NETWORK GRAPH

In order to describe the geometrical structure of the network, it is sufficient to replace the different power system components (of the corresponding power system network) such as generators, transformers and transmission lines etc. by a single line element irrespective of the characteristics of the power system components.

The geometrical interconnection of these line elements (of the corresponding power system network) is known as a graph (rather linear graph as the graph means always a linear graph). Each source and the shunt admittance across it are taken as a single element. The terminals of the elements are called the nodes.

A graph is connected if, and only if, there exists a path between every pair of nodes. A single edge or a single node is a connected graph. If every edge of the graph is assigned a direction, the graph is termed as an oriented graph. The direction is generally, so assigned as to coincide with the assumed positive direction of the current in the element.

Power networks are so structured that out of the m total nodes, one node (normally described by 0) is always at ground potential and the remaining n = (m – 1) nodes are the buses at which the source power is injected. Figure 6.5 shows the oriented graph of the network given in Fig. 6.2 (b).

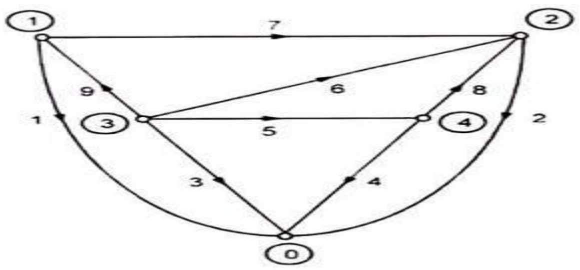


Fig. 6.5. ***Oriented Graph of The Network Given in Fig. 6.2 (b)***

A connected sub-graph containing all the nodes of the original graph but no closed path is called a tree. The tree branches form a sub-set of the elements of the connected graph. The number of branches b required to form a tree is equal to the number of buses in the network (the total number of nodes, including the reference node, is one more than the number of buses), i.e.,

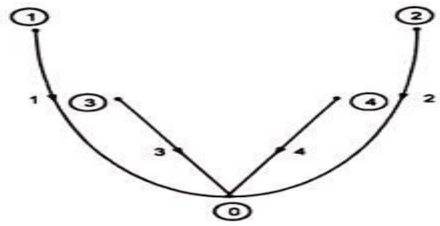
$$b = m - 1 = n \text{ (number of buses)} \qquad \ldots(6.12)$$

The elements of the original graph not included in the tree, form a sub-graph which may not necessarily be connected, is known as cotree. The cotree is a complement of a tree. The elements of a cotree are called the links.

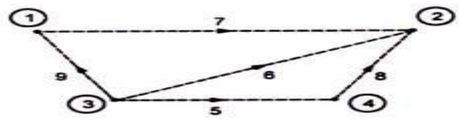
The number of links I of a connected graph with e elements is given as –

$$l = e - b = e - m + 1 \qquad \text{...(6.13)}$$

i.e., number of links equals number of elements less the number of tree branches. A tree and the corresponding cotree of graph shown in Fig. 6.5 are shown in Figs. 6.6 (a) and 6.6 (b) respectively.



(a) Tree



(b) Co-Tree

Fig. 6.6

If a link is added to the tree, the corresponding graph contains one closed path called a loop. Thus a graph has as many loops as the number of links.

The above system has 9 branches. So it has 18 variables (9 branch voltages and 9 branch currents). However, it can be easily seen that all these 18 variables are not independent. The number of independent variables is found from the concept of the tree.

The number of tree branches gives the number of independent voltages. For any system the number of tree branches is equal to the number of buses. The number of links gives the number of independent current variables.

BUS INCIDENCE MATRIX

If "G" is a graph with "n" nodes and "e" elements, then the matrix $\bar{A}$ whose n rows correspond to the "n" nodes (i.e., vertices) and "e" columns correspond to the "e" elements, i.e., edges, is known as an incidence matrix.

The matrix elements are:

a_{ik} = 1 if i^{th} element is incident to and directed away from the k^{th} node (bus).

= – 1 if the i^{th} element is incident to but directed towards the k^{th} node

= 0 if the ith element is not incident to the k^{th} node.

The dimension of this matrix is n x e and its rank is less than n.

Any node of the connected graph can be selected as the reference node and then the variables of the remaining n – 1 node which are termed as buses can be measured with respect to this assigned reference node.

The matrix "A" obtained from the incidence matrix $\bar{A}$ by deleting the reference row (corresponding to the reference node) is termed as reduced or bus incidence matrix (the number of buses in the connected graph is equal to n – 1 where n is the number of nodes). The order of this matrix is (n – 1) x e and its rows are linearly independent with rank equal to (n – 1).

