

1.2 METHOD OF JOINTS

In this method, after determining the reactions at the supports, the equilibrium of every joint is considered. This means the sum of all the vertical forces as well as the horizontal forces acting on a joint is equated to zero. The joint should be selected in such a way that at any time there are only two members, in which the forces are unknown. The force in the member will be compressive if the member pushes the joint to which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.

Example 1.2.1 A truss of 8m span consisting of seven members each of 4m length supported at its ends and loaded as shown in Fig.5.2. Determine the forces in the members by method of joints

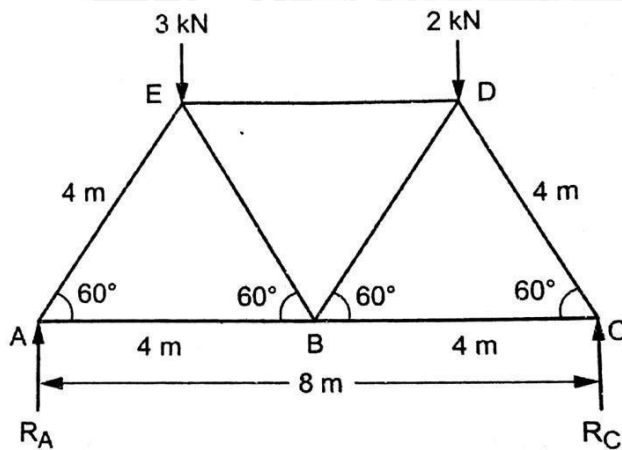


Fig.1.2.a

Solution:

Determine the reactions at A and C

Taking moment about A

$$R_C \times 8 = R_D \times 6 + R_E \times 2$$

$$\text{Or } R_C \times 8 = 2 \times 6 + 3 \times 2$$

$$\text{Or } R_C = 2.25 \text{ kN}$$

We know that ,

Upward vertical reaction = Downward vertical reaction

$$R_A + R_C = 3 + 2$$

$$\text{Or } R_A + 2.25 = 5$$

$$\text{Or } R_A = 2.75 \text{ kN}$$

Consider the joint A.

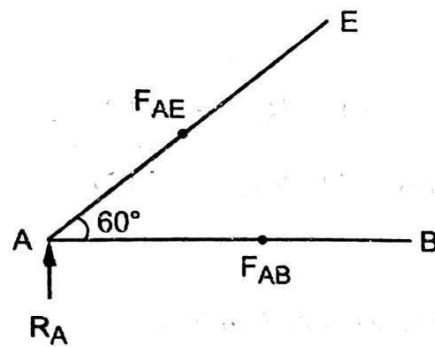


Fig.1.2.b

Assume the forces (F_{AE} and F_{AB}) acting on joint A are tensile forces (acting away from joint A). If we get negative value the force in that member is compressive.

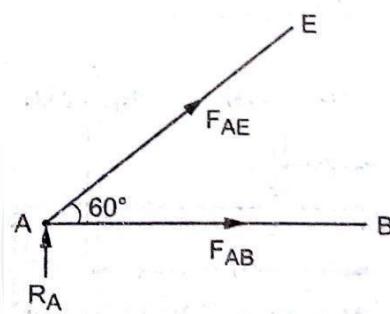


Fig.1.2.c

At joint A:

Resolving the force (F_{AE}) vertically, we know that the sum of vertical forces = 0.

$$R_A + F_{AE} \sin 60^\circ = 0$$

$$\text{Or } 2.75 = -F_{AE} \sin 60^\circ$$

$$\text{Or } F_{AE} = -3.17 \text{ kN}$$

(Compression)

Resolving the force (F_{AE}) horizontally,

Sum of horizontal forces = 0

$$F_{AB} + F_{AE} \cos 60^\circ = 0$$

$$\text{Or } F_{AB} = -F_{AE} \cos 60^\circ$$

$$\text{Or } F_{AB} = -1.58 \text{ kN (Tension)}$$

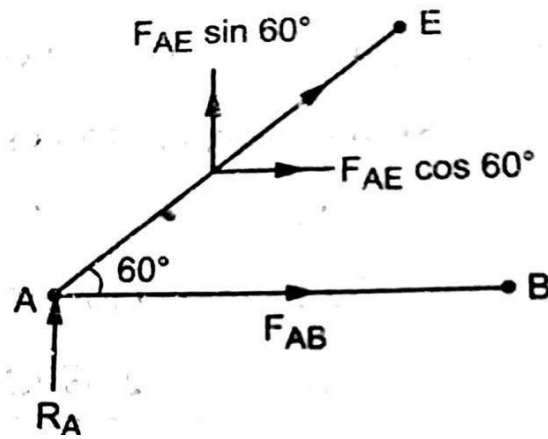


Fig.1.2.d

Consider the joint C.

Assume the forces (F_{DC} and F_{BC}) acting on joint A are tensile forces (acting away from joint A). If we get negative value the force in that member is compressive.

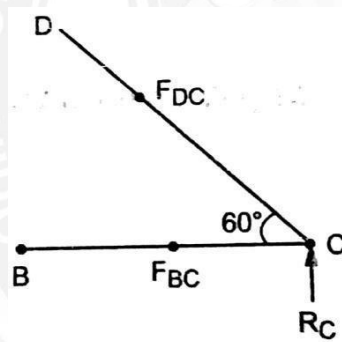


Fig.5.2.e

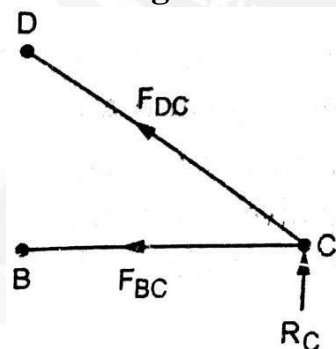


Fig.1.2.f

At joint C:

