NEWTON RAPHSON METHOD

Newton Raphson Method is an iterative technique for solving a set of various nonlinear equations with an equal number of unknowns. There are two methods of solutions for the load flow using Newton Raphson Method. The first method uses rectangular coordinates for the variables while the second method uses the polar coordinate form. Out of these two methods the polar coordinate form is used widely.

Procedure of Newton Raphson Method

The computational procedure for Newton Raphson Method using polar coordinate is given below.

The Newton-Raphson procedure is as follows:

Step-1:

Choose the initial values of the voltage magnitudes $|V|^{(0)}$ of all n_p load buses and $n - 1$ angles $\delta^{(0)}$ of the voltages of all the buses except the slack bus.

Step-2:

Use the estimated $|V|^{(0)}$ and $\delta^{(0)}$ to calculate a total $n - 1$ number of injected real power $P_{calc}^{(0)}$ and equal number of real power mismatch $\Delta P^{(0)}$.

Step-3:

Use the estimated $|V|^{(0)}$ and $\delta^{(0)}$ to calculate a total n_p number of injected reactive power $Q_{calc}^{(0)}$ and equal number of reactive power mismatch $\Delta Q^{(0)}$.

Step-3:

Use the estimated $|V|^{(0)}$ and $\delta^{(0)}$ to formulate the Jacobian matrix $J^{(0)}$.

Step-4:

Solve equation for $\delta^{(0)}$ and $\Delta |V|^{(0)} \div |V|^{(0)}$.

Step-5:

Obtain the updates from

$$\delta^{(1)} = \delta^{(0)} + \Delta\delta^{(0)}$$

$$|V|^{(1)} = |V|^{(0)}\left[1 + \frac{\Delta|V|^{(0)}}{|V|^{(0)}}\right]$$

Step-6:

Check if all the mismatches are below a small number. Terminate the process if yes. Otherwise go back to step-1 to start the next iteration with the updates given by the equation

We shall now discuss the formation of the sub matrices of the Jacobian matrix. To do that **we shall use the real and reactive power equations .**Let us rewrite them with the help of equation as

A. Formation of J11

Let us define J11 as

$$Q_i = -|V_i|^2 B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i)$$

$$P_i = |V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$J_{11} = \begin{bmatrix} L_{22} & \cdots & L_{2n} \\ \vdots & \ddots & \vdots \\ L_{n2} & \cdots & L_{nn} \end{bmatrix}$$

It can be seen from the equation that L_{ik}'s are the partial derivatives of P_i with respect to δ_k. The derivative P_i equation with respect to k for $i \neq k$ is given by

$$L_{ik} = \frac{\partial P_i}{\partial \delta_k} = -|Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i), \quad i \neq k$$

Similarly the derivative P_i with respect to k for $i = k$ is given by

$$L_{ii} = \frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i)$$

Comparing the above equation we can write

$$L_{ii} = \frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 B_{ii}$$

B. Formation of J$_{21}$

Let us define J_{21} as

$$J_{21} = \begin{bmatrix} M_{21} & \cdots & M_{2n} \\ \vdots & \ddots & \vdots \\ M_{n_p 2} & \cdots & M_{n_p n} \end{bmatrix}$$

it is evident that the elements of J_{21} are the partial derivative of Q with respect to δ . From the equation we can write

$$M_{ik} = \frac{\partial Q_i}{\partial \delta_k} = -|Y_{ik}V_iV_k|\cos(\theta_{ik} + \delta_k - \delta_i), \quad i \neq k$$

Similarly for i = k we have

$$M_{ii} = \frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}V_iV_k|\cos(\theta_{ik} + \delta_k - \delta_i) = P_i - |V_i|^2 G_{ii}$$

C. Formation of J12

Let us define J12 as

$$J_{12} = \begin{bmatrix} N_{12} & \cdots & N_{2n_i} \\ \vdots & \ddots & \vdots \\ N_{n2} & \cdots & N_{nn_i} \end{bmatrix}$$

As evident from equation the elements of J21 involve the derivatives of real power P with respect to magnitude of bus voltage |V| . For i ≠ k , we can write from equation

$$N_{ik} = |V_k|\frac{\partial P_i}{\partial |V_k|} = |Y_{ik}V_iV_k|\cos(\theta_{ik} + \delta_k - \delta_i) = -M_{ik} \quad i \neq k$$

