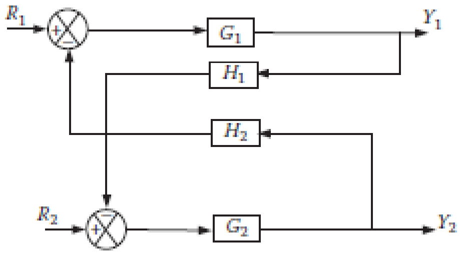
MULTIPLE-INPUT AND MULTIPLE-OUTPUT SYSTEMS

Most systems we have considered so far are single-input, single-output (SISO) systems, where there is one input and one output. For example, when a voltage is supplied to a motor, the motor rotates, and its angular velocity (output) can be measured. However, many systems have multiple degrees of freedom, where more than one variable controls the systems. In this case, multiple inputs and multiple outputs (MIMO) may be present.

$$\begin{cases} (R_1 - Y_2H_2)G_1 = Y_1 \\ (R_2 - Y_1H_1)G_2 = Y_2 \end{cases}$$



Substitute Y_2 into Y_1 to get:

$$(R_1 - (R_2 - Y_1H_1)G_2H_2)G_1 = Y_1$$

$$R_1G_1 - R_2G_1G_2H_2 = Y_1(1 - G_1G_2H_1H_2)$$

$$Y_1 = \frac{R_1G_1 - R_2G_1G_2H_2}{(1 - G_1G_2H_1H_2)}$$

Similarly, substitute Y_1 into Y_2 to get:

$$(R_2 - (R_1 - Y_2H_2)G_1H_1)G_2 = Y_2$$

$$R_2G_2 - R_1G_1G_2H_1 = Y_2(1 - G_1G_2H_1H_2)$$

$$Y_2 = \frac{R_2G_2 - R_1G_1G_2H_1}{(1 - G_1G_2H_1H_2)}$$

These two equations may be written in matrix form as:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \frac{G_1}{\mathbf{K}} & \frac{-G_1G_2H_2}{\mathbf{K}} \\ \frac{-G_1G_2H_1}{\mathbf{K}} & \frac{G_2}{\mathbf{K}} \end{bmatrix} \times \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

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$$\mathbf{K} = (1 - G_1G_2H_1H_2)$$

A multi-axis robot has multiple inputs and multiple outputs that must be controlled simultaneously. However, in most robots, each axis is controlled individually as a SISO unit. Although this introduces some error, the error is small for most practical purposes. The analysis of these systems is beyond the scope ofthis introduction to control theory. For further reading, please refer to related books and journal articles on

this topic.