For the specific system shown in Fig. 6.5, the 9-branch voltages (V_{b1}, V_{b2}, V_{b3}, … V_{b9}) can be expressed in terms of 4-bus voltages (V_1 …. V_4) as below:

$$\left.\begin{aligned} V_{b1} &= V_1 \\ V_{b2} &= V_2 \\ V_{b3} &= V_3 \\ V_{b4} &= V_4 \\ V_{b5} &= V_3 - V_4 \\ V_{b6} &= V_3 - V_2 \\ V_{b7} &= V_1 - V_2 \\ V_{b8} &= V_4 - V_2 \\ V_{b9} &= V_3 - V_1 \end{aligned}\right\} \qquad \text{...(6.19)}$$

Equation (6.19) can be written in matrix form as –

$$V = A\, V_{bus} \qquad \text{...(6.20)}$$

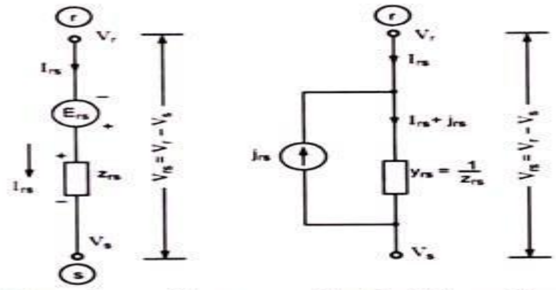
Where the bus incidence matrix A is –

$$A = \begin{array}{c} e \backslash \text{Bus} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{array}{c} e \backslash \text{Bus} \\ \text{Tree} \\ \text{Branches} \\ \\ = \\ \\ \text{Link} \\ \text{Branches} \end{array} \begin{bmatrix} \text{Buses} \\ \hline A_b \\ \hline A_l \end{bmatrix} \qquad \text{...(6.21)}$$

This matrix is rectangular and, therefore, singular. Its elements a_{ik} are found as per rules given above.

PRIMITIVE NETWORK:

A network is constituted by many branches and each branch consists of active and/or passive elements. Fig. 6.1(a) and 6.1(b) show a network branch, containing both active and passive elements in impedance and admittance representation. The impedance is a voltage source E_{rs} in series with and impedance, z_{rs}; while in admittance form there is a current source j_{rs} in parallel with an admittance y_{rs}. The element current is I_{rs} and element voltage, $V_{rs} = V_r - V_s$ where V_r and V_s are the voltages of the element nodes r and s, respectively.



(a) Impedance Form *(b) Admittance Form*

Fig. 6.7. ***Representation of a Network Element***

The noteworthy point is that for steady state ac performance, all element variables (V_{rs}, V_r, V_s, I_{rs}, J_{rs}) are phasors and element parameters z_{rs} and y_{rs} are complex numbers.

The performance equation for impedance representation, depicted in Fig. 6.7(a), can be written as –

$$V_{rs} + E_{rs} = z_{rs} I_{rs} \quad \ldots (6.14)$$

And for admittance representation depicted in Fig. 6.1(b) –

$$I_{rs} + J_{rs} = y_{rs} V_{rs} \quad \ldots (6.15)$$

The two representations shown in Figs. 6.7(a) and 6.7(b) are equivalent wherein the parallel source current in admittance form is related to the series voltage in impedance form by –

$$\text{and} \quad \left. \begin{aligned} J_{rs} &= -y_{rs} E_{rs} \\ y_{rs} &= \frac{1}{z_{rs}} \end{aligned} \right\} \quad \ldots (6.16)$$

A set of unconnected elements is known as primitive network. The performance equations in admittance (or impedance) form can be written for all branches.

The set of these equations in impedance form is –

$$V + E = ZI \quad \text{...(6.17)}$$

And in admittance form $I + J = YV$ ...(6.18)

where V and E are branch voltage and source voltage matrices, I and J are branch current and source current matrices, Z is primitive impedance matrix (i.e., a matrix whose elements are branch self-impedances) and Y is primitive admittance matrix (i.e., matrix whose elements are branch self-admittances). These are related as Z = 1/Y. If there is no coupling between elements, Z and Y are diagonal matrices.

Bus admittance matrix

Formulation of Y_{bus} and Z_{bus}:

Substituting Eq. (6.20) into Eq. (6.18), we have –

$$I + J = Y A V_{bus} \quad \text{... (6.22)}$$

Premultiplying Eq. (6.22) by A^T (i.e., transpose of the bus incidence matrix) we have –

$$A^T I + A^T J = A^T Y A V_{bus} \quad \text{... (6.23)}$$

Each component of the n-dimensional vector $A^T I$ is the algebraic sum of the element currents leaving the nodes 1, 2, 3, …, n.

Therefore, as per Kirchhoffs' current law –

$$A^T I = 0 \quad \text{... (6.24)}$$

Similarly, each component of vector $A^T J$ can be recognized as the algebraic sum of all source currents injected into nodes 1, 2, … n. These components are therefore the bus currents.

Hence we can write –

$$A^T J = J_{bus} \quad \text{... (6.25)}$$

Equation (6.23) is then simplified to –

$$J_{bus} = A^T Y A V_{bus} \quad \text{...(6.26)}$$

Comparing Eqs. (6.26) and (6.25), we have –

$$Y_{bus} = A^T Y A \quad \text{...(6.27)}$$

Above Eq. (6.27) suggests the formulation of Y_{bus}. Since matrix A is singular, $A^T YA$ is a singular transformation of Y. The bus incidence matrix can be obtained through a computer programme. Standard matrix multiplication and matrix transpose sub-routines can be employed to compute Y_{bus} using Eq. (6.27). Z_{bus} is the inverse of Y_{bus}.