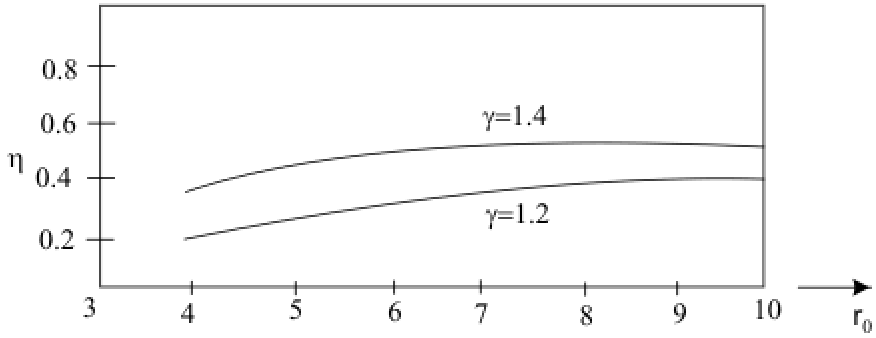


Figure5.2

Performance of an engine is evaluated in terms of the efficiency (see Figure 5.2). However, sometime it is convenient to describe the performance in terms of mean effective pressure, an imaginary pressure obtained by equating the cycle work to the work evaluated by the following the relation

$$W_{net} = P_m \int dV = P_m (V_1 - V_2) \qquad (5.15)$$

Themeaneffectivepressureisdefinedasthenetworkdividedbythedisplacementvolume. That is

$$P_m = \frac{W_{net}}{(V_1 - V_2)}$$

DIESEL CYCLE

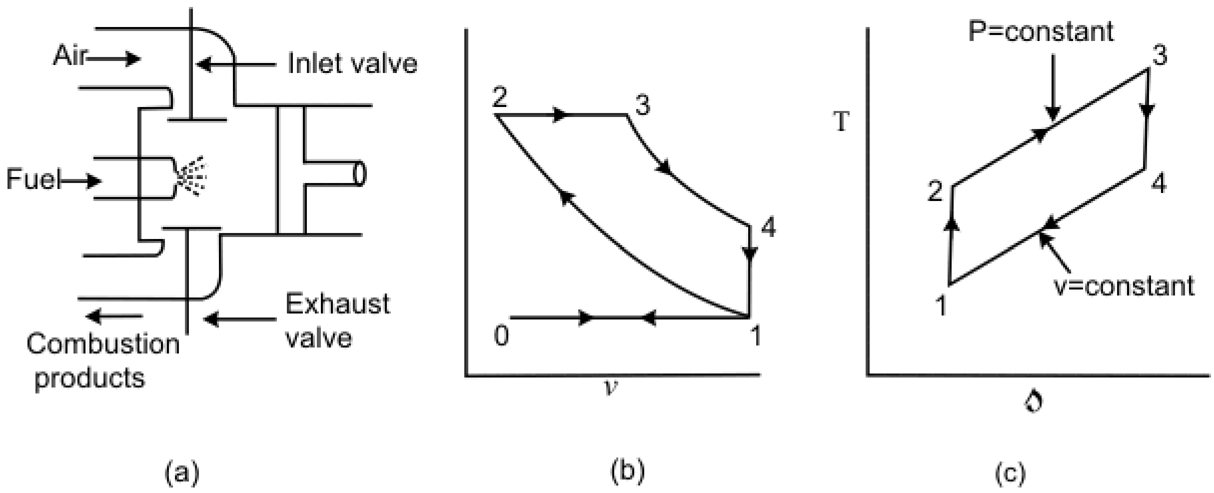


Figure5.3 (a),(b)and (c)

TheDieselcyclewas developedbyRudolfDieselinGermany.Figures5.3(a),(b)and (c) explaintheworkingprincipleofanAirStandardDieselcycle.Thefollowingarethe processes.

0-1: Constantpressure suctionduringwhich fresh air isdrawn into the cylinderas thepiston executes the outward motion.

1-2: The air is compressed isentropically. Usually the compression ratio in the **Diesel cycle is much higher them that of Otto cycle**. Because of the high compression ratio, the temperature of the gas at the end of isentropic compression is so high that when fuel is injected, it gets ignited immediately.

2-3: The fuel is injected into the hot compressed air at state 2 and the fuel undergoes a chemical reaction. The combustion of Diesel oil in air is not as spontaneous as the combustion of gasoline and the combustion is relatively slow. Hence the piston starts moving outward as combustion take place. The combustion processes is conveniently approximated as **a constant pressure energy addition process**.

3-4: The combustion products undergo isentropic expansion and the piston executes an outward motion.

4-1: The combustion products are exhausted at constant volume when the discharge port opens. This is replaced by a **constant-volume energy rejection process**.

1-0: The remaining combustion products are exhausted at constant pressure by the inward motion of the piston.

IntheanalysisofaDieselcycle,twoimportantparametersare:compression ratio $(r_0 = V_1/V_2)$ and the cut-off ratio (r_c). The cut-off ratio is defined as the ratio of thevolume at the end of constant-pressure energy addition process to the volume at the beginning of the energy addition process.

$$r_c = \frac{V_3}{V_2} \tag{5.16}$$

Energy added $= Q_1 = mc_p (T_3 - T_2)$ (5.17)

Energy rejected $= Q_2 = mc_v (T_4 - T_1)$ (5.18)

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T_4 - T_1}{\gamma (T_3 - T_2)} \tag{5.19}$$

or

$$\eta = 1 - \frac{\{(T_4/T_1) - 1\} T_1}{\{(T_3/T_2) - 1\} \gamma T_2} \tag{5.20}$$

1-2is Isentropic:

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1}{r_0}\right)^{\gamma-1}$$

4-1isConstant volume:

$$\frac{T_4}{T_1} = \frac{P_4}{P_1} = \frac{P_4}{P_3} \times \frac{P_3}{P_1}$$

But $P_2 = P_3$

Hence

$$\frac{T_4}{T_1} = \frac{P_4}{P_3} \times \frac{P_2}{P_1}$$

Since **1–2** and**3–4**areisentropicprocesses$\left(PV^{\gamma} = C\right)$

$$\frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^{\gamma}$$ and $$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

Hence

$$\frac{T_4}{T_1} = \left(\frac{V_3}{V_4}\right)^{\gamma} \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{V_3}{V_2}\right)^{\gamma} = (r_c)^{\gamma} \qquad (5.21)$$

$$\eta = 1 - \frac{\left\{(r_c)^{\gamma} - 1\right\}}{\gamma (r_0)^{\gamma-1} \left\{r_c - 1\right\}} \qquad (5.22)$$

Alsotobe noted

$$\frac{T_3}{T_2} = \frac{P_3 V_3}{P_2 V_2} = \frac{V_3}{V_2} = r_c \qquad (5.23)$$

ThecompressionratiosnormallyintheDieselenginesvarybetween14and17.

