# ROHININ COLLEGE OF ENGINEERING AND TECHNOLOGY

**Approved by AICTE & Affliated to anna university**

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**DEPARTMENT OF MECHANICAL ENGINEERING**

**NAME OF THE SUBJECT: ENGINEERING MECHANICS**

**SUBJECT CODE : ME3351**

**REGULATION 2021**

# UNIT III: DISTRIBUTED FORCES

Problem: 1

Find the product of inertia and principal moment of inertia of the section about the centroidal axis

Product of inertia

$I_{xy} = I_{x_1y_1} + I_{x2y2} + I_{x3y3}$

$I = I_{xy} + a_1(x_1 - \bar{X})(y_1 - \bar{Y})$

$I_{x_{1y1}} = a_1 x^1_1 y^1_1$ $\quad X^1_1 = x_1 - \bar{X}$ $\quad Y^1_1 = y_1 - \bar{Y}$

$I_{x_{2y2}} = a_2 x^1_2 y^1_2$ $\quad X^1_2 = x_2 - \bar{X}$ $\quad Y^1_2 = y_2 - \bar{Y}$

$I_{x_{3y3}} = a_3 x^1_3 y^1_3$ $\quad X^1_3 = x_3 - \bar{X}$ $\quad Y^1_3 = y_3 - \bar{Y}$

Section (1)

$a_1 = 40 \times 8 = 320mm^2$

$x_1 = 25 + \frac{30}{2} = 52mm$

$y_1 = \frac{8}{2} = 4mm$

Section (2)

$a_3 = 40 \times 8 = 320mm^2$

$x_3 = \frac{40}{2} = 20mm$

$y_2 = 8 + \frac{44}{2} = 30mm$

Section (3)

$a_1 = 40 \times 8 = 320mm^2$

$x_3 = \frac{40}{2} = 20mm$

$y_3 = 8 + 44 + \frac{8}{2} = 56mm$

$\bar{X} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3} = \frac{(320 \times 52) + (352 \times 36) + (320 \times 20)}{320 + 352 + 320} = 36mm$

$\bar{Y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3} = \frac{(320 \times 4) + (352 \times 30) + (320 \times 56)}{320 + 352 + 320} = 30mm$

$I_{x_{1y1}} = a_1 x_1^1 y_1^1$  $X_1^1 = x_1 - \bar{X} = 52 - 36 = 16$

$Y_1^1 = y_1 - \bar{Y} = 4 - 30 = -26$

$I_{x_{1y1}} = 320 \times 16 \times (-26)$

$I_{x_{1y1}} = -133120mm^4$

$I_{x_{2y2}} = a_2 x_2^1 y_2^1$ $\quad X_2^1 = x_2 - \bar{X} = 36 - 36 = 0$

$Y_2^1 = y_2 - \bar{Y} = 30 - 30 = 0$

$I_{x_{2y2}} = 0$

$I_{x_{3y3}} = a_3 x_3^1 y_3^1$ $\quad X_3^1 = x_3 - \bar{X} = 20 - 36 = -16$

$Y_3^1 = y_3 - \bar{Y} = 56 - 30 = 26mm$

$I_{x_{3y3}} = -133120mm^4$

Product of Inertia:

$I_{xy} = I_{X1Y1} + I_{X2Y2} + I_{X3Y3}$

$= -133120 + 0 + (-133120)$

$I_{xy} = -266240mm^4$

Principal Moment of Inertia:

Maximum Principal moment of inertia:

$$I_{Max} = \frac{I_{XX} + I_{YY}}{2} \pm \sqrt{\left(\frac{I_{XX} + I_{YY}}{2}\right)^2 + I^2XY}$$

$I_{XX} = I_{XX1} + I_{XX2} + I_{XX3}$

$I_{YY} = I_{YY1} + I_{YY2} + I_{YY3}$

$$I_{XX1} = \frac{bd^3}{12} + a_1(Y_1 - \bar{Y_1})^2$$

$$I_{XX2} = \frac{bd^3}{12} + a_2(Y_1 - \bar{Y_1})^2$$

$$I_{XX3} = \frac{bd^3}{12} + a_3(Y_1 - \bar{Y_1})^2$$

$I = I_{XX1} + a_1(Y_1 - \bar{Y}_1)^2 + I_{XX2} + a_2(\bar{Y} - Y_2)^2 + I_{XX3} + a_3(\bar{Y} - Y_3)^2$

$= 1706.66 + 320(30 - 4)^2 + 28394.66 + 352(30 - 30)^2 + 1706.66 + 320(30 - 56)^2$

