

Analysis and design of singly and doubly reinforced rectangular beams by Limit state method

The Concrete beam whose only tension zone of cross-section area is covered with steel rod is known as a **singly reinforced beam**.

ANALYSIS OF SINGLY REINFORCED BEAMS PROBLEMS:

TYPE 1 PROBLEM:

GIVEN DATA: A_{st} in mm^2 or number of bars with diameter, size of beam (b, D), type of concrete (f_{ck}), type of steel (f_y), if load to be calculated then span is given. REQUIRED: Ultimate moment or factored moment or moment of resistance (M_u) or M_u & w .

Note:

1. Ultimate moment or factored moment (M_u) = $1.5 \times$ working moment = $1.5 \times M$
2. Ultimate load or factored load (w_u) = $1.5 \times$ working load = $1.5 \times w$

DESIGN STEPS:

STEP 1: Note down the value for $X_{u,max}/d$ by referring IS: 456-2000

F_y in N/mm^2	$X_{u,max}/d$
250	0.53
415	0.48
500	0.46

STEP 2: Determine depth of neutral axis X_u/d $X_u/d = (0.87.f_y.A_{st})/(0.36.f_{ck}.b.d)$

Where,

X_u = depth of neutral axis

F_y = characteristic tensile strength of steel in N/mm^2 A_{st} = area of steel in tension in mm^2

F_{ck} = characteristic compressive strength of concrete in N/mm^2 b = breadth or width of member

d = effective depth in mm

Effective depth (d) = overall depth (D) – effective cover (d')
Effective cover (d') = clear cover + diameter of bar/2

Clear cover for beam = 25mm.

STEP 3: Compare X_u/d and $X_{u,max}/d$

If $X_u/d < X_{u,max}/d$, then section is under reinforced. The moment of resistance is calculated by

$$M_u = 0.87.f_y.A_{st}.d. [1 - (f_y.A_{st})/(f_{ck}.b.d)]$$

If $X_u/d > X_{u,max}/d$, then section is over reinforced. The moment of resistance is calculated by

$$M_{u,lim} = 0.149.f_{ck}.b.d^2 \text{ for Fe250 steel.}$$

$$M_{u,lim} = 0.138.f_{ck}.b.d^2 \text{ for Fe415 steel.}$$

$$M_{u,lim} = 0.133.f_{ck}.b.d^2 \text{ for Fe500 steel.}$$

If the section is balanced that is $X_u/d = X_{u,max}/d$ then the limiting moment of resistance ($M_{u,lim}$) is calculated.

STEP 4: Working moment = $M = M_u/1.5$

The maximum bending moment for simply supported beam carrying UDL = $wl^2/8$ Now equating maximum bending moment and working moment

$$M = wl^2/8, w = 8M/l^2$$

Where w = total load = dead load + live load. DL

$$\text{self-weight of beam} = \rho.b.D = 25.b.D \text{ kN/m}$$

Live load = w -DL in kN/m.

PROBLEM 1. Find the depth of neutral axis of a singly reinforced R.C beam of 230mm width and 450mm effective depth. It is reinforced with 4 bars of 16mm diameter. Use M20 concrete and Fe415 bars. Also comment on the type of beam.

Given data: $b=230\text{mm}$,

$d=450\text{mm}$,

$A_{st}=4\text{-}\#16$,

$f_{ck}=20\text{ N/mm}^2$,

$f_y=415\text{ N/mm}^2$

Required: X_u

Solution:

Step1: As per IS: 456-2000 $X_{u,max}/d = 0.48$ for Fe415

Step2: $X_u/d = (0.87 \cdot f_y \cdot A_{st}) / (0.36 \cdot f_{ck} \cdot b \cdot d)$

$A_{st} = \text{no of bars} \times \pi(\text{diameter})^2/4 = 4 \times \pi(16)^2/4$

$$= 504.24 \text{ mm}^2$$

$$X_u/d = (0.87 \times 415 \times 504.24) / (0.36 \times 20 \times 230 \times 450)$$

$$= 0.39$$

$$X_u = 0.39 \times d = 0.39 \times 450 = 175.5\text{mm}.$$

Step3: By comparing $X_u/d < X_{u,max}/d$

Section is under-reinforced.

PROBLEM 2.A singly reinforced concrete beam 250mm width is reinforced with 4 bars of 25mm diameter at an effective depth 400mm. If M₂₀ grade concrete and Fe₄₁₅ bars are used. Compute moment of resistance of the section.

Given data: b=250mm, d=400mm, A_{st}=4-#25, f_{ck}=20 N/mm², f_y=415 N/mm² Required: Mu

Solution:

Step1: As per IS: 456-2000 $X_{u,max}/d = 0.48$ for Fe₄₁₅

Step2: $X_u/d = (0.87.f_y.A_{st})/(0.36.f_{ck}.b.d)$

$$A_{st} = \text{no of bars} \times \pi(\text{diameter})^2/4$$

$$= 4 \times \pi(25)^2/4 = 1963.75 \text{ mm}^2$$

$$X_u/d = (0.87 \times 415 \times 1963.75) / (0.36 \times 20 \times 250 \times 400)$$

$$= 0.98$$

Step3: By comparing $X_u/d > X_{u,max}/d$

Section is Over-reinforced.