Resolving the force (F_{DC}) vertically,

Sum of horizontal forces = 0

$$\text{Or } R_C + F_{DC} \sin 60^\circ = 0$$

$$\text{Or } 2.25 = -F_{DC} \sin 60^\circ$$

$$\text{Or } F_{DC} = -2.59 \text{ kN}$$

(Compression)

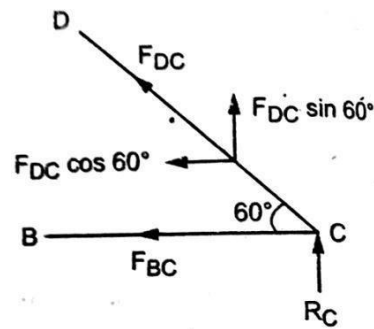


Fig.1.2.h

Resolving the force horizontally,

Sum of horizontal forces = 0

$$-F_{BC} - F_{DC} \cos 60^\circ = 0$$

(Force acting towards left side is -ve)

Or $F_{BC} = -F_{DC} \cos 60^\circ$

Or $F_{BC} = -2.59 \times \cos 60^\circ$

Or $F_{BC} = 1.295 \text{ kN (Tension)}$

Consider the joint B.

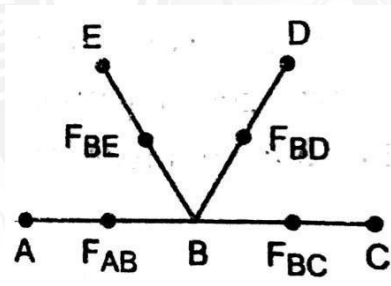


Fig.1.2.i

Assume the forces (F_{AB} , F_{BC} , F_{BD} and F_{BE}) acting on joint B are tensile forces (acting away from joint B). If we get negative value the force in that member is compressive.

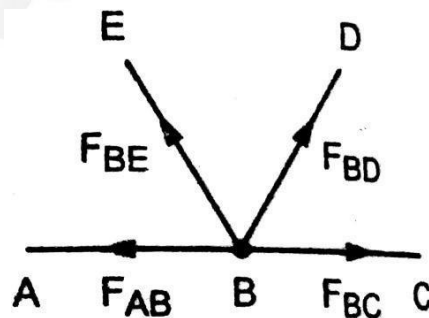


Fig.1.2.j

At joint B:

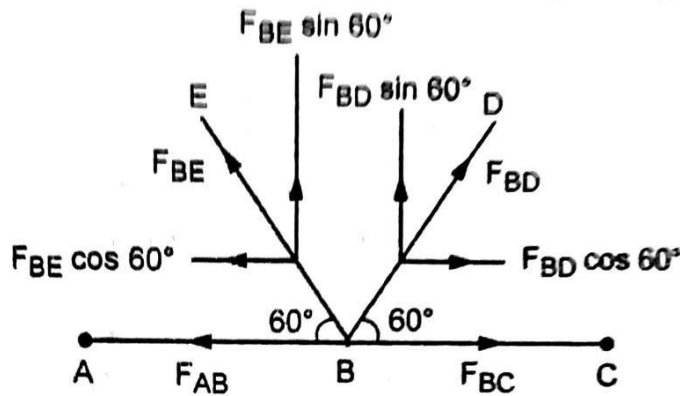


Fig.1.2.k

Resolving the force (F_{BE} & F_{BD}) vertically,

$$\text{Sum of horizontal forces} = 0$$

$$F_{BE} \sin 60^\circ + F_{BD} \sin 60^\circ = 0$$

Or

$$F_{BE} \sin 60^\circ = -F_{BD} \sin 60^\circ$$

Or

$$F_{BE} = -F_{BD}$$

Resolving the force (F_{BE} & F_{BD}) horizontally,

$$\text{Sum of horizontal forces} = 0$$

$$\Sigma H = 0$$

$$-F_{AB} - F_{BE} \cos 60^\circ + F_{BC} + F_{BD} \cos 60^\circ = 0$$

(Force acting towards right side is -ve, force towards left side is -ve)

$$\text{Or } -1.58 - F_{BE} \cos 60^\circ + 1.29 + F_{BD} \cos 60^\circ = 0$$

Or

$$-0.29 + 2F_{BD} \cos 60^\circ = 0$$

(Since, $F_{BE} = -F_{BD}$)

Or

$$F_{BD} = 0.29 \text{ kN}$$

(Tension)

Or

$$F_{BE} = -0.29 \text{ kN (Compression)}$$

Consider the joint D.

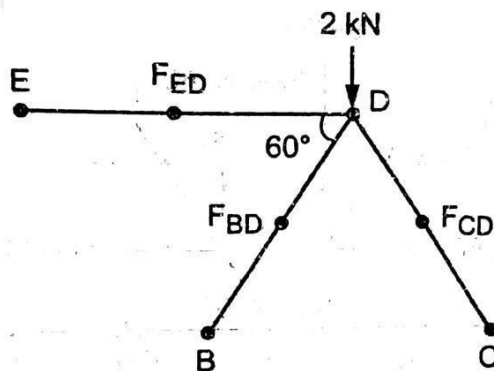


Fig.1.2.l

Assume the forces (F_{ED} , F_{BD} , and F_{CD}) acting on joint D are tensile forces (acting away from joint D). If we get negative value, the force on that member is compressive.