For $i = k$ we have

$$N_{ii} = |V_i|\frac{\partial P_i}{\partial |V_i|} = |V_i|\left[2|V_i|G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}V_k|\cos(\theta_{ik} + \delta_k - \delta_i)\right]$$

$$= 2|V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}V_iV_k|\cos(\theta_{ik} + \delta_k - \delta_i) = 2|V_i|^2 G_{ii} + M_{ii}$$

Formation of J_{22}

For the formation of J_{22} let us define

$$J_{22} = \begin{bmatrix} O_{22} & \cdots & O_{2n_u} \\ \vdots & \ddots & \vdots \\ O_{n_u 2} & \cdots & O_{n_u n_u} \end{bmatrix}$$

For $i \neq k$ we can write from equation

$$O_{ik} = |V_i| \frac{\partial Q_i}{\partial |V_k|} = -|V_i| |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) = L_{ik}, \quad i \neq k$$

Finally for $i = k$ we have

$$O_{ii} = |V_i| \frac{\partial Q_i}{\partial |V_k|} = V_i \left[-2|V_i| B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \right]$$

$$= -2|V_i|^2 B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) = -2|V_i|^2 B_{ii} - L_{ii}$$

We therefore see that once the sub matrices J_{11} and J_{21} are computed, the formation of the sub matrices J_{12} and J_{22} is fairly straightforward. For large system this will result in considerable saving in the computation time.

Newton Raphson Method Flow Chart

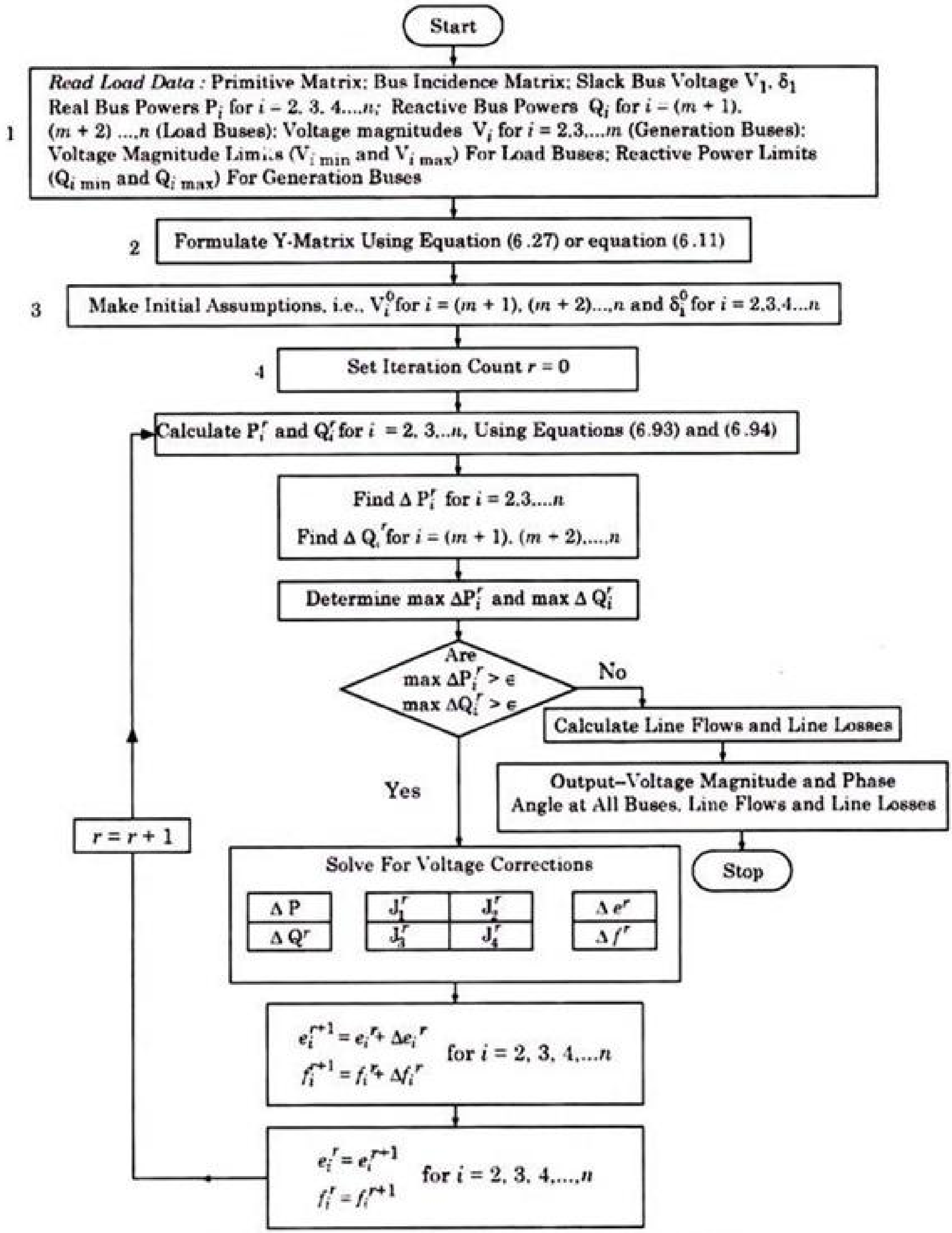
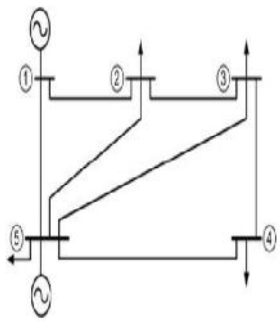