$I_{XX} = 464.44 \times 10^3 mm^4$

$I_{yy} = I_{yy1} + a_1(X - X_1)^2 + I_{yy2} + a_2(X - X_2)^2 + I_{yy3} + a_3(X - X_3)^2$

$= 42666.66 + 320(36 - 52)^2 + 1877.33 + 352(36 - 36)^2 + 42666.66 + 320(36 - 20)^2$

$I_{yy} = 251.05 \times 10^3 mm^4$

$I_{Max}$

$$I_{Max} = \frac{I + I_{yy}}{2} \sqrt{+\left(\frac{I_{xx} + I}{2}\right)^2 + I^2xy}$$

$$= \frac{464.44\times10^3+251.05\times10^3}{2} + \sqrt{\left(\frac{464.44\times10^3-251.05\times10^3}{2}\right)^2 + (-266240)^2}$$

$I_{Max} = 357745 + \sqrt{1.138 \times 10^{10} + 7.083 \times 10^{10}}$

$I_{Max} = 357745 + 286.81 \times 10^3$

$I_{Max} = 644.55 \times 10^3 mm^4$

$$I_{Min} = \frac{I + I_{yy}}{2} \sqrt{+\left(\frac{I_{xx} + I}{2}\right)^2 + I^2xy}$$

$$= \frac{464.44 \times 10^3 + 251.05 \times 10^3}{2} - \sqrt{\left(\frac{464.44 \times 10^3 + 251.05 \times 10^3}{2}\right) + (-266240)^2}$$

$I_{Min} = 357745 - 268.81 \times 10^3$

$I_{Min} = 88.935 \times 10^3 mm^4$

The position of Principal Axes is given by

$$tan2\theta = \left(\frac{I_{xy}}{\frac{I_{x}-I_{yy}}{2}}\right) = \left(\frac{266240}{\frac{464.44\times10^3-251.05\times10^3}{2}}\right)$$

$$tan2\theta = \left(\frac{-2I_{xy}}{I_{xx}-I_{yy}}\right)$$

$$tan2\theta = 2.49$$

$$2\theta = tan^{-1}(2.49)$$

$$2\theta = 68.16 = \frac{68.16}{2}$$

$$\theta = 34°4'$$

Principal Moment of inertia:

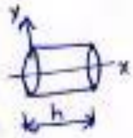
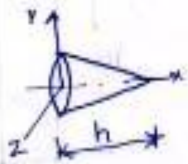
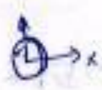
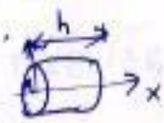
The perpendicular axis above which product of inertia is zero called principal axis and the moment of inertia with respect to these axis are called principal moment of inertia.

$$I_{max} \& I_{min} = \left(\frac{I + I_{yy}}{2}\right) \pm \left(\frac{I_{xx} + I}{2}\right)^2 + I_{xy}^2$$

Location of principal Axes

$$tan2\theta = \frac{-2Ixy}{I_{xx} - I_{yy}}$$

Centre of gravity of common volume

| Shape | Fig | volume | Centroid |
|---|---|---|---|
| ~~Cylinder~~ | | | |
| cylinder |  | $V = \pi r^2 h$ | $\bar{x} = h/2$, $q_1$ = hay |
| cone |  | $V = \frac{1}{3}\pi r^2 h$ | $\bar{x} = h/4$, [illegible] |
| Pyramid |  | $V = \frac{1}{3}\, abh$ | $\bar{x} = h/4$ |
| Hemisphere |  | $V = \frac{2}{3}\pi r^2$ | $\bar{x} = \frac{3r}{8}$ |
| parabola |  | $V = \frac{1}{2}\pi a^2 h$ | $\bar{x} = h/3$ |
| Semi ellipse |  | $V = \frac{2}{3}\pi a^2 h$ | $\bar{x} = \frac{3h}{8}$ |

1. Find the position of the centroid of the solid combine shown in fig. consist of a solid cone of height 50 mm and base diameter 80 mm and a cylinder of height 100 mm and diameter 80mm with a semicircular cut as shown.