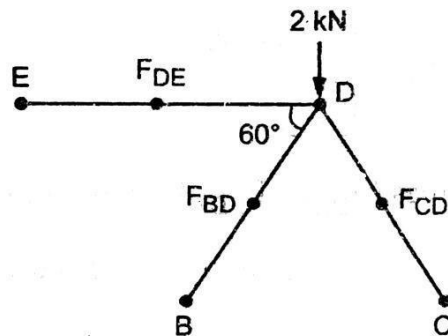


Fig.1.2.m

At joint D:

Resolving the force (F_{BD} & F_{CD}) horizontally, sum of horizontal forces = 0

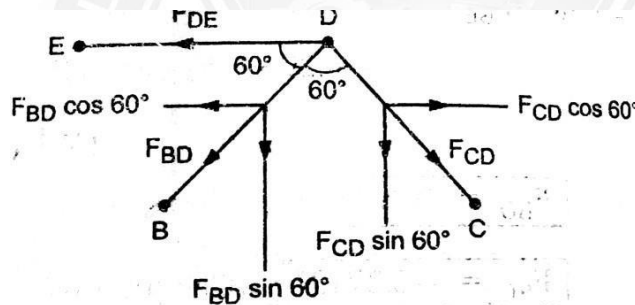


Fig.1.2.n

$$-F_{DE} - F_{BD} \cos 60^\circ + F_{CD} + F_{BD} \cos 60^\circ = 0$$

(Force towards right side +ve, force towards left side -ve)

$$\text{Or } -F_{DE} - 0.29 \times \cos 60^\circ - 2.59 \times \cos 60^\circ = 0$$

$$\text{Or } F_{DE} = -1.44 \text{ kN (Compression)}$$

Result:

Sl.No.	Member	Force (kN)	Nature of force.
1	AE	-3.17	compression
2	AB	1.58	Tension
3	CD	-2.59	compression
4	BC	1.29	Tension
5	BD	0.29	Tension
6	BE	-0.29	compression

7	DE	-1.44	compression
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Example 1.2.2 Determine the forces in the truss shown in Fig 5.3. It carries a horizontal load of 16 kN and vertical load of 24 kN.

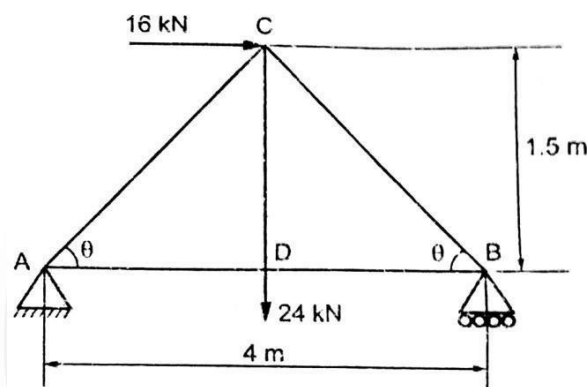


Fig.1.3.a

Solution: The truss is supported on rollers at B and hence the reaction at B should be vertical (R_B).

The truss is hinged at A and hence end A consists of a horizontal reaction (H_A) and vertical reaction (R_A).

Determine the reaction at A and B (R_A and R_B).

Taking moment about A.

$$R_B \times 4 = 24 \times 2 + 16 \times 1.5$$

$$R_B = 18 \text{ kN}$$

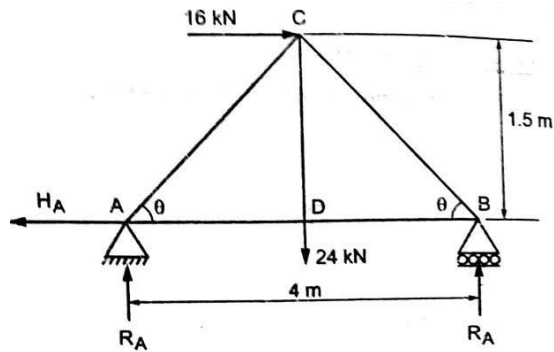
We know that,

Upward vertical load = Downward vertical load

$$R_A + R_B = 24$$

$$R_A + 18 = 24$$

$$R_A = 6 \text{ kN}$$


Fig.1.3.b

Right side horizontal load = Left side horizontal load

$$16 = H_A$$

$$H_A = 16 \text{ kN}$$

In the triangle BCD

$$BC^2 = CD^2 + BD^2$$

$$BC^2 = (1.5)^2 + 2^2$$

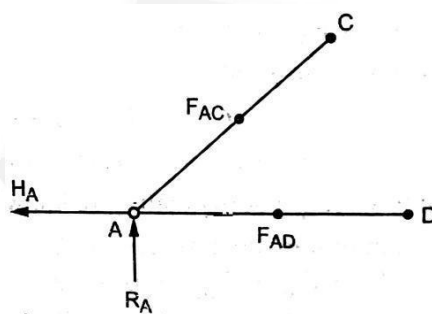
$$BC = 2.5 \text{ m}$$

$$\sin \theta = \frac{DC}{BC} = \frac{1.5}{2.5}$$

$$\sin \theta = 0.6$$

$$\theta = 36.8^\circ$$

Consider the joint A.


Fig.1.3.c

Assume the forces F_{AC} and F_{AD} acting on joint A are tensile forces (acting away from joint A). If we get negative value force on that member is compressive.

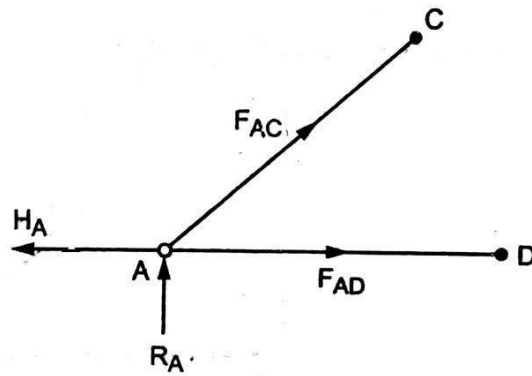


Fig.1.3.d

At joint A:

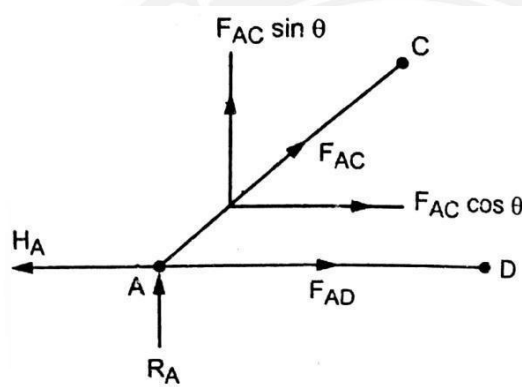


Fig.1.3.e

Resolving the force (F_{AC}) vertically, we know that

Sum of vertical forces = 0

$$R_A + F_{AC} \sin \theta = 0$$

$$6 + F_{AC} \sin 36.8^\circ = 0$$

$$6 = -F_{AC} \sin 36.8^\circ$$

$$F_{AC} = -10 \text{ kN (Compressive)}$$

Resolving the force (F_{AC}) horizontally, we know that,

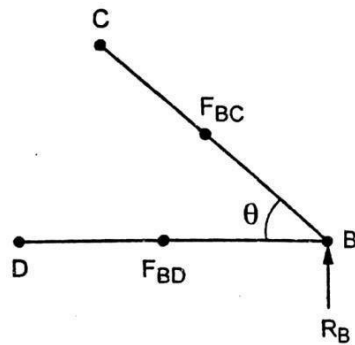
Sum of horizontal forces = 0

$$-H_A + F_{AD} + F_{AC} \cos \theta = 0$$

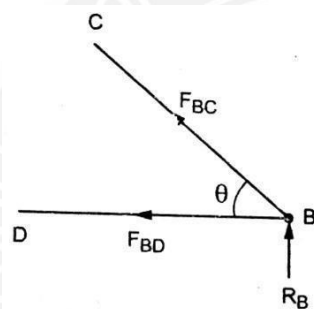
$$-16 + F_{AD} + -10 \times \cos 36.8^\circ = 0$$

$$F_{AD} = 24 \text{ kN (Tension)}$$

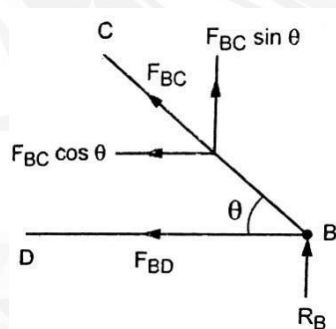
Consider the joint B.

**Fig.1.3.f**

Assume the forces F_{BC} and F_{BD} acting on joint B are tensile forces (acting away from B). If we get negative value, force in that member is compressive.

**Fig.1.3.g**

At joint B:

**Fig.1.3.g**

Resolving the force (F_{BC}) vertically, we know that,

Sum of vertical forces = 0

$$R_B + F_{BC} \sin \theta = 0$$

$$18 + F_{BC} \sin \theta = 0$$

$$F_{BC} = -\frac{18}{\sin 36.8}$$

$$\mathbf{F_{BC} = -30kN \text{ (Compressive)}}$$

Resolving the force (F_{BC}) horizontally,

we know that,

Sum of horizontal forces = 0

$$-F_{BC} \cos \theta - F_{BD} = 0$$

$$-F_{BC} \cos \theta = F_{BD}$$

$$30 \cos \theta = F_{BD}$$

$$30 \cos 36.8^\circ = F_{BD}$$

$$F_{BD} = 24 \text{ kN (Tension)}$$

Result:

Sl.No.	Member	Force (kN)	Nature of force
1	AC	-10	Compression
2	AD	24	Tension
3	BC	-30	Compression
4	BD	24	Tension

Example 1.2.3 A truss is loaded as in Fig 5.4. Determine the forces in all the members of that truss.

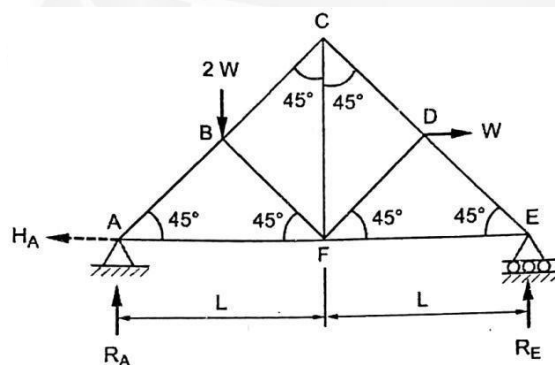


Fig 1.4.a

Solution: To solve the above problem, consider $W = 1 \text{ kN}$
To find the reactions at the support:

$$R_A + R_E = 2W = 2$$

$$H_A = W = 1$$

Taking moment about point A,

$$i.e., \Sigma MA = 0$$

$$-(RE \times 2L) \times \left(2 \times \frac{L}{2}\right) + \left(1 \times \frac{L}{2}\right) = 0$$

$$\left(L \times \frac{L}{2}\right) = R_E \times 2L$$

3

$$\frac{1}{2} L = R_E \times 2L$$

$$R_E = 0.75$$

$$R_A + R_E = 2$$

$$R_A = 2 - R_E = 2 - 0.75$$

$$R_A = 1.25$$

Solving the above problem using method of joints:

At joint A:

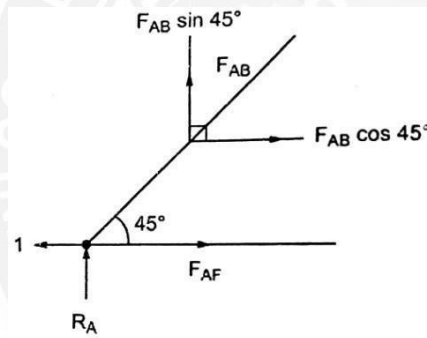


Fig 1.4.b

Sum of horizontal forces = $\Sigma H = 0$

Sum of vertical forces = $\Sigma V = 0$

$$\Sigma V = 0, \quad R_A + F_{AB} \sin 45^\circ = 0$$

$$F_{AB} \sin 45^\circ = -R_A$$

$$F_{AB} = \frac{-1.25}{\sin 45^\circ} = \frac{-1.25}{\frac{1}{\sqrt{2}}} = -1.77$$