Fig. 6.21. ***Flow Chart For Power Flow Solution Using N–R Method***

Solution of Newton-Raphson Load Flow

The Newton-Raphson load flow program is tested on the system of Fig. with the system data and initial conditions given in Tables we can write



Line (bus to bus)	Impedance	Line charging (Y/2)
1-2	$0.02 + j\,0.10$	$j\,0.030$
1-5	$0.05 + j\,0.25$	$j\,0.020$
2-3	$0.04 + j\,0.20$	$j\,0.025$
2-5	$0.05 + j\,0.25$	$j\,0.020$
3-4	$0.05 + j\,0.25$	$j\,0.020$
3-5	$0.08 + j\,0.40$	$j\,0.010$
4-5	$0.10 + j\,0.50$	$j\,0.075$

$$L_{23}^{(0)} = -\left|Y_{23}V_2^{(0)}V_3^{(0)}\right|\sin\left(\theta_{23} + \delta_3 - \delta_2\right) = -\left|Y_{23}\right|\sin\theta_{23} = -B_{23} = -4.8077$$

Similarly from the equation we have

$$Q_2^{(0)} = -\left|V_2^{(0)}\right|^2 B_{22} - \sum_{\substack{k=1\\k\neq 2}}^{n}\left|Y_{2k}V_2^{(0)}V_k^{(0)}\right|\sin\left(\theta_{2k} + \delta_k - \delta_2\right)$$
$$= -B_{22} - 1.05B_{21} - B_{23} - B_{24} - 1.02B_{25} = -0.6327$$

Hence from the equation we get

$$L_{22}^{(0)} = -Q_2^{(0)} - \left|V_2^{(0)}\right|^2 B_{22} = -0.6327 - B_{22} = 18.8269$$

In a similar way the rest of the components of the matrix $J_{11}^{(0)}$ are calculated. This matrix is given by

$$J_{11}^{(0)} = \begin{bmatrix} 18.8269 & -4.8077 & 0 & -3.9231 \\ -4.8077 & 11.1058 & -3.8462 & -2.4519 \\ 0 & -3.8462 & 5.8077 & -1.9615 \\ -3.9231 & -2.4519 & -1.9615 & 12.4558 \end{bmatrix}$$

For forming the off diagonal elements of J_{21} we note from equation that

$$M_{23}^{(0)} = -\left|Y_{23}V_2^{(0)}V_3^{(0)}\right|\cos\left(\theta_{23} + \delta_2 - \delta_3\right) = -G_{23} = 0.9615$$

Also from equation the real power injected at bus-2 is calculated as

$$P_2^{(0)} = \left|V_2^{(0)}\right|^2 G_{22} + \sum_{\substack{k=1 \\ k\neq 2}}^{n} \left|Y_{2k} V_2^{(0)} V_k^{(0)}\right| \cos\left(\theta_{2k} + \delta_k - \delta_2\right)$$

$$= G_{22} + 1.05G_{21} + G_{23} + G_{24} + 1.02G_{25} = -0.1115$$

Hence from equation we have

$$M_{22} = P_2^{(0)} - \left|V_2^{(0)}\right|^2 G_{22} = -3.7654$$

Similarly the rest of the elements of the matrix J_{21} are calculated. This matrix is then given as

$$J_{21}^{(0)} = \begin{bmatrix} -3.7654 & 0.9615 & 0 & 0.7846 \\ 0.9615 & -2.2212 & 0.7692 & 0.4904 \\ 0 & 0.7692 & -1.1615 & 0.3923 \end{bmatrix}$$

