5.1 Lateral Earth Pressure: Types and Derivation

When a soil mass is retained at a higher level by a retaining wall, the retained mass of the soil tends to slide and assume a flat slope for equilibrium, which is resisted by the retaining wall. This exerts pressure on the retaining wall, which is known as lateral earth pressure. Usually, the retaining wall is constructed first and then the soil behind the wall is backfilled; hence, the retained soil is often called backfill. The back of the wall is either vertical or slightly inclined to the vertical and the lateral earth pressure is slightly inclined to the horizontal due to wall friction and inclination of the back of the wall.

The magnitude of the lateral earth pressure depends on the following factors:

i. Type and extent of the movement of the wall and the resulting horizontal strain in the backfill.

ii. Properties of the backfill material, including the density (γ), cohesion (c), and angle of shearing resistance (ϕ).

iii. Groundwater conditions in the backfill such as depth of water table and provision for drainage.

iv. Degree of roughness of the surface of the back of the retaining wall.

v. Slope of the back of the retaining wall.

vi. Depth of the retaining wall, that is, the height of the backfill to be retained.

vii. Inclination of the backfill surface with the horizontal.

viii. Additional loads on the backfill surface such as traffic loads or additional constructions, if any.

Types of Lateral Earth Pressure:

There are three basic types of lateral earth pressure.

They are:

1. Active earth pressure.
2. Passive earth pressure.
3. Earth pressure at rest.

These three basic types of lateral earth pressures are discussed below:

1. Active Earth Pressure:

Figure 1(a) shows a retaining wall of height H with a backfill having a horizontal surface. If the retaining wall were not there, the backfill would assume a stable flat slope. We know that cohesion less soils assume a stable slope equal to the angle of internal friction without any lateral support. Hence, when a backfill is retained, the wedge of soil above a certain slope tends to slide and move away from the rest of the backfill for equilibrium. This tends to push or rotate the wall away from the backfill if the wall is free to move or rotate.

The movement of the wall away from the backfill causes expansion of the backfill, resulting in stress release, thereby reducing the lateral earth pressure. Thus, the more is the movement of the wall away from the backfill, the more is the horizontal strain in the backfill, in the form of expansion, and the less is the lateral earth pressure. Initially when the wall is in a state of rest, a typical element of backfill at any depth is subjected to vertical stress due to self-weight of soil above the element and lateral earth pressure in the horizontal direction. The state of stress for the soil element is represented by Mohr's circle (I) in Fig. 1(b), where OB is the vertical stress and OA1 is the lateral earth pressure at rest.

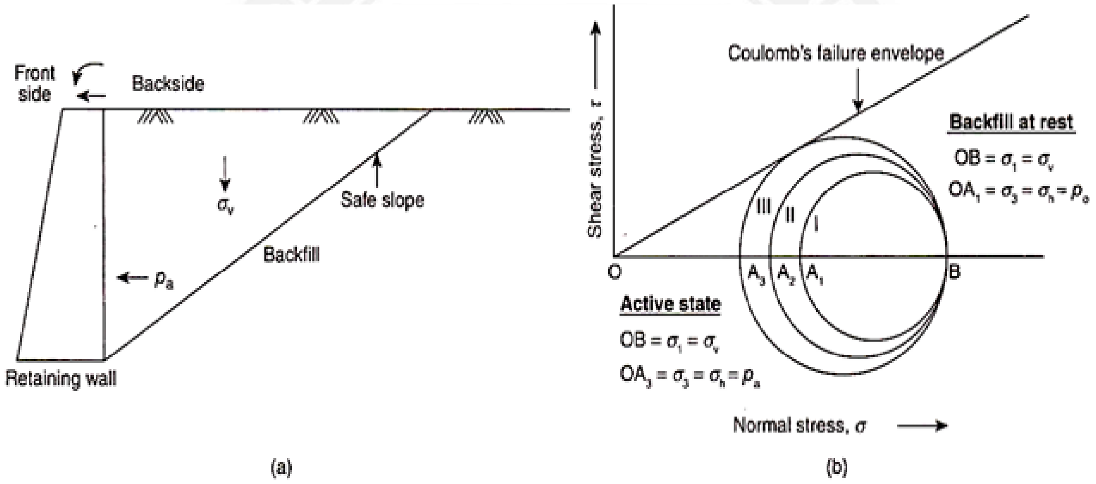

Fig 1(a)Retaining wall with a backfill Fig(b)Mohrs circle showing the gradual decrease of lateral earth pressure in active case