$F_{AB} = -1.77$ (Compression)

$$\Sigma H = 0, \quad -1 + F_{AB} \cos 45^\circ + F_{AF} = 0$$

$$-1 + (-1.77 \times \cos 45^\circ) + F_{AF} = 0$$

$$F_{AF} = 2.25 \text{ (Tension)}$$

At joint E:

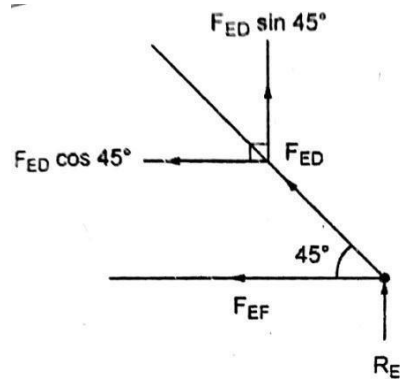


Fig 1.4.c

$$\sum V = 0$$

$$F_{ED} \sin 45^\circ + R_E = 0$$

$$F_{ED} = \frac{R_E}{\sin 45^\circ} = \frac{0.75}{\sin 45^\circ}$$

$$F_{ED} = -1.06 \text{ (Compressive)}$$

$$\sum H = 0$$

$$-F_{EF} - F_{ED} \cos 45^\circ = 0$$

$$-F_{EF} - (-1.06 \times \cos 45^\circ) = 0$$

$$-F_{EF} + 0.75 = 0$$

$$F_{EF} = 0.75 \text{ (Tensile)}$$

At joint B:

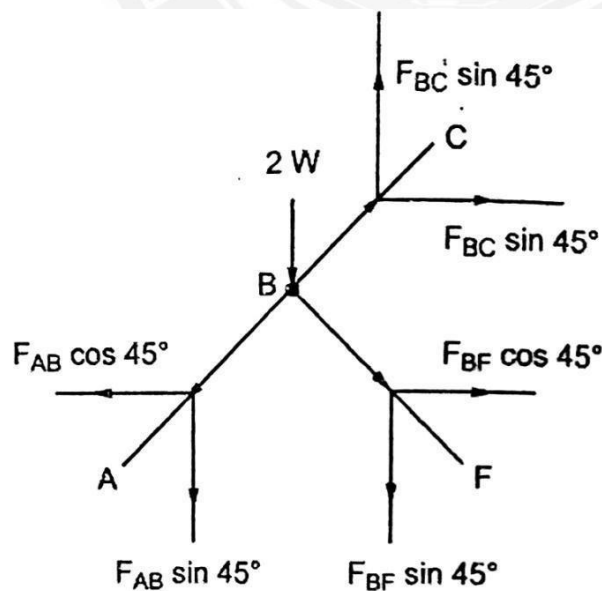


Fig 1.4.d

$$\sum H = 0$$

$$-F_{AB} \cos 45^\circ + F_{BC} \cos 45^\circ + F_{BF} \cos 45^\circ = 0$$

$$\cos 45^\circ [-F_{AB} + F_{BC} + F_{BF}] = 0$$

$$F_{BC} + F_{BF} = F_{AB} = -1.77$$

$$F_{BC} + F_{BF} = -1.77 \quad \dots (1)$$

$$\Sigma V = 0$$

$$F_{BC} \sin 45^\circ - F_{AB} \sin 45^\circ - F_{BF} \sin 45^\circ - 2 = 0$$

$$F_{BC} \sin 45^\circ - F_{BF} \sin 45^\circ = 2 + F_{BA} \cdot \sin 45^\circ$$

$$F_{BC} \sin 45^\circ - F_{BF} \sin 45^\circ = 2 + (-1.77 \sin 45^\circ)$$

$$(F_{BC} - F_{BF}) \sin 45^\circ = 0.75$$

$$F_{BC} - F_{BF} = \frac{0.75}{\sin 45^\circ} = 1.06$$

$$F_{BC} - F_{BF} = 1.06 \quad \dots (2)$$

Solving equations (1) and (2), we get

$$F_{BC} = -0.36 \text{ (Compression)}$$

$$F_{BF} = -1.42 \text{ (Compression)}$$

At joint D:

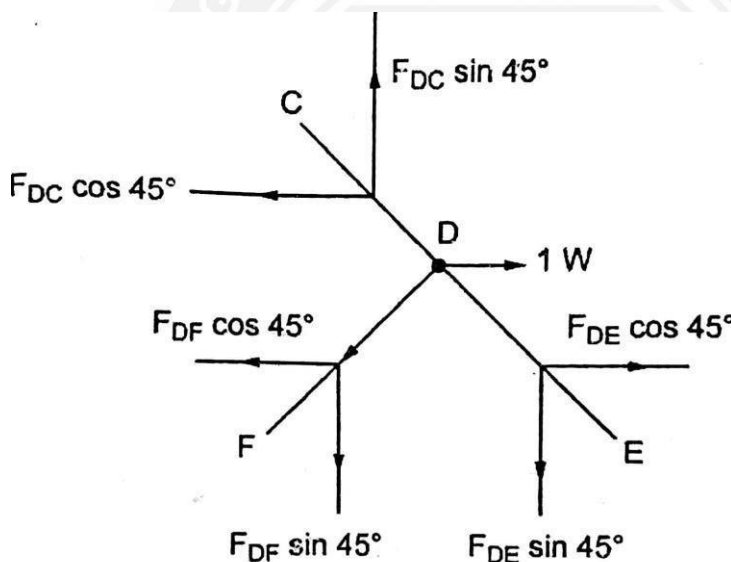


Fig 1.4.d

$$\Sigma H = 0$$

$$-F_{DC} \cos 45^\circ - F_{DF} \cos 45^\circ + F_{DE} \cos 45^\circ + 1 = 0$$

$$-F_{DC} \cos 45^\circ - F_{DF} \cos 45^\circ - (1.06 \times \cos 45^\circ) + 1 = 0$$