For calculating the off diagonal elements of the matrix J12 we note from (4.47) that they are negative of the off diagonal elements of J21 . However the size of J21 is (3 X 4) while the size of J12 is (4 X 3). Therefore to avoid this discrepancy we first compute a matrix M that is given by

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}$$

The elements of the above matrix are computed in accordance with equation . We can then define

$$J_{21} = M(1:3, 1:4) \text{ and } J_{12} = -M(1:4, 1:3)$$

Furthermore the diagonal elements of J_{12} are overwritten in accordance with equation. This matrix is then given by

$$J_{12}^{(0)} = \begin{bmatrix} 3.5423 & -0.9615 & 0 \\ -0.9615 & 2.2019 & -0.7692 \\ 0 & -0.7692 & 1.1462 \\ 0.7846 & -0.4904 & -0.3923 \end{bmatrix}$$

Finally it can be noticed from equation that $J_{22} = J_{11}$ (1:3, 1:3). However the diagonal elements of J_{22} are then overwritten in accordance with equation.This gives the following matrix

$$J_{22}^{(0)} = \begin{bmatrix} 17.5615 & -4.8077 & 0 \\ -4.8077 & 10.8996 & -3.8462 \\ 0 & -3.8462 & 5.5408 \end{bmatrix}$$

From the initial conditions the power and reactive power are computed as

$$P_{calc}^{(0)} = \begin{bmatrix} -0.1115 & -0.0096 & -0.0077 & -0.0098 \end{bmatrix}^T$$
$$Q_{calc}^{(0)} = \begin{bmatrix} -0.6327 & -0.1031 & -0.1335 \end{bmatrix}^T$$

Consequently the mismatches are found to be

$$\Delta P^{(0)} = \begin{bmatrix} -0.8485 & -0.3404 & -0.1523 & 0.2302 \end{bmatrix}^T$$
$$\Delta Q^{(0)} = \begin{bmatrix} 0.0127 & -0.0369 & 0.0535 \end{bmatrix}^T$$

Then the updates at the end of the first iteration are given as

$$\begin{bmatrix} \delta_2^{(0)} \\ \delta_3^{(0)} \\ \delta_3^{(0)} \\ \delta_4^{(0)} \end{bmatrix} = \begin{bmatrix} -4.91 \\ -6.95 \\ -7.19 \\ -3.09 \end{bmatrix} \text{deg} \quad \begin{bmatrix} |V_2|^{(0)} \\ |V_3|^{(0)} \\ |V_4|^{(0)} \end{bmatrix} = \begin{bmatrix} 0.9864 \\ 0.9817 \\ 0.9913 \end{bmatrix}$$

The load flow converges in 7 iterations when all the power and reactive power mismatches are below 10−6.

Load Flow Results

It is to be noted here that both Gauss-Seidel and Newton-Raphson methods yielded the same result. However the Newton-Raphson method converged faster than the Gauss-Seidel method. The bus voltage magnitudes, angles of each bus along with power generated and consumed at each bus are given in Table 4.4. It can be seen from this table that the total power generated is 174.6 MW whereas the total load is 171 MW. This indicates that there is a line loss of about 3.6 MW for all the lines put together. It is to be noted that the real and reactive power of the slack bus and the reactive power of the P-V bus are computed from equation after the convergence of the load flow.

Bus no.	Bus voltage		Power generated		Load	
	Magnitude (pu)	Angle (deg)	P (MW)	Q (MVAr)	P (MW)	P (MVAr)
1	1.05	0	126.60	57.11	0	0
2	0.9826	– 5.0124	0	0	96	62
3	0.9777	– 7.1322	0	0	35	14
4	0.9876	– 7.3705	0	0	16	8
5	1.02	– 3.2014	48	15.59	24	11

The current flowing between the buses *i* and *k* can be written as

$$I_{ik} = -Y_{ik}(V_i - V_k), \quad i \neq k$$

Therefore the complex power leaving bus- *i* is given by

$$P_i + jQ_i - V_i I_i^*$$

Similarly the complex power entering bus- *k* is

$$P_k + jQ_k - V_k I_k^*$$

Therefore the $I^2 R$ loss in the line segment *i*-*k* is

$$P_{loss,i-k} = P_i - P_k$$

The real power flow over different lines is listed in Table 4.5. This table also gives the $I^2 R$ loss along various segments. It can be seen that all the losses add up to 3.6 MW, which is the net difference between power generation and load. Finally we can compute the line I^2X drops in a similar fashion. This drop is given by

$$Q_{drop,i-k} = Q_i - Q_k$$

However we have to consider the effect of line charging separately.