AIRSTANDARDDUALCYCLE

Figures 5.4 (a) and (b) shows the working principles of a Dual cycle. In the dual cycle, the energy addition is accomplished in two stages: Part of the energy is added at constant volume and part of the energy is added at constant pressure. The remaining processes are similar to those of the Otto cycle and the Diesel cycle. The efficiency of the cycle can be estimated in the following way

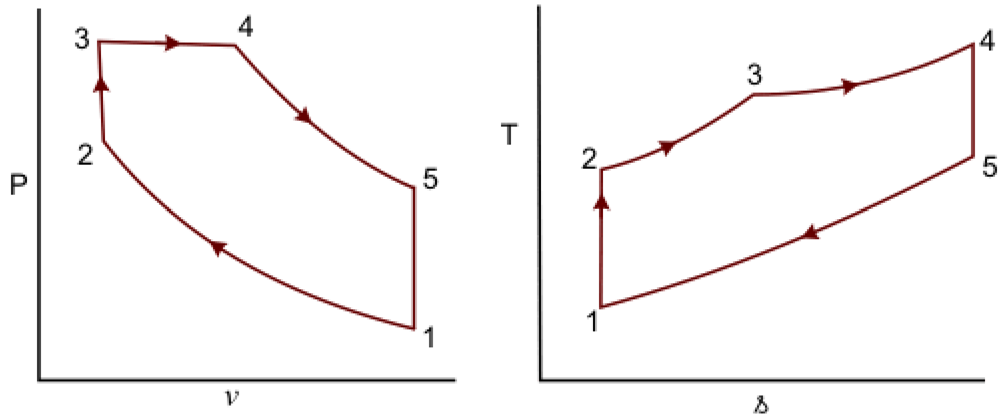


Figure5.4.1(a)and (b)

Energy added

$$q_1 = c_v(T_3 - T_2) + c_p(T_4 - T_3) \tag{5.24}$$

Energy rejected

$$q_2 = c_v(T_5 - T_1) \tag{5.25}$$

$$\eta = 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)} \qquad (5.25)$$

or

$$\eta = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \tag{5.26}$$

Theefficiencyofthecyclecanbeexpressedintermsofthefollowingratios

Compressionratio, $r_0 = \frac{V_1}{V_2}$ (5.27)

Cut-off ratio, $r_c = \frac{V_4}{V_3}$ (5.28)

Expansionratio, $r_e = \frac{V_5}{V_4}$ (5.28)

Constantvolumepressureratio, $r_{vp} = \frac{p_3}{p_2}$ (5.29)

$$\eta_{Dual} = 1 - \frac{1}{(r_0)^{\gamma-1}} \frac{r_{vp}(r_c)^{\gamma} - 1}{(r_{vp} - 1) + \gamma r_{vp}(r_c - 1)} \quad (5.30)$$

If $r_c = 1,\ \eta_{Dual} \rightarrow \eta_{Otto}$

If $r_{vp} = 1,\ \eta_{Dual} \rightarrow \eta_{Diesel}$

ComparisonofOtto,Diesel&Dual Cycles

Forsamecompressionratioandheatrejection(Figures5.5(a)and(b))

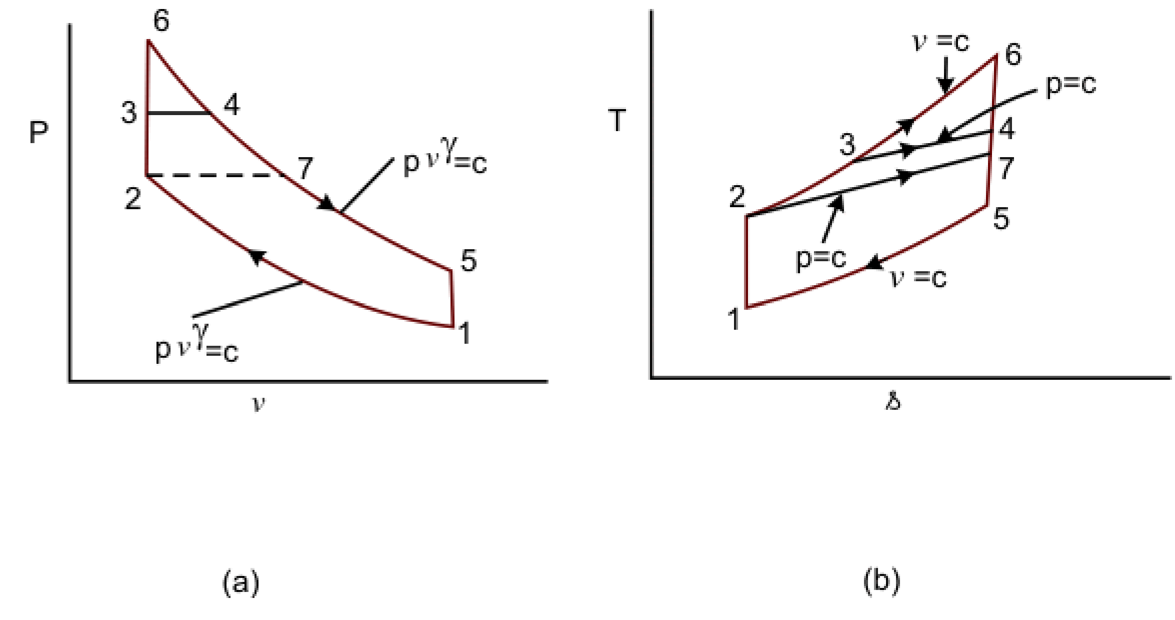


Figure5.5 (a)and(b)

1-6-4-5:Ottocycle

1-7-4-5:Dieselcycle

1-2-3-45Dualcycle

Forthe same Q_2,thehighertheQ$_1$,thehigheristhecycle efficiency

$$\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$$

Forthesamemaximumpressureand temperature(Figures5.6(a)and(b))

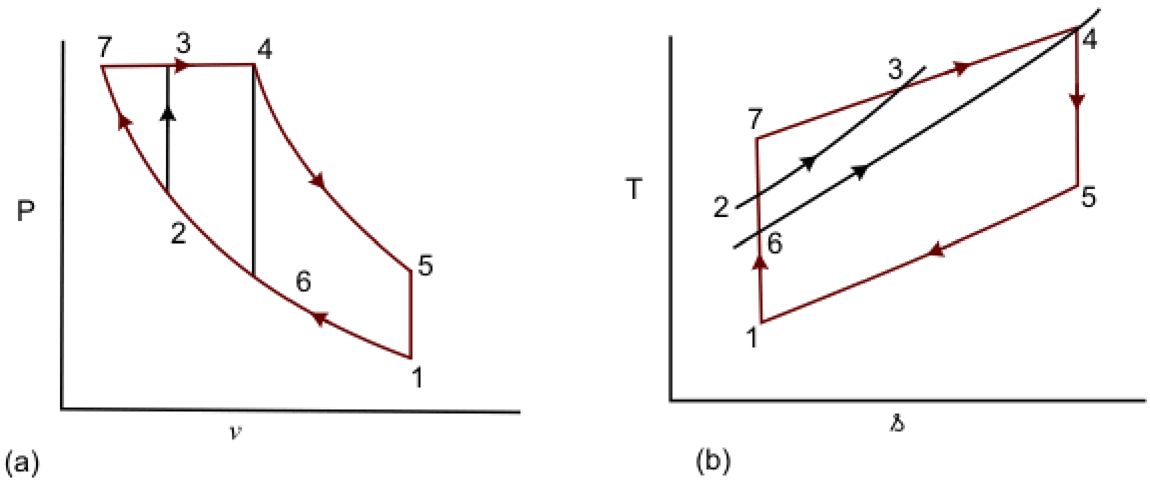


Figure5.6 (a)and(b)