$$-F_{DC} \cos 45^\circ - F_{DF} \cos 45^\circ = -0.25$$

$$-(F_{DC} + F_{DF}) \cos 45^\circ = -0.25$$

$$F_{DC} + F_{DF} = +\frac{0.25}{\cos 45^\circ}$$

$$F_{DC} + F_{DF} = 0.35 \quad \dots (3)$$

$$\Sigma V = 0$$

$$+F_{DC} \sin 45^\circ - F_{DF} \sin 45^\circ - F_{DE} \sin 45^\circ = 0$$

$$F_{DC} - F_{DF} - F_{DE} = 0$$

$$F_{DC} - F_{DF} = +F_{DE}$$

$$F_{DC} - F_{DF} = -1.06 \quad \dots (4)$$

Solving (3) and (4), we get

$$F_{DC} = -0.35$$

$$F_{DF} = 0.71$$

(Compression)

(Tension)

Member	Force (kN)	Nature of force
AB	-1.77W	Compression
BC	-0.36W	Compression
CD	-0.35W	Compression
DE	-1.06W	Compression
EF	0.75W	Tension
FA	2.25W	Tension
FD	0.71W	Tension
CF	0.5W	Tension
BF	-1.42W	Compression

Example 1.2.4 Analyse the truss shown in Fig.5.5. using methods of joints.

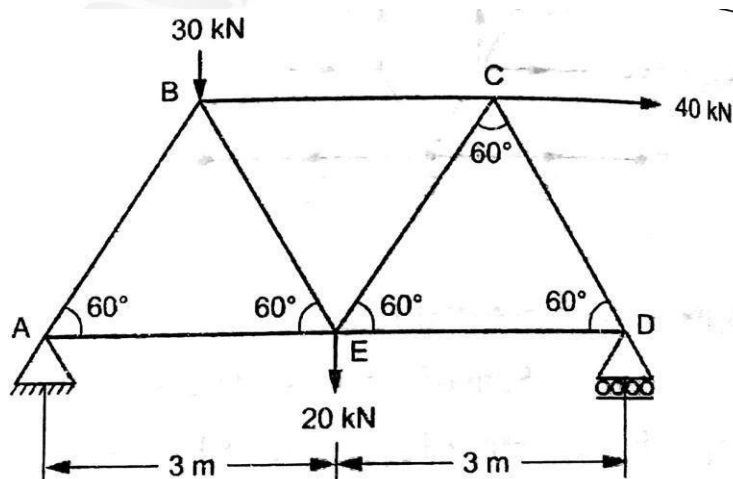
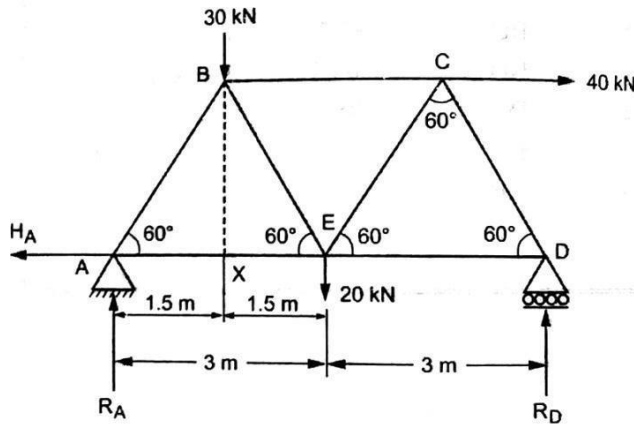


Fig 1.5.a**Solution:**

The truss is supported on rollers at D and hence the reaction at D should be vertical (R_D).

The truss is hinged at A and hence end A consists of horizontal reaction (H_A) and vertical reaction (R_A).

**Fig 1.5.b**

From $\Delta^{le} BEX$,

$$\tan 60^\circ = \frac{BX}{XE} = \frac{BX}{1.5}$$

$$BX = \tan 60^\circ \times 1.5$$

$$\mathbf{BX = 2.6 \text{ m}}$$

Determine the reactions. Taking moment about A

$$\begin{aligned} R_D \times 6 &= 40 \times BX + 20 \times 3 + 30 \times 1.5 \\ &= 40 \times 2.6 + 20 \times 3 + 30 \times 1.5 \end{aligned}$$

$$\mathbf{R_D = 34.8}$$

kN

We know that,

Upward vertical forces = Downward vertical forces

$$R_A + R_D = 30 + 20$$

$$R_A = 50 - R_D$$

$$R_A = 50 - 34.8$$

$$\mathbf{R_A = 15.2 \text{ kN}}$$

We know that,

Horizontal forces towards right side = Horizontal forces towards left side

$$40 \text{ kN} = H_A, \quad \therefore \mathbf{H_A = 40 \text{ kN}}$$

Consider the joint A.

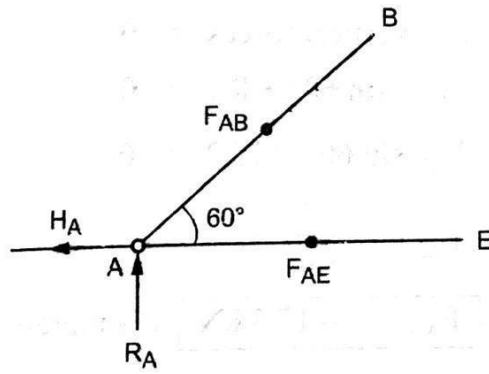


Fig 1.5.c

Assume the forces F_{AB} and F_{AE} acting on joint A are tensile forces (acting away from A). If we get negative value, the force in that member is compressive.

