Non Linear Optimisation

The problem of minimizing a function of several variables, possibly subject to constraints on these variables, is what optimization is about. So the main problem is easy to state! And, more importantly, such problems arise in many applications in natural science, engineering, economics and business as well as in mathematics itself.

Nonlinear optimization differs from Fourier analysis and wavelet theory in that classical multivariate analysis also is an important ingredient. It contains a significant amount of fixed point theory, nonlinear equations, and optimization.

The basic concepts Optimization deals with finding optimal solutions! So we need to define what this is. Let $f : Rn \rightarrow R$ be a real-valued function in n variables. The function value is written as f(x), for $x \in Rn$, or f(x1, x2,...,xn). This is the function we want to minimize (or maximize) and it is often called the objective function. Let $x* \in Rn$. Then $x*$ is a local minimum (or local minimizer) of f if there is an " > 0 such that

$$f(x*) \leq f(x) \text{ for all } x \in B(x*; ").$$

Applications

It is useful to see some application areas for optimization. They are many, and here we mention a few in some detail.

Portfolio optimization

The following optimization problem was introduced by Markowitz in order to find an optimal portfolio in a financial market; he later received the Nobel prize in economics1 (in 1990) for his contributions in this area.

The decision variables are x1, x2, ...,xn where xi is the fraction of a total investment that is made in (say) stock i. Thus one has available a set of stocks in different companies (Statoil, IBM, Apple etc.) or bonds. The fractions xi must be nonnegative (so we consider no short sale) and add up to 1.

Fitting a model

In many applications one has a mathematical model of some phenomenon where the model has some parameters. These parameters represent a flexibility of the model, and they may be adjusted so that the model explains the phenomenon best possible.

Maximum likelihood

A very important problem in statistics, arising in many applications, is parameter estimation and, in particular, maximum likelihood estimation. It leads to optimization. Let Y be a “continuous”

real-valued random variable with probability densisty px(y). Here x is a parameter (often one uses other symbols for the parameter, like ξ, θ etc.).

Where do optimization problems arise?

Economics: Consumer theory / supplier theory Finance: Optimal hedging / pricing

Statistics: data fitting, regression, pattern recognition

Science / Engineering: Aerospace, product design, data mining

Other Business decisions: scheduling, production, organizational decisions

Government: Military applications, fund allocation, etc

Other Personal decisions: Sports, on-field decisions, player acquisition, marketing

Example

Portfolio optimization

variables: amount invested in different assets

constraint: budget, max/min investment per asset, minimum return

objective: overall risk, or return variance

Device sizing in electronic circuits

variables: device width and lengths

constraints: manufacturing limits, max area power consumption

Data fitting

variable: model parameters

contraints: prior information, parameter limits

objective: measure of misfit, prediction error