1-6-4-5:Ottocycle

1-7-4-5:Dieselcycle

1-2-3-45 Dual cycle

Q_1isrepresentedby:

Area under6-4 $\rightarrow$ for Otto cycle
area under7-4 $\rightarrow$ for Diesel cycle

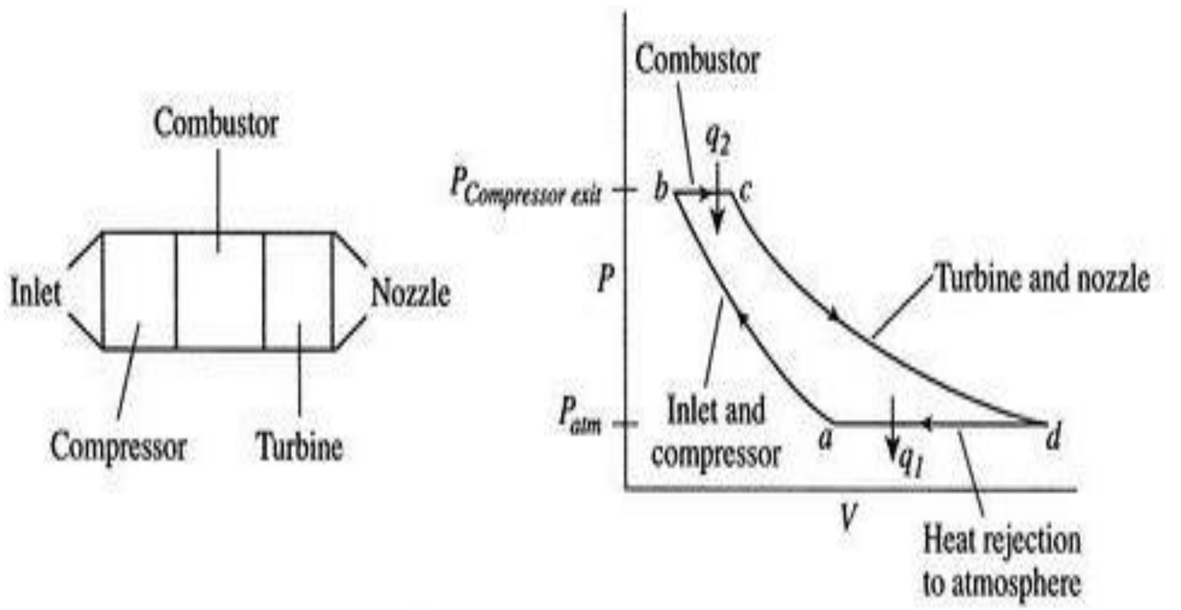
and
areaunder2-3-4$\rightarrow$forDualcycle and Q_2issame for allthecycles

$$\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$$

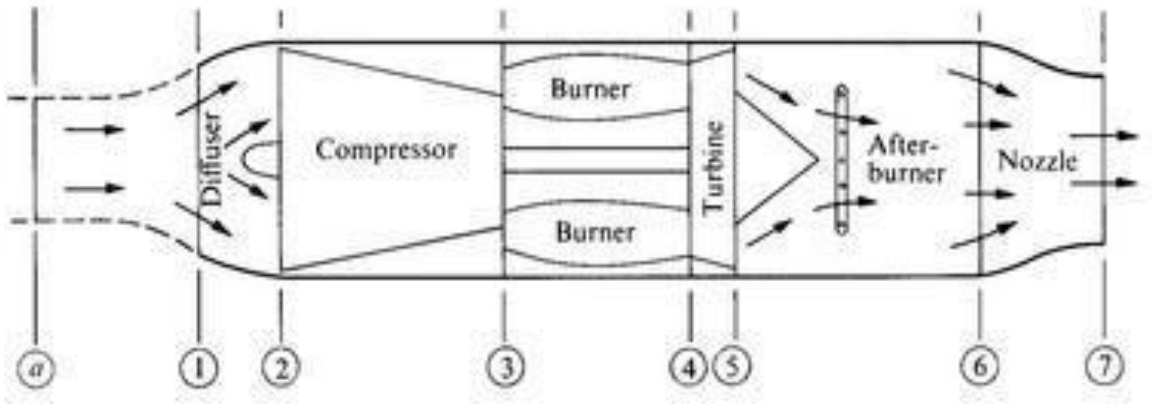
TheBrayton cycle

TheBraytoncycle(orJoulecycle)representstheoperationofagasturbineengine.Thecycle consists of four processes, as shown in Figure alongside a sketch of an engine:

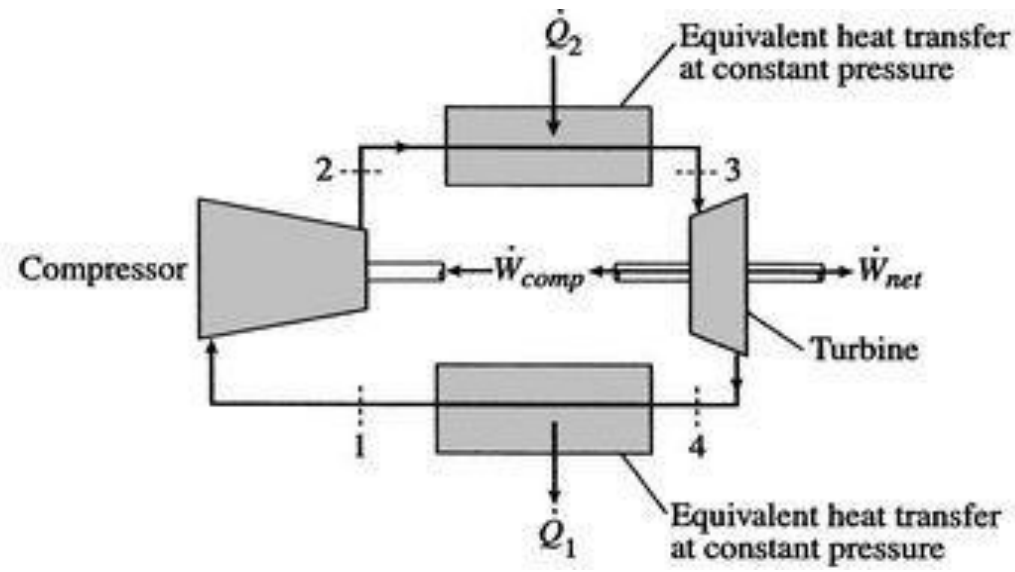
a-bAdiabatic,quasi-static(orreversible)compressionintheinletandcompressor;
b-cConstantpressurefuelcombustion(idealizedasconstantpressureheat addition);
c-dAdiabatic,quasi-static(orreversible)expansioninttheturbineandexhaustnozzle,with which we takesomeworkoutoftheairanduseittodrivethecompressor,and taketheremainingwork out and use it to accelerate fluid for jet propulsion, or to turn a generator for electrical power generation;
d-aCooltheair atconstantpressurebacktoitsinitialcondition.