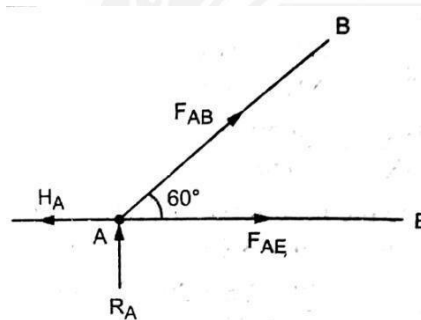


Fig 1.5.d

At joint A:

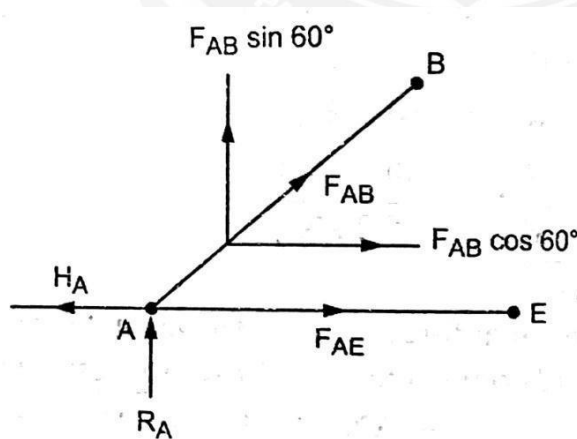


Fig 1.5.e

Resolving the force vertically,

$$\text{Sum of vertical forces} = 0$$

$$F_{AB} \sin 60^\circ + R_A = 0$$

$$F_{AB} \sin 60^\circ + 15.2 = 0 \quad 15.2$$

$$F_{AB} = \frac{-}{\sin 60^\circ}$$

$$F_{AB} = -17.5 \text{ kN (Compressive)}$$

Resolving the force horizontally,

$$\text{Sum of horizontal forces} = 0$$

$$F_{AB} \cos 60^\circ - H_A + F_{AE} = 0$$

$$-17.5 \cos 60^\circ - 40 + F_{AE} = 0$$

$$-17.5(0.5) - 40 + F_{AE} = 0$$

$$F_{AE} = 48.75 \text{ kN (Tension)}$$

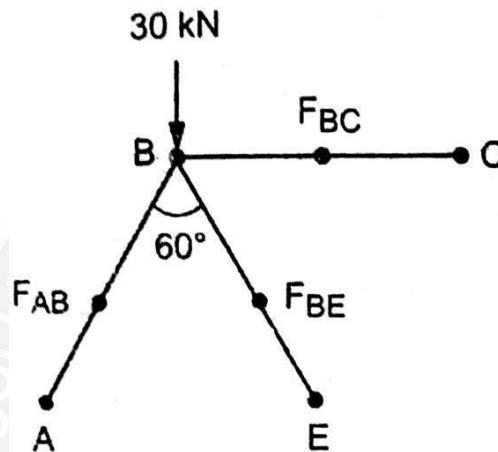


Fig 1.5.f

Consider the joint B.

Assume the forces acting on joint B are tensile forces. If we get negative value, force in that member is compressive.

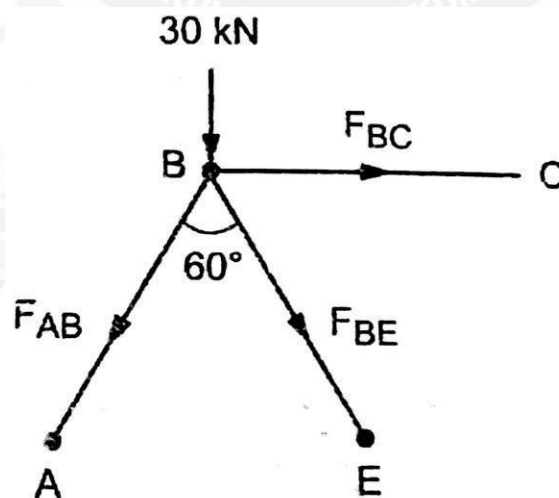
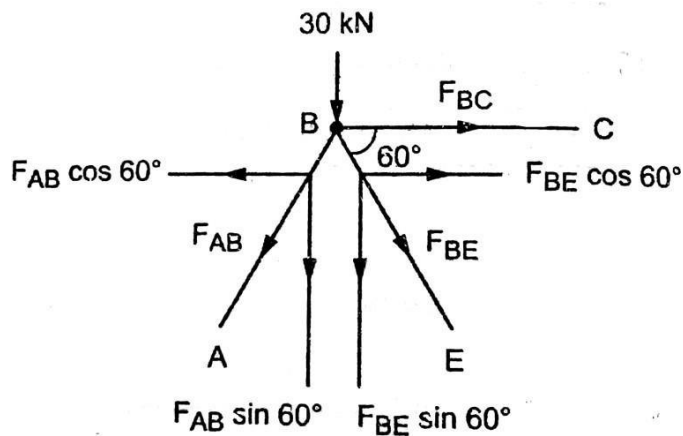


Fig 1.5.f

At joint B

Resolving the force vertically, Sum of vertical forces = 0


Fig 1.5.g

$$30\text{kN} - F_{AB} \sin 60^\circ - F_{BE} \sin 60^\circ = 0$$

$$-30 + 17.5 \times \sin 60^\circ = F_{BE} \sin 60^\circ$$

$$F_{BE} = -17.1\text{kN (Compression)}$$

Resolving the force horizontally,

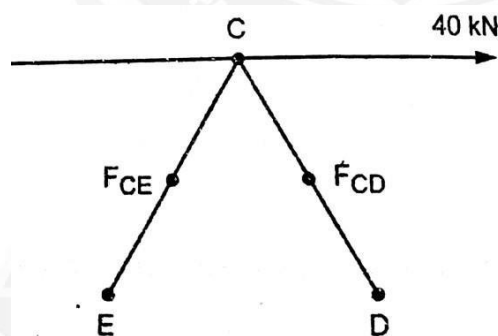
$$\text{Sum of horizontal forces} = 0$$

$$F_{BC} + F_{BE} \cos 60^\circ - F_{AB} \cos 60^\circ = 0$$

$$F_{BC} - 17.1(0.5) + 17.1(0.5) = 0$$

$$F_{BC} = -0.2\text{kN (Compression)}$$

Consider the joint C:


Fig 1.5.h

Assume the forces acting on C are tensile forces . If we get negative value, the force on that member is compressive.