[Fig 1 https://www.soilmanagementindia.com/lateral-earth-pressure/lateral-earth-pressure-types-and-derivation-soil/13925]

When the lateral earth pressure tends to push or rotate the wall away from the backfill, the movement of the wall away from the backfill causes expansion of the backfill, resulting in stress release, thereby reducing the lateral earth pressure. Thus, the more is the movement of the wall away from the backfill, the more is the horizontal strain in the backfill, in the form of expansion, and the less is the lateral earth pressure.

This is shown in Fig.1 (b), by Mohr's circle (II), in which $\sigma_h = \sigma_3 = OA_2$ is the reduced lateral earth pressure while the vertical stress, equal to $\sigma_v = \sigma_1 = OB$, remains constant. The decrease in the lateral earth pressure thus causes increase in the diameter of Mohr's circle, causing it to approach the Coulomb's failure envelope.

The decrease in the lateral earth pressure due to movement of wall away from the backfill and consequent expansion and stress release continues until Mohr's circle touches the Coulomb's failure envelope of the backfill material. When Mohr's circle touches the failure envelope, as shown by Mohr's circle (III) in Fig.1(b), the backfill material is on the verge of failure (limiting equilibrium) and no further decrease in the lateral earth pressure can take place. The minimum lateral earth pressure exerted on the retaining wall, when the wall moves away from the backfill, and the backfill material is in the limiting equilibrium, is known as active earth pressure.

When the wall moves away from the backfill, the backfill is said to be in the active state and the minimum lateral earth pressure exerted by the backfill in the active state in its limiting equilibrium condition is known as active earth pressure. Active earth pressure occurs when Mohr's circle of stresses at any point in the backfill touches the Coulomb's failure envelope.

Active earth pressure is denoted by the symbol pa, and its units are kN/m^2, t/m^2, or kgf/cm^2. All retaining walls, which are free to move or rotate, are by default subjected to active earth pressure and are designed to resist the same.

2. Passive Earth Pressure:

All retaining walls are usually not placed on the ground surface on the front side but are laid at some depth. Hence, the retaining wall has soil to some depth on its front side. When the wall moves away from the backfill due to active earth pressure, it actually moves towards the soil on the front side.

The movement of the wall is resisted by the front soil and exerts a lateral pressure on the wall, in a direction opposite to that of active earth pressure, as shown in Fig.2. Also, the movement of the wall towards the front soil causes compression of the soil, which, in turn, increases the lateral pressure from the front soil.

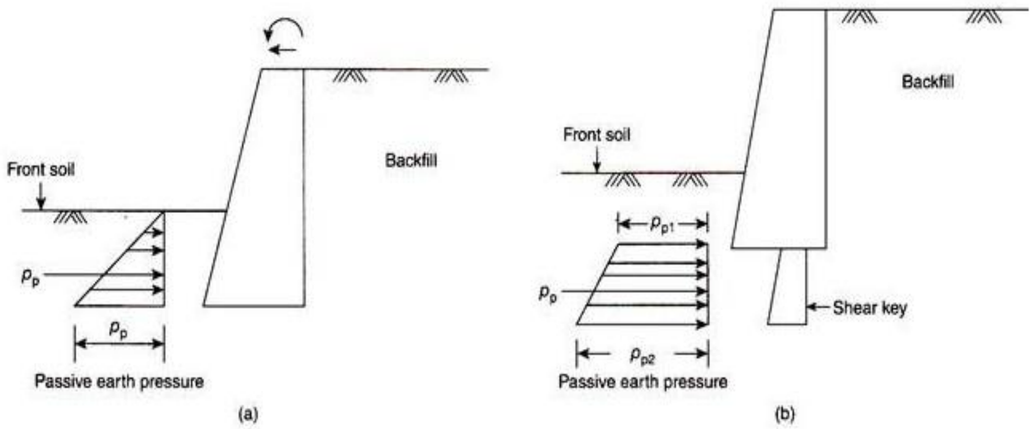

Fig 2 Practical cases of passive earth pressure a) In front of a retaining wall and b) on shear key below a retaining wall

