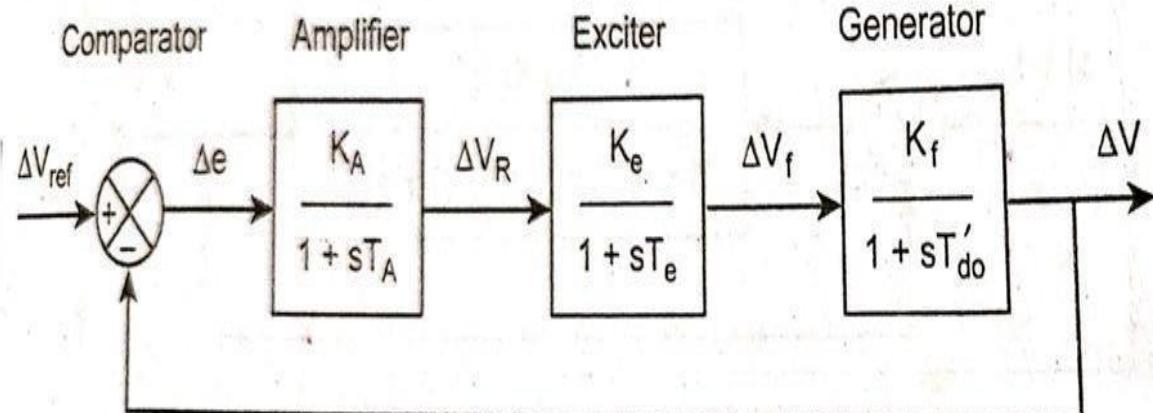


## STATIC ANALYSIS OF AUTOMATIC VOLTAGE REGULATOR LOOP

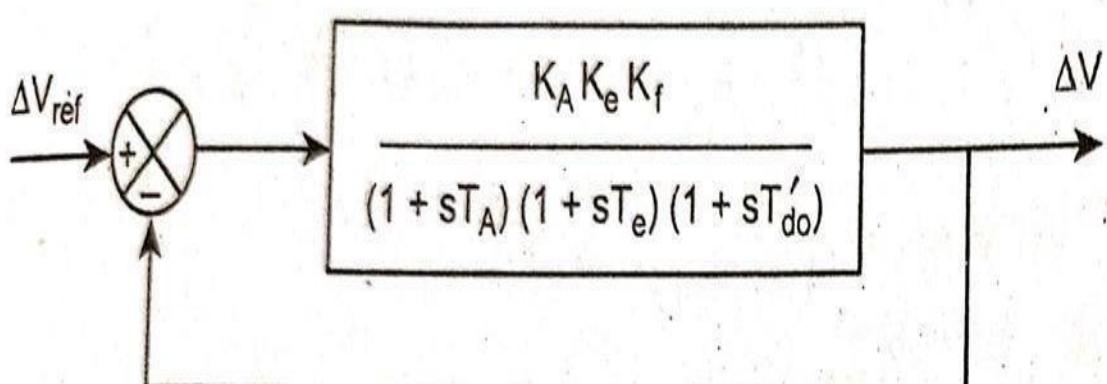
- The automatic voltage regulator must regulate the terminal voltage  $|V|$  within the required static accuracy limit.
- It must have sufficient speed response.
- It must be stable.

The block diagram of AVR is as shown in Fig.



$$\text{Initial error, } \Delta e_0 = \Delta|V|_{\text{ref}0} - \Delta|V|_0$$

$$\text{Open loop T.F, } G(s) = \frac{K_A \ K_e \ K_f}{(1 + sT_A) \ (1 + sT_e) \ (1 + sT'_{d0})}$$



$$\text{At initial condition, } \Delta |V|_0 = \frac{G(s)}{1 + G(s)} \Delta |V|_{ref0} \quad \dots \dots \dots (1)$$

$\Delta e_0$  must be less than some specified percentage  $P$  of reference voltage  $\Delta |V|_{ref0}$ . The static accuracy specification is :

$$\therefore \Delta e_0 < \frac{P}{100} \Delta |V|_{ref0} \quad \dots \dots \dots (2)$$

For a constant input, the transfer function is obtained by setting  $s=0$

Substituting equation (1) in (2) we get,

$$\Delta e_0 = (\Delta |V|_{ref0}) - \left( \frac{G(s)}{1 + G(s)} \Delta |V|_{ref0} \right)$$

$$\Delta e_0 = \Delta |V|_{ref0} \left[ \frac{1}{1 + G(s)} \right]$$

Putting  $S = 0$ ,

$$\Delta e_0 = (\Delta |V|_{ref0}) \left[ \frac{1}{1 + \lim_{s \rightarrow 0} G(s)} \right] = \frac{\Delta |V|_{ref0}}{1 + k_p}$$

Position error constant,  $K_p = \lim_{s \rightarrow 0} G(s)$

$$K_p = \lim_{s \rightarrow 0} G(s)$$

$$= \lim_{s \rightarrow 0} G(s) = \lim_{s \rightarrow 0} \frac{K_A \ K_e \ K_f}{(1 + sT_A) \ (1 + sT_e) \ (1 + sT_{d0}')} \quad \dots \dots \dots$$

$$K_p = K_A \ K_e \ K_f$$

$$\Delta e_0 = \frac{\Delta |V|_{ref0}}{1 + K}$$

If  $K$  increases,  $\Delta e_0$  decreases, so static error decreases with an increased loop gain.