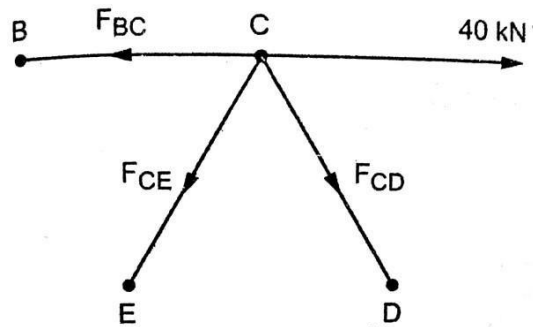


Fig 1.5.i

At joint B:

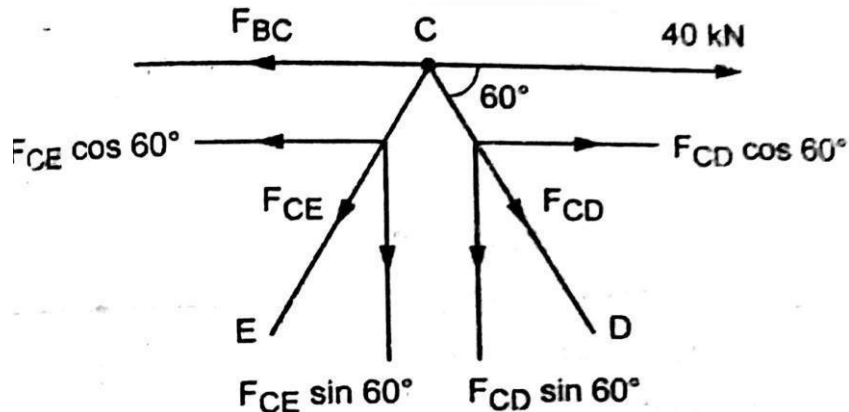


Fig 1.5.j

Resolving the force vertically,

$$\begin{aligned} \text{Sum of vertical forces} &= 0 \\ -F_{CE} \sin 60^\circ - F_{CD} \sin 60^\circ &= 0 \\ F_{CE} &= -F_{CD} \end{aligned} \quad \dots (A)$$

Resolving the force horizontally,

$$\begin{aligned} \text{Sum of horizontal forces} &= 0 \\ -F_{BC} + 40 - F_{CE} \cos 60^\circ + F_{CD} \cos 60^\circ &= 0 \\ 0.2 + 40 + 2 F_{CD} \cos 60^\circ &= 0 \\ (\text{Since, } F_{CE} &= -F_{CD}) \\ F_{CD} &= -40.2 \text{ kN (Compression)} \end{aligned}$$

Apply in (A),

$$F_{CE} = 40.2 \text{ kN (Tension)}$$

Consider the joint D:

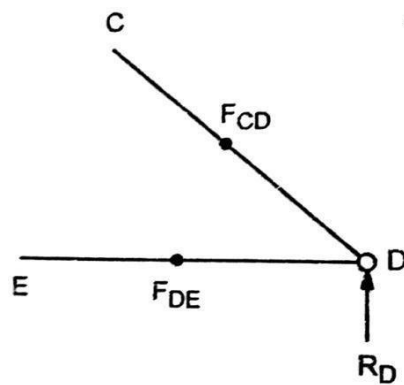


Fig 1.5.k

Assume the forces acting on joint D are tensile forces if we get negative value, the force in that member is compressive.

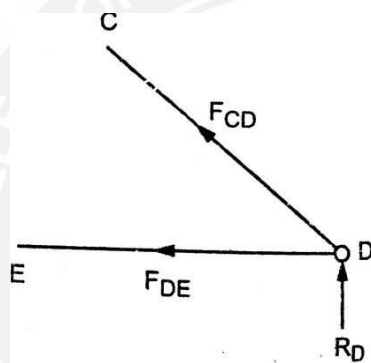


Fig 1.5.l

Resolving the force horizontally,

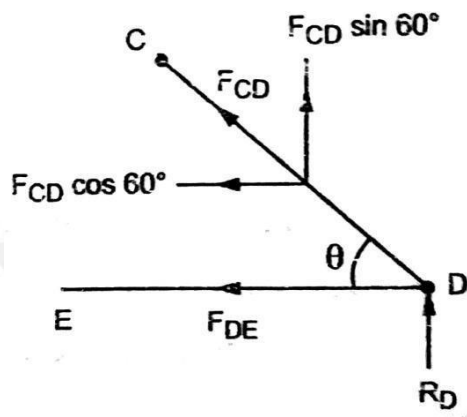


Fig 1.5.m

Sum of horizontal forces = 0

$$- F_{DE} - F_{CD} \cos 60^\circ = 0$$

$$- F_{DE} + 40.2 \cos 60^\circ = 0$$

$$F_{DE} = 20.1 \text{ kN (Tension)}$$

Sl.No.	Member	Force (kN)	Nature of force
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1	AB	-17.5	Compression
2	AE	48.75	Tension
3	BE	-17.1	Compression
4	BC	-0.2	Compression
5	CD	-40.2	Compression
6	CE	40.2	Tension
7	DE	20.1	Tension