[Fig 2 https://www.soilmanagementindia.com/lateral-earth-pressure/lateral-earth-pressure-types-and-derivation-soil/13925]

Thus, the more is the movement of the wall toward the front soil, the more is the horizontal strain in the front soil, in the form of compression, and the more is the lateral earth pressure from the front soil opposite to that of active earth pressure. This is shown in Fig.3, by Mohr's circle (II), in which $\sigma_h = \sigma_3 = OA_2$ is the increased lateral earth pressure while the vertical stress, equal to $\sigma_v = \sigma_1 = OB$, remains constant. The increase in the lateral earth pressure causes decrease in the diameter of Mohr's circle as shown by Mohr's circles (II) and (III), and Mohr's circle reduces to a point, as represented by points A_4 and B, which become concurrent.

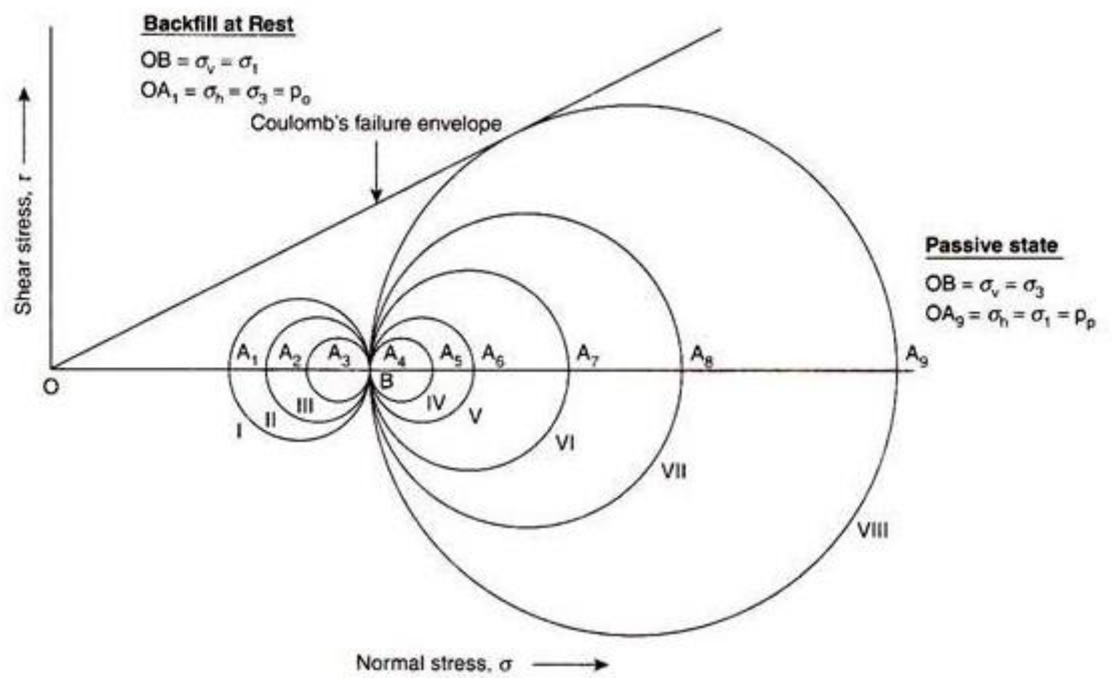

Fig 3 Mohrs circle showing the gradual decrease and then increase in lateral earth pressure in the passive case

[Fig 3 https://www.soilmanagementindia.com/lateral-earth-pressure/lateral-earth-pressure-types-and-derivation-soil/13925]

Further increase of the lateral earth pressure from the front soil makes it higher than the vertical stress. At this stage, the lateral earth pressure becomes the major principal stress and the vertical stress becomes the minor principal stress. This is shown by Mohr's circles (IV), (V), (VI), etc., causing again an increase in the diameter of Mohr's circle.

The increase in the diameter of Mohr's circle leads it to approach the Coulomb's failure envelope. The increase in the lateral earth pressure due to the movement of wall towards the front soil and the consequent compression continues until Mohr's circle touches the Coulomb's failure envelope of the front soil.