Power dispatched		Power received		Line loss (MW)
from (bus)	amount (MW)	in (bus)	amount (MW)	
1	101.0395	2	98.6494	2.3901
1	25.5561	5	25.2297	0.3264
2	17.6170	3	17.4882	0.1288
3	0.7976	4	0.7888	0.0089
5	15.1520	2	14.9676	0.1844
5	18.6212	3	18.3095	0.3117
5	15.4566	4	15.2112	0.2454
				Total = 3.5956

Consider the line segment 1-2. The voltage of bus-1 is $V_1 = 1.05 < 0°$ per unit while that of bus-2 is $V_2 = 0.9826 < -5.0124°$ per unit. From the equation we have

$$I_{12} = 0.9623 - j0.5187 = 1.0932\angle -28.33°$$

Therefore the complex power dispatched from bus-1 is

$$S_{12} = V_1 I_{12}^* \times 100 = -101.0395 - j54.4645$$

where the negative signal indicates the power is leaving bus-1. The complex power received at bus-2 is

$$S_{21} = V_2 I_{12}^* \times 100 = 98.6494 + j42.5141$$

Therefore out of a total amount of 101.0395 MW of real power is dispatched from bus-1 over the line segment 1-2, 98.6494 MW reaches bus-2. This indicates that the drop in the line segment is 2.3901 MW. Note that

$$|I_{12}|^2 \times R_{12} \times 100 = 1.0932^2 \times 0.02 \times 100 = 2.3901$$

where R_{12} is resistance of the line segment 1-2. Therefore we can also use this method to calculate the line loss.

Now the reactive drop in the line segment 1-2 is

$$|I_{12}|^2 \times X_{12} \times 100 = 1.0932^2 \times 0.1 \times 100 = 10.9508$$

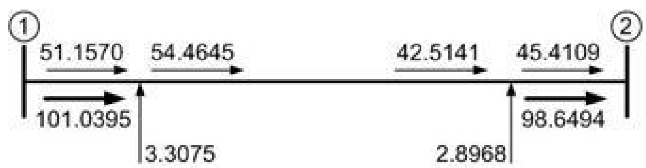
We also get this quantity by subtracting the reactive power absorbed by bus-2 from that supplied by bus-1. The above calculation however does not include the line charging. Note that since the line is modeled by an equivalent- p , the voltage across the shunt capacitor is the bus voltage to which the shunt capacitor is connected. Therefore the current *I* 12 flowing through line segment is not the current leaving bus-1 or entering bus-2 - it is the current flowing in between the two charging capacitors. Since the shunt branches are purely reactive, the real power flow does not get affected by the charging capacitors. Each charging capacitor is assumed to inject a reactive power that is the product of the half line charging admittance and square of the magnitude of the voltage of that at bus. The half line charging admittance of this line is 0.03. Therefore line charging capacitor will inject

$$0.03 \times 100 \times |V_1|^2 = 3.3075$$

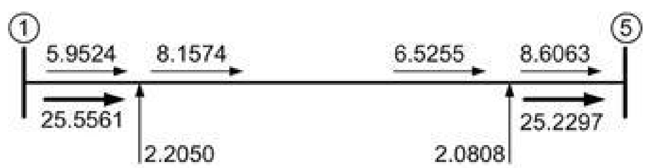
at bus-1. Similarly the reactive injected at bus-2 will be

$$0.03 \times 100 \times |V_2|^2 = 2.8968$$

The power flow through the line segments 1-2 and 1-5 are shown in Fig..



(a)



(b)

Advantages of Newton Raphson Method

The various advantages of Newton Raphson Method are as follows:-

- It possesses quadratic convergence characteristics. Therefore, the convergence is very fast.
- The number of iterations is independent of the size of the system. Solutions to a high accuracy are obtained nearly always in two to three iterations for both small and large systems.
- The Newton Raphson Method convergence is not sensitive to the choice of slack bus.
- Overall, there is a saving in computation time since less number of iterations are required.

Limitations of Newton Raphson Method

The various limitations are given below.

- This solution technique is difficult.
- It takes longer time as the elements of the Jacobian are to be computed for each iteration.
- The computer memory requirement is large.