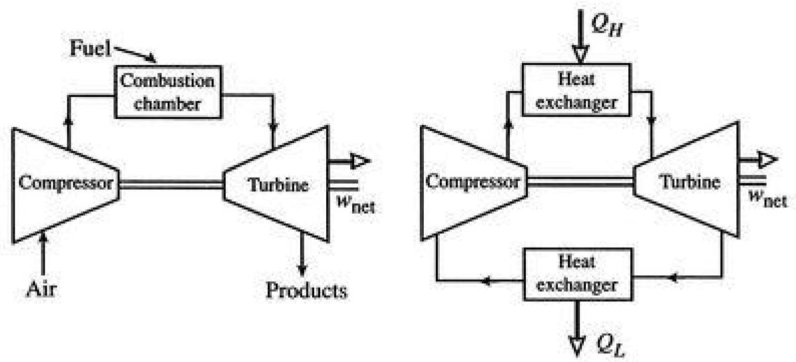


The components of a Brayton cycle device for jet propulsion are shown in Figure 3.14. We will typically represent these components schematically, as in Figure 3.15. In practice, real Brayton cycles take one of two forms. Figure 3.16(a) shows an ``open'' cycle, where the working fluid enters and then exits the device. This is the way a jet propulsion cycle works. Figure 3.16(b) shows the alternative, a closed cycle, which re circulates the working fluid. Closed cycles are used, for example, in space power generation.



Schematicsoftypicalmilitarygasturbineengines.





Open cycle operation Closed cycle operation

OptionsforoperatingBraytoncyclegasturbineengines

WorkandEfficiency

The objective now is to find the work done, the heat absc$a$͏eb, ꜫcd de thermal efficiency ofthe cycle.Tracingthepath shownaroundthe cycle from---and backto ,thefirst law gives (writing the equation in terms of a unit mass),

$$\Delta u_{a\text{-}b\text{-}c\text{-}d\text{-}a} = 0 = q_2 + q_1 - w.$$

Here$\Delta u$is zero because$u$is a function of state, and any cycle returns the system to itsstarting state[3.2]. The net work done is therefore

$$w = q_2 + q_1,$$

q_1 q_2 q_1
where,are defined as heat received **by**the system (is negative). We thus need to evaluate the heat transferred in processes$b$-$c$and$d$-a.

Foraconstantpressure,quasi-staticprocesstheheatexchangeperunitmassis

$$dh = c_p dT = dq, \quad \text{or} \quad [dq]_{\text{constant P}} = dh.$$

Wecanseethisbywritingthefirstlawin termsofenthalpy orbyrememberingthedefinition of c_p.

Theheatexchangecanbeexpressedintermsofenthalpydifferencesbetweentherelevant states. Treating the working fluid as a perfect gas with constant specific heats, for the heat addition from the combustor,

$$q_2 = h_c - h_b = c_p(T_c - T_b).$$

Theheatrejectedis,similarly,

$$q_1 = h_a - h_d = c_p(T_a - T_d).$$

Thenetworkperunitmassisgivenby

$$\text{Net work per unit mass } = q_1 + q_2 = c_p[(T_c - T_b) + (T_a - T_d)].$$

ThethermalefficiencyoftheBraytoncyclecannowbeexpressedintermsofthe temperatures:

$$\eta = \frac{\text{Net work}}{\text{Heat in}} = \frac{c_p[(T_c - T_b) - (T_d - T_a)]}{c_p[T_c - T_b]} = 1 - \frac{(T_d - T_a)}{(T_c - T_b)} = 1 - \frac{T_a(T_d/T_a - 1)}{T_b(T_c/T_b - 1)}.$$

To proceed further, we need to examinethe relationshipsbetween the different temperatures. Weknowthatpoints$a$and$d$areonaconstantpressureprocessasarepoints$b$and$c$, and $P_a = P_d$; $P_b = P_c$ .Theothertwolegsofthecycleareadiabaticandreversible,so

$$\frac{P_d}{P_c} = \frac{P_a}{P_b} \qquad \Rightarrow \qquad \left(\frac{T_d}{T_c}\right)^{\gamma/(\gamma-1)} = \left(\frac{T_a}{T_b}\right)^{\gamma/(\gamma-1)}.$$

Therefore $T_d/T_c = T_a/T_b$,or,finally, $T_d/T_a = T_c/T_b$. Using this relation in the expression for thermal efficiency, Eq. (3.8) yields an expression for the thermal efficiency of a Brayton cycle

$$\text{Ideal Brayton cycle efficiency: } \eta_B = 1 - \frac{T_a}{T_b} = 1 - \frac{T_{\text{atmospheric}}}{T_{\text{compressor exit}}}.$$

Thetemperatureratioacrossthecompressor, $T_b/T_a = TR$. In terms of compressor temperature ratio, and using the relation for an adiabatic reversible process we can write the efficiency in terms of the compressor (and cycle) pressure ratio, which is the parameter commonly used:

$$\eta_B = 1 - \frac{1}{TR} = 1 - \frac{1}{PR^{(\gamma-1)/\gamma}}.$$

THE RANKINE CYCLE

The Rankine cycle is an ideal cycle for vapour power cycles. Many of the impracticalities

a

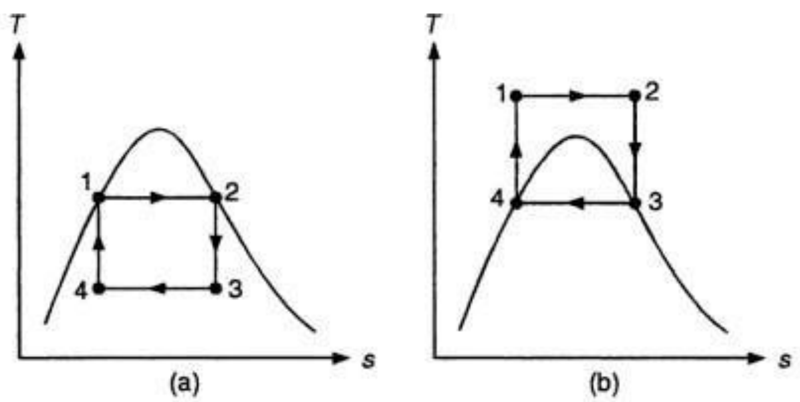


Figure 9.1 *T–s* diagram of two Carnot vapour cycles.

associated with the Carnot cycle can be eliminated by superheating the steam in the boiler and condensing it completely in the condenser, as shown in Fig. 9.2. The processes involved are:

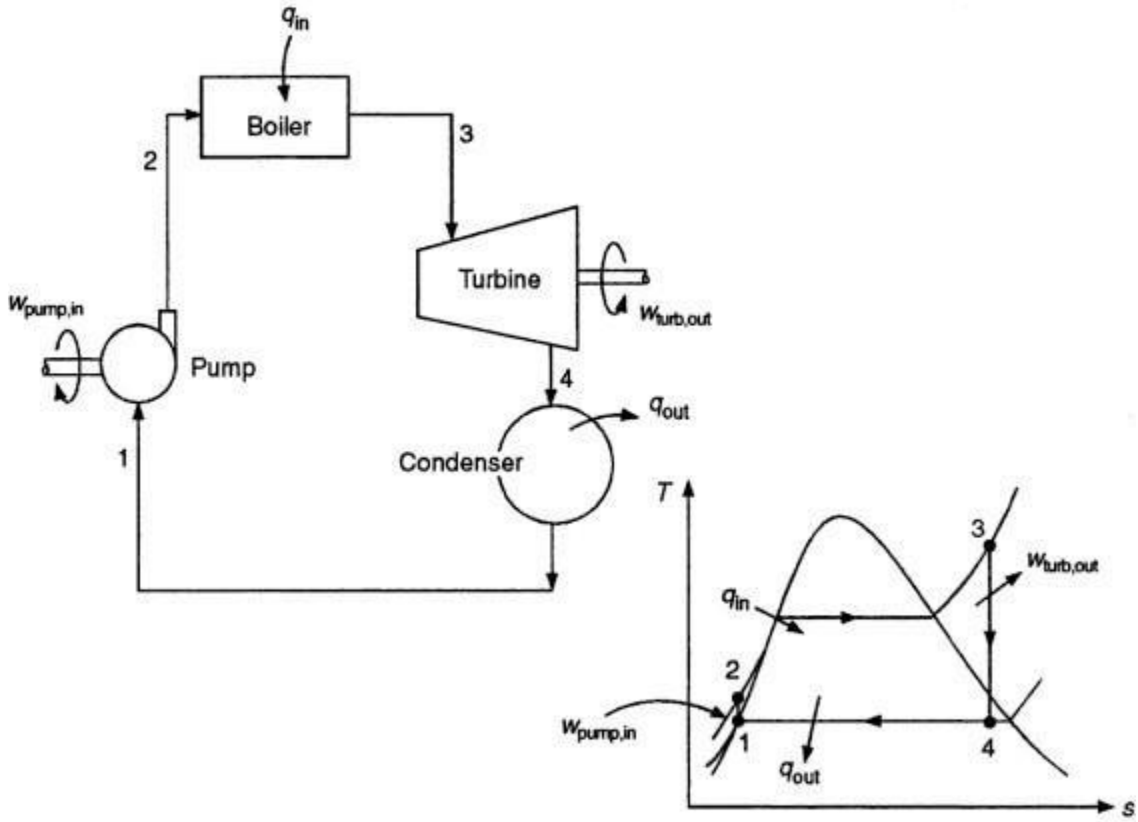


Figure 9.2 The Rankine cycle.

isentropic compression in a pump (1–2), constant pressure heat addition in a boiler (2–3), isentropic expansion in a turbine (3–4), and constant pressure heat rejection in a condenser (4–1). The cycle that results in these processes is the *Rankine cycle*.

The pump, boiler, and condenser associated with a Rankine cycle are steady-flow devices, and thus all the processes of this cycle can be analysed as steady-flow processes. The Δke and

Example 3.30. *(a) With the help of p-v and T-s diagram compare the cold air standard otto, diesel and dual combustion cycles for same maximum pressure and maximum temperature.* **(AMIE Summer, 1998)**

Solution. Refer Fig. 3.29. (*a*, *b*).

The air-standard Otto, Dual and Diesel cycles are drawn on common *p-v* and *T-s* diagrams for the same maximum pressure and maximum temperature, for the purpose of comparison.

Otto 1-2-3-4-1, Dual 1-2′-3′-3-4-1, Diesel 1-2″-3-4-1 (Fig 3.29 (*a*)).

Slope of constant volume lines on *T-s* diagram is higher than that of constant pressure lines. (Fig. 3.29 (*b*)).

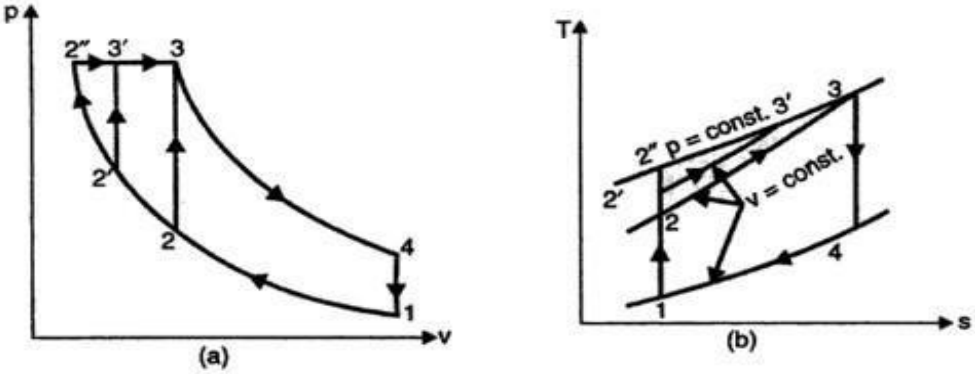


Fig. 3.29

Here the otto cycle must be limited to a low compression ratio (*r*) to fulfill the condition that point 3 (same maximum pressure and temperature) is to be a common state for all the three cycles.

The construction of cycles on *T-s* diagram proves that for the given conditions the heat rejected is same for all the three cycles (area under process line 4-1). Since, by definition,

$$\eta = 1 - \frac{\text{Heat rejected, } Q_r}{\text{Heat supplied, } Q_s} = 1 - \frac{\text{Const.}}{Q_s}$$

the cycle, with greater heat addition will be more efficient. From the *T-s* diagram,

$$Q_{s(\text{diesel})} = \text{Area under } 2''\text{-}3$$

$$Q_{s(\text{dual})} = \text{Area under } 2'\text{-}3'\text{-}3$$

$$Q_{s(\text{otto})} = \text{Area under } 2\text{-}3.$$

It can be seem that, $Q_{s(\text{diesel})} > Q_{s(\text{dual})} > Q_{s(\text{otto})}$

and thus, $\eta_{\text{diesel}} > \eta_{\text{dual}} > \eta_{\text{otto}}$.

In an engine working on Dual cycle, the temperature and pressure at the beginning of the cycle are 90°C and 1 bar respectively. The compression ratio is 9. The maximum pressure is limited to 68 bar and total heat supplied per kg of air is 1750 kJ. Determine :

(i) Pressure and temperatures at all salient points

(ii) Air standard efficiency

(iii) Mean effective pressure.

Solution. Refer Fig. 3.22.