$\bar{X} = \frac{v_1x_1 + v_2x_2 - v_3x_3}{v_1 + v_2 - v_3}$

$\bar{Y} = \frac{v_1y_1 + v_2y_2 - v_3y_3}{v_1 + v_2 - v_3}$

$v_1 = {}^1/_3\,\pi r^2 h = {}^1/_3 \times \pi \times 40^2 \times 50 = 83775.80mm^3$

$v_2 = \pi r^2 h = \pi \times 40^2 \times 100 = 502654.82mm^3$

$v_3 = \frac{\pi r^2 h}{2} = \frac{\pi \times 40^2 \times 100}{2} = 251327.41mm^3$

$X_1 = {}^b/_2 = {}^{80}/_2 = 40mm \qquad Y_1 = 100 + {}^{50}/_3 = 116.66mm$

$X_2 = {}^b/_2 = {}^{80}/_2 = 40mm \qquad Y_2 = {}^{100}/_2 = 50mm$

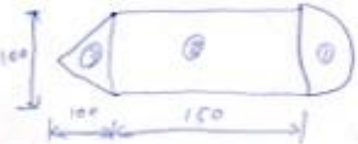
$X_3 = {}^d/_2 = {}^{80}/_2 = 40mm \qquad Y_3 = \frac{4r}{3\pi} + \frac{4\times40}{3\pi} = 16.97mm$

$\bar{X} = \frac{(83775.80 \times 40) + (502654.82 \times 40) - (251327.4 \times 40)}{83775.5 + 502654.82 - 251327.41}$

$\bar{X} = 40mm$

$\bar{Y} = \frac{(83775.80 \times 416.66) + (502654.82 \times 50) - (251327.41 \times 16.97)}{83775.80 + 502654.82 - 251325.41}$

2. A hemisphere of diameter 100 mm is fixed to cylinder is OA hemisphere diameter 100mm and cone is fixed another and of the cylinder its length is 100mm as shown fig. Locate the centroid of combine fig.



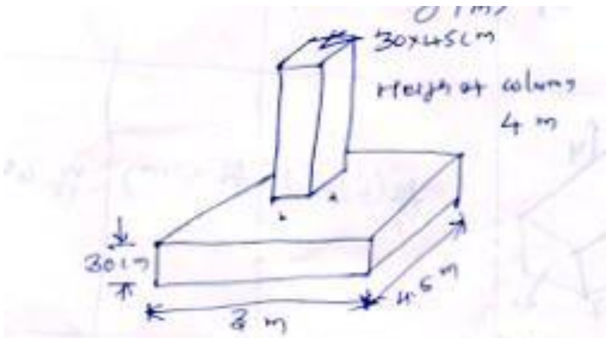

Mass Moment of irertia:

| Figure | Moment of Irertia about xx | about yy | about zz | Mass |
|---|---|---|---|---|
| Thin rod | $\frac{M}{12}L^2$ | $\frac{M}{12}L^2$ | $\frac{M}{6}L^2$ | |
| thin rectangle plate | $\frac{M}{12}(b^2+c^2)$ | $\frac{M}{12}L^2$ | $\frac{M}{12}b^2$ | |
| Rectangular prism | $\frac{M}{12}(b^2+a^2)$ | $\frac{M}{12}(a^2+b^2)$ | $\frac{M}{12}(a^2+b^2)$ | $\rho(a \times b \times c)$ |
| cylinder | $\frac{M}{2}r^2$ | $\frac{M}{12}(3r^2+h^2)$ | $\frac{M}{12}(3r^2+h^2)$ | $\rho(\pi r^2 h)$ |
| sphere | $\frac{2}{5}ma^2$ | $\frac{2}{5}ma^2$ | $\frac{2}{5}Ma^2$ | $\rho\left(\frac{4}{3}\pi a^3\right)$ |

3. A rectangular RCC column is centrally cast over a concrete bed R.C.C. in Fig. column is of section 30$\times$ 45 $cm$ and height 4m.the concrete bed is of size

3$\times$ 4.5$m$ and thickness 30cm. find the mass moment of inertia of the column and bed combination about its vertical centroidal axis.

Mass density of concrete=2500 kg/m$^3$



Soln:

$I_{yy}$ Combosite body= $(I_{yy})lumn + (I_{yy})bed$

$$(I_{yy})_{column} = \frac{M}{12}(b^2 + d^2)$$

M=mass volume$\times$ mass density

M=$(0.3 \times 0.45 \times 4)m^3 \times 2500\ {kg}/{m^3}$

M=3500 kg

$I_{yy}$ Column $=\frac{M}{12}(b^2 + d^2)$

$$= \frac{1350}{12}(0.3^2 + 0.45^2)$$

$I_{yy}$ Column=32.91kg.$m^2$

$I_{yy}$bed $= \frac{M}{12}(b^2 + d^2)$ M=massvolume$\times$ $density$

$= \frac{10125}{12}(3^2 + 4.5^2)$ $(3 \times 0.3 \times 4.5) \times 2500m^3 \times {kg}/{m^3}$

M=10125kg

$I_{yy}$ bed = 24679.69kg$m^2$

$I_{yy}$Composite body $= Iyy$ column +$I_{yy}$ bed

= 32.91+24679.69

$I_{YY}$= 24712.6kg.$m^2$