When Mohr's circle touches the failure envelope, as shown by Mohr's circle (VIII) in Figure 3, the front soil is on the verge of failure (limiting equilibrium) and no further increase in the lateral earth pressure can take place. The maximum lateral earth pressure exerted on the retaining wall, when the wall moves towards the front soil, while it reaches its limiting equilibrium, is known as passive earth pressure.

When the wall moves towards the front soil, the front soil is said to be in the passive state and the maximum lateral earth pressure exerted by the front soil in the passive state in its limiting equilibrium condition is known as passive earth pressure. Passive earth pressure occurs when Mohr's circle of stresses at any point in the front soil touches the Coulomb's failure envelope.

Another practical example of passive earth pressure is the case of shear key provided below the base of a retaining wall. A shear key shown in Fig.3 is provided to improve the stability of the wall against sliding. When the retaining wall moves away from the backfill due to active pressure, the shear key also moves in the same direction but toward the soil below the base of the wall on the front side.

This generates passive earth pressure on the shear key. It is denoted by the symbol p_P, and its units are kN/m^2, t/m^2, or kgf/cm^2. Passive earth pressure is actually a stabilizing force improving the stability of the retaining wall, unlike active earth pressure.

3. Earth Pressure at Rest:

Figure 4 shows a basement retaining wall in which the wall is rigidly fixed to the basement slab. The basement retaining wall is therefore fixed in position and cannot move away from the backfill when subjected to lateral earth pressure. The lateral earth pressure exerted by the backfill on a retaining wall which is fixed in position and cannot move is known as earth pressure at rest.

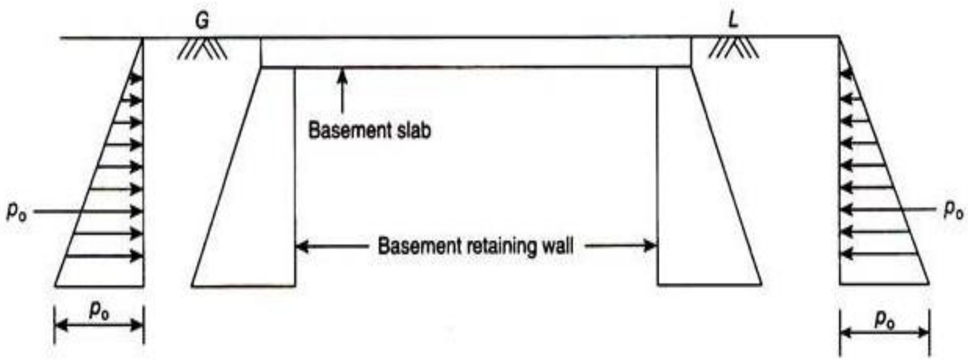

Fig 4 Earth pressure at rest on basement retaining walls

[Fig 4 https://www.soilmanagementindia.com/lateral-earth-pressure/lateral-earth-pressure-types-and-derivation-soil/13925]

It is denoted by the symbol p_0, and its units are kN/m^2, t/m^2, or kgf/cm^2. As the wall does not move, the earth pressure exerted does not cause any lateral strain, and hence, there is no expansion of the backfill and no stress release. Earth pressure at rest is therefore always more than active earth pressure for the same depth of soil.

The abutment of a bridge is rigidly attached to the deck slab of the bridge and is also similarly fixed in position and hence subjected to earth pressure at rest.

Thus, lateral earth pressure exerted on a retaining wall depends on the direction and extent of the movement of the wall. Figure 5 shows the variation in lateral earth pressure on the y-axis as a function of the wall movement. When the wall moves away from the backfill, lateral pressure decreases with the increase in the movement of the wall; the minimum lateral earth pressure exerted on the wall is known as active earth pressure.