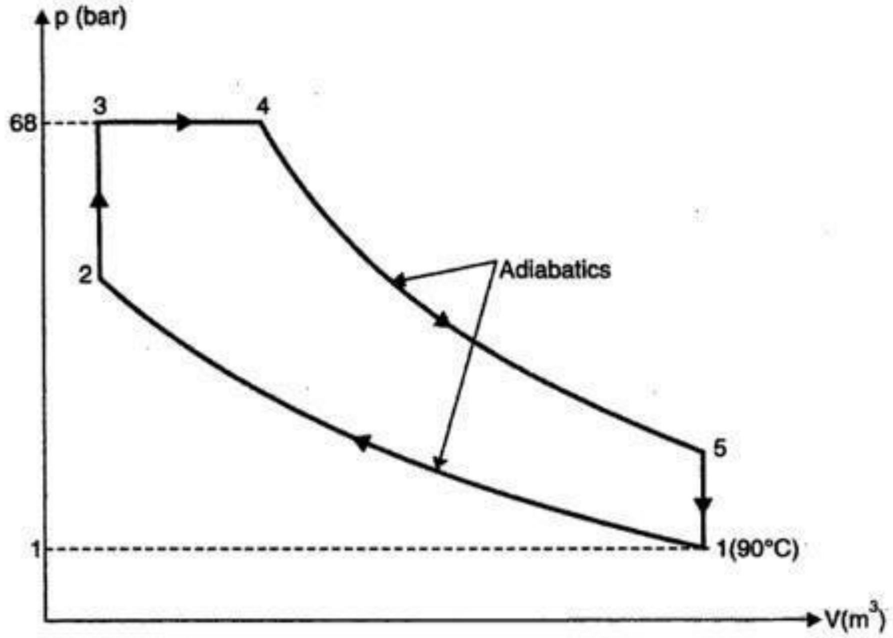


Fig. 3.22

Initial pressure,	$p_1 = 1$ bar
Initial temperature,	$T_1 = 90 + 273 = 363$ K
Compression ratio,	$r = 9$
Maximum pressure,	$p_3 = p_4 = 68$ bar
Total heat supplied	= 1750 kJ/kg

(*i*) **Pressures and temperatures at salient points :**

For the *isentropic process* 1-2,

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$\mathbf{p_2} = p_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times (r)^{\gamma} = 1 \times (9)^{1.4} = \mathbf{21.67\ bar.} \quad \textbf{(Ans.)}$$

Also,
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} = (9)^{1.4 - 1} = 2.408$$

∴
$$\mathbf{T_2} = T_1 \times 2.408 = 363 \times 2.408 = \mathbf{874.1\ K.} \quad \textbf{(Ans.)}$$

$$\mathbf{p_3 = p_4 = 68\ bar.} \quad \textbf{(Ans.)}$$

For the *constant volume process* 2-3,

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

$\therefore$ $$\mathbf{T_3} = T_2 \times \frac{p_3}{p_2} = 874.1 \times \frac{68}{21.67} = \mathbf{2742.9\ K.} \quad \textbf{(Ans.)}$$

Heat added at constant volume

$$= c_v\,(T_3 - T_2) = 0.71\,(2742.9 - 874.1) = 1326.8 \text{ kJ/kg}$$

$\therefore$ Heat added at constant pressure

= Total heat added – Heat added at constant volume

= 1750 – 1326.8 = 423.2 kJ/kg

$\therefore$ $$c_p(T_4 - T_3) = 423.2$$

$$1.0(T_4 - 2742.9) = 423.2$$

$\therefore$ $$\mathbf{T_4} = \mathbf{3166\ K.} \quad \textbf{(Ans.)}$$

For *constant pressure process* 3-4,

$$\rho = \frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{3166}{2742.9} = 1.15$$

For *adiabatic (or isentropic) process* 4-5,

$$\frac{V_5}{V_4} = \frac{V_5}{V_2} \times \frac{V_2}{V_4} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{r}{\rho} \qquad \left(\because \ \rho = \frac{V_4}{V_3}\right)$$

Also $$p_4V_4^{\gamma} = p_5V_5^{\gamma}$$

$\therefore$ $$\mathbf{p_5} = p_4 \times \left(\frac{V_4}{V_5}\right)^{\gamma} = 68 \times \left(\frac{\rho}{r}\right)^{\gamma} = 68 \times \left(\frac{1.15}{9}\right)^{1.4} = \mathbf{3.81\ bar.} \quad \textbf{(Ans.)}$$

Again, $$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} = \left(\frac{\rho}{r}\right)^{\gamma-1} = \left(\frac{1.15}{9}\right)^{1.4-1} = 0.439$$

$\therefore$ $$\mathbf{T_5} = T_4 \times 0.439 = 3166 \times 0.439 = \mathbf{1389.8\ K.} \quad \textbf{(Ans.)}$$

(*ii*) **Air standard efficiency :**

Heat rejected during constant volume process 5-1,

$$Q_r = C_v(T_5 - T_1) = 0.71(1389.8 - 363) = 729 \text{ kJ/kg}$$

$\therefore$ $$\eta_{\text{air-standard}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s}$$

$$= \frac{1750 - 729}{1750} = \mathbf{0.5834\ or\ 58.34\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Mean effective pressure, $\mathbf{p_m}$:**

Mean effective pressure is given by

$$p_m = \frac{\text{Work done per cycle}}{\text{Stroke volume}}$$

$$p_m = \frac{1}{V_s}\left[p_3\,(V_4 - V_3) + \frac{p_4V_4 - p_5V_5}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1}\right]$$

$$V_1 = V_5 = r\,V_c,\ V_2 = V_3 = V_c,\ V_4 = \rho V_c, \qquad \left[\because \ r = \frac{V_s + V_c}{V_c} = 1 + \frac{V_s}{V_c}\right.$$

$$V_s = (r-1)\,V_c \qquad \left.\therefore \ V_s = (r-1)\,V_c\right]$$

$$\therefore \quad p_m = \frac{1}{(r-1)V_c}\left[p_3(\rho V_c - V_c) + \frac{p_4\rho V_c - p_5 \times rV_c}{\gamma - 1} - \frac{p_2V_c - p_1rV_c}{\gamma - 1}\right]$$

$r = 9, \rho = 1.15, \gamma = 1.4$

$p_1 = 1$ bar, $p_2 = 21.67$ bar, $p_3 = p_4 = 68$ bar, $p_5 = 3.81$ bar

Substituting the above values in the above equation, we get

$$p_m = \frac{1}{(9-1)}\left[68(1.15-1) + \frac{68 \times 1.15 - 3.81 \times 9}{1.4-1} - \frac{21.67-9}{1.4-1}\right]$$

$$= \frac{1}{8}\ (10.2 + 109.77 - 31.67) = 11.04 \text{ bar}$$

Hence, ***mean effective pressure*** **= 11.04 bar. (Ans.)**

A Diesel engine working on a dual combustion cycle has a stroke volume of 0.0085 m^3 and a compression ratio 15 : 1. The fuel has a calorific value of 43890 kJ/kg. At the end of suction, the air is at 1 bar and 100°C. The maximum pressure in the cycle is 65 bar and air fuel ratio is 21 : 1. Find for ideal cycle the thermal efficiency. Assume c_p = 1.0 kJ/kg K and c_v = 0.71 kJ/kg K.

Solution. Refer Fig. 3.24.