To find the value of  $K$ ;

$$\therefore \Delta e_0 < \frac{P}{100} \Delta |V|_{ref0}$$

$$\frac{\Delta |V|_{ref0}}{1 + K} < \frac{P}{100} \Delta |V|_{ref0} = \frac{1}{1 + K} < \frac{P}{100}$$

$$1 + K > \frac{100}{P}$$

$$K > \frac{100}{P} - 1$$

If  $\Delta e_0$  is less than 1%, K must exceed 99%.

### Steady state response for a closed loop Transfer Function

$$\text{Closed loop T.F.} = \frac{\Delta V(s)}{\Delta V_{ref}(s)} = \frac{\frac{K_A \ K_e \ K_f}{(1 + sT_A) \ (1 + sT_e) \ (1 + sT'_{d0})}}{1 + \frac{K_A \ K_e \ K_f}{(1 + sT_A) \ (1 + sT_e) \ (1 + sT'_{d0})}}$$

$$\Delta V(s) = \frac{K_A \ K_e \ K_f \Delta V_{ref}(s)}{(1 + sT_A) \ (1 + sT_e) \ (1 + sT'_{d0}) + K_A \ K_e \ K_f}$$

$$\text{For a step input } \Delta V_{ref}(s) = \frac{1}{s}$$

Applying final value theorem,

$$\Delta V_{stat} = \lim_{s \rightarrow 0} s \Delta V(s)$$

$$\Delta V_{stat} = \lim_{s \rightarrow 0} \frac{s \times K_A \ K_e \ K_f \Delta V_{ref}(s) s^{\frac{1}{s}}}{(1 + sT_A) \ (1 + sT_e) \ (1 + sT'_{d0}) + K_A \ K_e \ K_f}$$

$$\Delta V_{stat} = \frac{K_A \ K_e \ K_f}{1 + K_A \ K_e \ K_f}$$

$$\Delta V_{stat} = \frac{K}{1 + K}$$

## Dynamic Analysis of AVR Loop

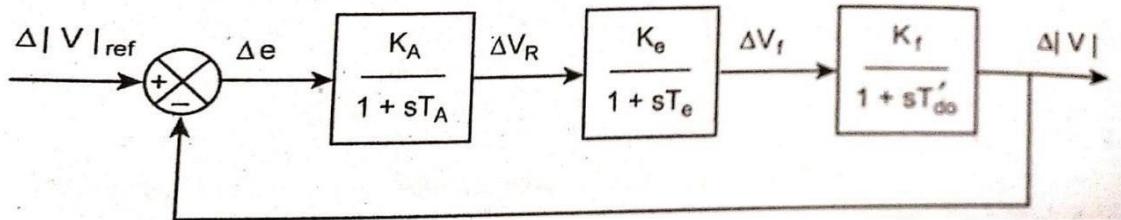


Fig Block diagram of AVR

$$\text{Open loop T.F } G(s) = \frac{K_A \ K_e \ K_f}{(1 + sT_A) \ (1 + sT_e) \ (1 + sT'_{do})}$$

$$\Delta V(s) = \frac{G(s)}{1 + G(s)} \Delta V_{ref}(s)$$

Taking inverse Laplace transform

$$\Delta V(s) = L^{-1} [\Delta V(s)]$$

The response depends upon the eigen values or closed loop poles, which are obtained from the characteristic equation

$$1 + G(s) = 0.$$

Find roots of characteristic equation [Eigen values]  $s_1, s_2, s_3$ .

### Case I : Roots are real and distinct

The open loop transfer function  $G(s)$  is of 3<sup>rd</sup> order. There are three eigen values  $s_1, s_2, s_3$ .

$$\Delta V(t) = L^{-1} \left[ \frac{k_1}{s - s_1} + \frac{k_2}{s - s_2} + \frac{k_3}{s - s_3} \right]$$

$$\text{Transient response} = k_1 e^{s_1 t} + k_2 e^{s_2 t} + k_3 e^{s_3 t}$$

### Case II : Two roots (Eigen values) are complex conjugate ( $\sigma \pm j\omega$ )

The transient response is  $A e^{\sigma t} \sin(\omega t + \beta)$

For AVR loop to be stable, the transient components must vanish with time.

All the eigen values are located in left half of s-plane. Then the loop possesses good tracking ability i.e the system is stable.

For high speed response, the loop possesses eigen values located far away to the left from origin in s-plane.

The closer the eigen value is located to the  $j\omega$  axis, the more dominant it becomes.