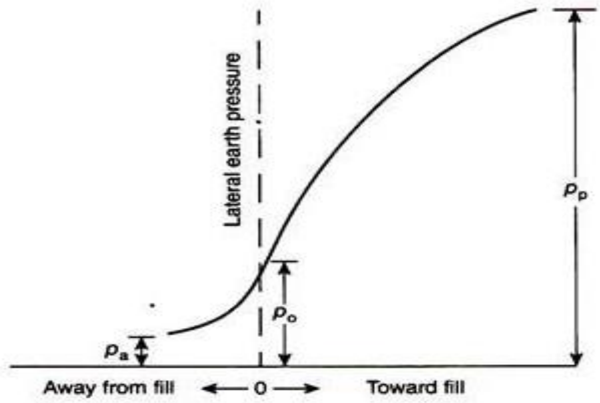

Fig 5 Variation in lateral earth pressure with the movement of the wall relative to the soil

[Fig 5 https://www.soilmanagementindia.com/lateral-earth-pressure/lateral-earth-pressure-types-and-derivation-soil/13925]

When the wall moves toward the soil, the lateral earth pressure generated increases with the increase in the movement of the wall; the maximum lateral earth pressure generated on the wall is known as passive earth pressure. The lateral earth pressure exerted on the wall when the wall is fixed in position is known as earth pressure at rest.

Derivation of Expression for Earth Pressure at Rest:

When a material is subjected to three-dimensional (3D) stresses, σ_x, σ_y and σ_z, along the three coordinate axes, x, y, and z, respectively, the strain along the x-axis can be computed from the principles of mechanics of materials as –

$$e_x = 1/E[\sigma_x - \mu(\sigma_y + \sigma_z)] \text{--------(1)}$$

where e_x is the horizontal strain (in the X-direction), E is the modulus of elasticity of soil, and μ is the Poisson's ratio. In the case of earth pressure at rest –

$$e_x = 0 \text{ ---------(2)}$$

$$\sigma_x = \sigma_y = P_0 \text{--------(3)}$$

Substituting these values in Eq. (1), we have –

$$e_x = 1/E\ [(p_0 - \mu(p_0 + \sigma_z)] = 0$$

$$\text{or } p_0 - \mu(p_0 + \sigma_z) = 0 \Rightarrow p_0 - \mu p_0 - \mu\sigma_z = 0 \Rightarrow p_0 - (1 + \mu) = \mu\sigma_z$$

$$p_0 = [\mu/(1 - \mu)]\sigma_z \ \ldots(4)$$

$$p_0 = K_0\sigma_z \text{--------}(5)$$

where K_0 is the coefficient of the earth pressure at rest and σ_z is the vertical stress due to the self-weight of the soil at depth z, where the earth pressure at rest is to be computed

$$K_0 = \mu/(1 - \mu) \text{-------}(6)$$

Equation (6) is valid for elastic materials but not for soils, since soils are not elastic. Table 1 gives typical values of K_0 for different types of backfills, as obtained from actual measurement of earth pressure at rest.

Table 1 Coefficient of earth Pressure at rest for different soil

S.No	Type of soil	K_0
1	Loose Sand	0.5-0.6
2	Dense Sand	0.3-0.5
3	Undrained Clay	0.8-1.1
4	Over consolidated Clay	1.0-3.0

Coefficients of earth pressure - Earth Pressure Coefficient:

On a small unit at depth Z in the back there are two kinds of pressure.

i)Vertical Earth pressure:

The pressure applied in the vertical direction due to the back fill lying above it.

ii)Horizontal Earth pressure:

The pressure applied in the horizontal direction due to backfill is called the horizontal pressure or lateral earth pressure

Coefficient of active earth pressure at rest:

When the retaining wall is at rest then the ratio between the lateral earth pressure and the vertical pressure is called the co-efficient of the earth pressure at rest,

$$K_o = \text{lateral pressure / vertical pressure}$$

Co-efficient of active earth pressure:

When the retaining wall is moving away from the backfill the ratio between lateral earth pressure and vertical earth pressure is called coefficient of active earth pressure.

Ka = lateral pressure / vertical pressure

(or)

It is the ratio of horizontal and vertical principal effective stress when a retaining wall moves away from the retained soil

$$k_a = \frac{1 - sin\varphi}{1 + sin\varphi} = \tan^2(45 - \frac{\varphi}{2})$$

Coefficient of passive earth pressure:

When the retaining wall is moving towards the backfill, then the ratio between the lateral earth pressure and the vertical earth pressure is called the coefficient of passive earth pressure.

Kp = lateral pressure / vertical pressure

(or)

It is the ratio of horizontal and vertical principal effective stress when a retaining wall is forced against a soil mass.

$$k_p = \frac{1 + sin\varphi}{1 - sin\varphi} = \tan^2(45 + \frac{\varphi}{2})$$