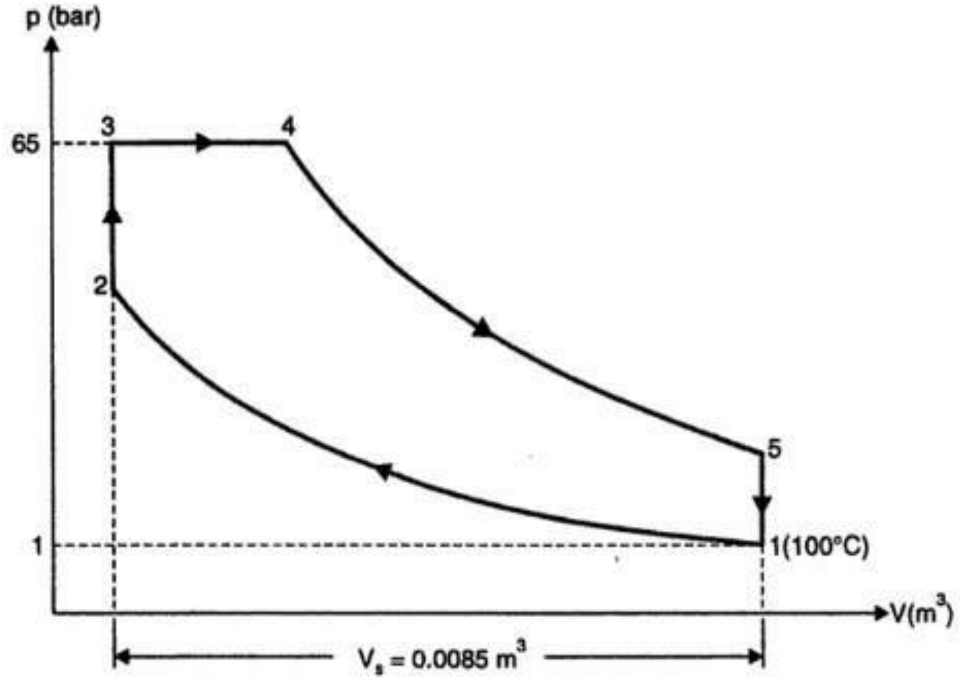


Fig. 3.24

Initial temperature, $T_1 = 100 + 273 = 373$ K

Initial pressure, $p_1 = 1$ bar

Maximum pressure in the cycle, $p_3 = p_4 = 65$ bar

Stroke volume, $V_s = 0.0085 \text{ m}^3$

Air-fuel ratio $= 21 : 1$

Compression ratio, $r = 15 : 1$

Calorific value of fuel, $C = 43890$ kJ/kg

$c_p = 1.0$ kJ/kg K, $c_v = 0.71$ kJ/kg K

Thermal efficiency :

$$V_s = V_1 - V_2 = 0.0085 \text{ m}^3$$

and as $$r = \frac{V_1}{V_2} = 15, \text{ then } V_1 = 15V_2$$

$\therefore$ $$15V_2 - V_2 = 0.0085$$

or $$14V_2 = 0.0085$$

or $$V_2 = V_3 = V_c = \frac{0.0085}{14} = 0.0006 \text{ m}^3$$

For *adiabatic compression process 1-2,*

$$p_1V_1^\gamma = p_2V_2^\gamma$$

or $$p_2 = p_1 \cdot \left(\frac{V_1}{V_2}\right)^\gamma = 1 \times (15)^{1.41} \qquad \left[\gamma = \frac{c_p}{c_v} = \frac{1.0}{0.71} = 1.41\right]$$

$$= 45.5 \text{ bar}$$

Also, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (15)^{1.41-1} = 3.04$$

$\therefore$ $$T_2 = T_1 \times 3.04 = 373 \times 3.04 = 1134 \text{ K or } 861°\text{C}$$

For *constant volume process 2-3,*

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

or $$T_3 = T_2 \times \frac{p_3}{p_2} = 1134 \times \frac{65}{45.5} = 1620 \text{ K or } 1347°\text{C}$$

According to characteristic equation of gas,

$$p_1V_1 = mRT_1$$

$\therefore$ $$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.009}{287 \times 373} = 0.0084 \text{ kg (air)}$$

Heat added during constant volume process 2-3,

$$= m \times c_v (T_3 - T_2)$$
$$= 0.0084 \times 0.71 (1620 - 1134)$$
$$= 2.898 \text{ kJ}$$

Amount of fuel added during the *constant volume process 2-3,*

$$= \frac{2.898}{43890} = 0.000066 \text{ kg}$$

Also as air-fuel ratio is 21 : 1.

$\therefore$ Total amount of fuel added $= \frac{0.0084}{21} = 0.0004$ kg

Quantity of fuel added during the process 3-4,

$= 0.0004 - 0.000066 = 0.000334$ kg

$\therefore$ *Heat added during the constant pressure operation 3-4*

$= 0.000334 \times 43890 = 14.66$ kJ

But $(0.0084 + 0.0004)\, c_p\, (T_4 - T_3) = 14.66$

$0.0088 \times 1.0\, (T_4 - 1620) = 14.66$

$\therefore$ $T_4 = \frac{14.66}{0.0088} + 1620 = 3286$ K or 3013°C

Again for operation 3-4,

$$\frac{V_3}{T_3} = \frac{V_4}{T_4} \quad \text{or} \quad V_4 = \frac{V_3 T_4}{T_3} = \frac{0.0006 \times 3286}{1620} = 0.001217 \text{ m}^3$$

For adiabatic expansion operation 4-5,

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{\gamma - 1} = \left(\frac{0.009}{0.001217}\right)^{1.41 - 1} = 2.27$$

or $$T_5 = \frac{T_4}{2.27} = \frac{3286}{2.27} = 1447.5 \text{ K or } 1174.5°\text{C}$$

Heat rejected during constant volume process 5-1,

$= m\, c_v\, (T_5 - T_1)$

$= (0.00854 + 0.0004) \times 0.71\, (1447.5 - 373) = 6.713$ kJ

Work done = Heat supplied – Heat rejected

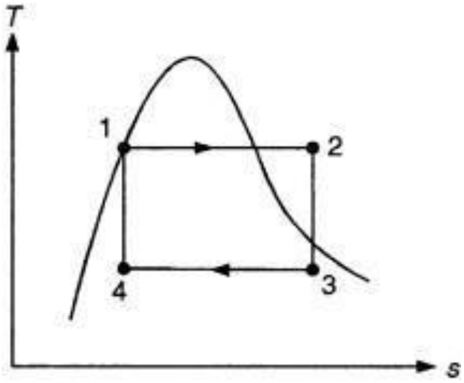
$= (2.898 + 14.66) - 6.713 = 10.845$ kJ

$\therefore$ *Thermal efficiency,*

$$\eta_{th} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{10.845}{(2.898 + 14.66)} = \mathbf{0.6176 \text{ or } 61.76\%.} \quad \textbf{(Ans.)}$$

EXAMPLE 9.1

A steam power plant operates between a boiler pressure of 4 MPa and 300°C and a condenser pressure of 50 kPa. Determine the thermal efficiency of the cycle, the work ratio, and the specific steam flow rate, assuming (a) the cycle to be a Carnot cycle, and (b) a simple ideal Rankine cycle.



Solution

(a) The *T–s* diagram of a Carnot cycle is shown in the adjacent figure.

Process 1–2 is reversible and isothermal heating of water in the boiler.

Process 2–3 is isentropic expansion of steam at state 2 in the turbine.

Process 3–4 is reversible and isothermal condensation of steam in the condenser.

Process 4–1 is isentropic compression of steam to initial state.

At state 1: $P_1 = 4$ MPa, $T_1 = 300°C$

At state 2: $P_2 = 50$ kPa, the steam is in a saturated state.

From the saturated water-pressure table (Table 4 of the Appendix), at 50 kPa, we get $T_2 = T_{min} = T_{sat} = 81.33°C$

Δpe of the steam are usually small compared with the work and heat transfer terms and are, therefore, neglected. Thus, the steady-flow energy equation per unit mass of steam is

$$q - w = h_e - h_i \quad \text{(kJ/kg)} \tag{9.1}$$

Assuming the pump and turbine to be isentropic and noting that there is no work associated with the boiler and the condenser, the energy conservation relation for each device becomes

$$w_{pump,in} = h_2 - h_1 = v(P_2 - P_1) \tag{9.2}$$

$$q_{boi,in} = h_3 - h_2 \tag{9.3}$$

$$w_{turb,out} = h_3 - h_4 \tag{9.4}$$

$$q_{cond,out} = h_4 - h_1 \tag{9.5}$$

The thermal efficiency of the Rankine cycle is given by

$$\boxed{\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}} \tag{9.6}$$

where $w_{net} = q_{in} - q_{out} = w_{turb,out} - w_{pump,in}$

The η_{th} can also be interpreted as the ratio of the area enclosed by the cycle on a *T–s* diagram to the area under the heat addition process.

Therefore, the thermal efficiency for the given Carnot cycle is

$$\eta_{th,carnot} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{81.33 + 273.15}{300 + 273.15} = 0.3815$$

$$= \boxed{38.15 \text{ per cent}}$$

The work ratio = $\dfrac{\text{net work output}}{\text{gross work output}} = \dfrac{w_{net,out}}{w_{gross,out}}$

Heat supplied = $h_2 - h_1 = h_{fg\,@\,4MPa}$ = 1714.1 kJ/kg (From Table 4 of the Appendix)

$$\eta_{th,carnot} = \frac{w_{net,out} - w_{net,in}}{\text{gross heat supplied}} = 0.3815$$

Therefore,

$$w_{net,\,out} - w_{net,\,in} = 0.3815 \times 1714.1 = 653.9 \text{ kJ/kg}$$

That is, the net work output = 653.9 kJ/kg.

To find the expansion work for the process 2–3, h_3 is required.

From Table 4, h_2 = 2801.4 kJ/kg and $s_2 = s_3$ = 6.0701 kJ/(kg K)

But

$$s_3 = 6.0701 = s_{f3} + x_3 s_{fg3} = 1.0910 + x_3(7.5939 - 1.0910)$$

or

$$x_3 = 0.766$$

Now,

$$h_3 = h_{f3} + x_3 h_{fg3} = 340.49 + 0.766(2645.9 - 340.49) = 2106.4 \text{ kJ/kg}$$

Therefore,

$$w_{32} = h_2 - h_3 = 2801.4 - 2106.4 = 695 \text{ kJ/kg}$$

That is, the gross work output, $w_{gross,out}$ = 695 kJ/kg
Therefore,

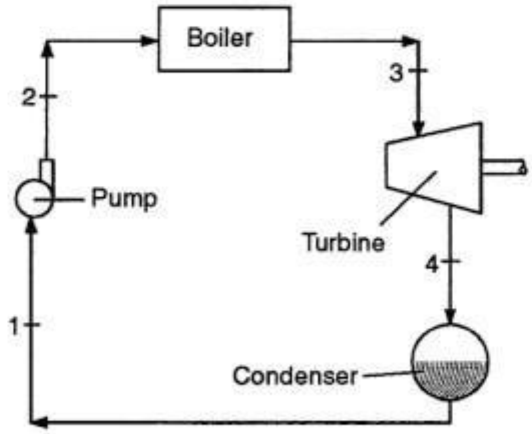
$$\text{Work ratio} = \frac{w_{net,out}}{w_{gross,out}} = \frac{653.9}{695} = \boxed{0.94}$$

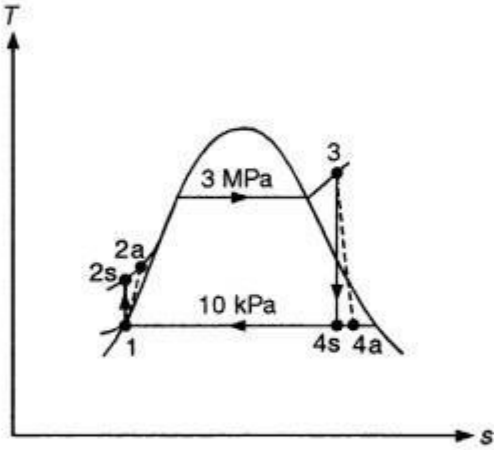
The specific steam flow rate (ssfr) is the steam flow required to develop unit power output. That is,

$$\text{ssfr} = \frac{\dot{m}_{steam}}{\dot{m}_s w_{out}} = \frac{1}{w_{net,\,out}}$$

$$= \frac{1}{653.9} = \boxed{0.00153 \text{ kg/kW}}$$

A steam power plant operates on the cycle shown below with 3 MPa and 400°C at the turbine inlet and 10 kPa at the turbine exhaust. The adiabatic efficiency of the turbine is 85 per cent and that of the pump is 80 per cent. Determine (a) the thermal efficiency of the cycle, and (b) the mass flow rate of the steam if the power output is 20 MW.





Solution

All the components are treated as steady-flow devices. The changes, if any, in the kinetic and potential energies are assumed to be negligible. Losses other than those in the turbine and pump are neglected.

(a) $$w_{\text{pump,in}} = \frac{v_1(P_2 - P_1)}{\eta_P} = \frac{0.001010\,(3000 - 10)}{0.80} = 3.77 \text{ kJ/kg}$$

Turbine work output is

$$w_{\text{turb,out}} = \eta_T w_{\text{turb,in}} = \eta_T (h_3 - h_{4s})$$
$$= 0.85(3230.90 - 2192.21) = 882.89 \text{ kJ/kg}$$

Boiler heat input is

$$q_{\text{in}} = h_3 - h_2 = 3230.9 - 195.59 = 3035.31 \text{ kJ/kg}$$

Thus,

$$w_{\text{net,out}} = w_{\text{turb,out}} - w_{\text{pump,in}} = 882.89 - 3.77 = 879.12 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net,out}}}{q_{\text{in}}} = \frac{879.12}{3035.31} = 0.2896 = \boxed{28.96 \text{ per cent}}$$

If there are no losses in the turbine and the pump, the thermal efficiency would be 28.99 per cent.

(b) The power generated by the power plant is

$$\dot{W}_{\text{net,out}} = \dot{m} w_{\text{net,out}} = 20{,}000 \text{ kW}$$

Therefore, the mass flow rate, $\dot{m} = \dfrac{20{,}000}{879.12} = \boxed{22.75 \text{ kg/s}}$