Step4:

$$M_{u,lim} = 0.138.f_{ck}.b.d^2$$

$$= 0.138 \times 20 \times 250 \times 400^2$$

$$= 110400000 \text{ N-mm} = 110.4 \text{ kN-m}$$

PROBLEM 3.A simply supported singly reinforced beam having 250mm wide and 500mm effective depth provided with Fe₄₁₅ steel and M₂₀ grade of concrete. Determine the ultimate moment of resistance of beam.

Given data: b=250mm,

d=500mm,

f_{ck}=20 N/mm²,

f_y=415 N/mm²

Required: Mu,lim

Solution:

$$\begin{aligned} \text{Mu,lim} &= 0.138 \cdot f_{ck} \cdot b \cdot d^2 \\ &= 0.138 \times 20 \times 250 \times 500^2 \\ &= 172500000 \text{ N-mm} \\ &= 172.5 \text{ kN-m} \end{aligned}$$

PROBLEM 4.A rectangular section of 230x500mm is used as a simply supported beam for effective span of 6m. The beam consists of tensile reinforcement of 4000 mm² and center of reinforcement is placed at 35mm from the bottom edge. What maximum total UDL can be allowed on the beam? Given M20 concrete and Fe415 steel.

Given data:

b=230mm,

D=500mm,

simply supported beam, l=6m,

A_{st}=4000mm², d'=35mm,

f_{ck}=20 N/mm²,

$$f_y = 415 \text{ N/mm}^2$$

Required: w

Solution:

Step1: As per IS: 456-2000

$$X_{u,\max}/d = 0.48 \text{ for Fe415}$$

Step2:

$$X_u/d = (0.87 \cdot f_y \cdot A_{st}) / (0.36 \cdot f_{ck} \cdot b \cdot d)$$

$$d = D - d' = 500 - 35 = 465 \text{ mm}$$

$$X_u/d = (0.87 \times 415 \times 4000) / (0.36 \times 20 \times 230 \times 465)$$

$$= 1.875$$

Step3: by comparing $X_u/d > X_{u,\max}/d$

Section is Over-reinforced.

$$\text{Step4: } \mu_{u,\lim} = 0.138 \cdot f_{ck} \cdot b d^2 = 0.138 \times 20 \times 230 \times 465^2$$

$$= 137259630 \text{ N-mm}$$

$$= 137.26 \text{ kN-m}$$

$$\text{Step5: } M = \mu_u / 1.5$$

$$= 137.26 / 1.5 = 91.506 \text{ kN-m}$$

Maximum bending moment for simply supported beam with UDL

$$M = w l^2 / 8,$$

$$w = 8M / l^2$$

$$= 8 \cdot 91.506 / 6^2 = 20.33 \text{ kN-m}$$

ANALYSIS OF DOUBLY REINFORCED BEAM

Definition: the RCC beam section in which steel reinforcement is provided to resist both compression and tension is called doubly reinforced beam.

The circumstances under which doubly reinforced sections are provided:

1. When there are architectural restrictions on the depth of otherwise singly reinforced section.
2. Restriction in the depth at the location of beam at plinth level, along with the provision of ventilator between the ground level and the bottom of plinth beam.
3. In a continuous beam floor system, where the beam acts as a T-beam in the midspan and acts as a rectangular beam at the supports where the B.M may be much greater than at the mid span.
4. Where it is required to increase the stiffness of the beam.
5. It is found that the compression steel increases the rotation capacity and ductility

TYPE I PROBLEMS (M_u):

Given data: A_{st} , A_{sc} , size of beam, effective cover for compression steel (d'), type of concrete (f_{ck}) and steel (f_y). If load to be calculated then span is given.

Required: ultimate moment or factored moment or moment of resistance (M_u) and super imposed load (w).

Solution:

Step1: calculate $A_{sc} = \text{no of bars} \times \pi (\phi_c)^2 / 4 \text{ mm}^2$, $A_{st} = \text{no of bars} \times \pi (\phi_t)^2 / 4 \text{ mm}^2$ Where, $\phi_c =$ diameter of compression steel, $\phi_t =$ diameter of tension steel

Step2: $X_{u,max} = 0.46d$ for Fe500

0.48d for Fe415

0.53d for Fe250

Step3: stress in compression (f_{sc}): from the table F of SP16 by linear interpolation Calculate d'/d

For Fe250 $f_{sc}=0.87.f_y$

Step4: $A_{st2} = A_{sc}.f_{sc} / (0.87.f_y)$

Step5: $A_{st} = A_{st1} + A_{st2}$

$A_{st1} = A_{st} - A_{st2}$

Step 6: depth of neutral axis

$X_u/d = (0.87.f_y.A_{st1}) / (0.36.f_{ck}.b.d)$ or $X_u = 0.87.f_y.A_{st} - f_{sc}.A_{sc} / (0.36.f_{ck}.b)$

If, $X_u < X_{u,max}$ section is under reinforced.

Therefore, calculate the moment of resistance by the following expression $M_u - M_{u,lim} = f_{sc}.A_{sc}.(d-d')$

$M_u = 0.36.f_{ck}.b.x_u.(d-0.42.x_u) + f_{sc}.A_{sc}(d-d')$ If, $X_u > X_{u,max}$ section is over reinforced.

Put $X_u = X_{u,max}$ value and calculate moment of resistance by the following expression, $M_u = 0.36.f_{ck}.b.x_{u,max}.(d-0.42.x_{u,max}) + f_{sc}.A_{sc}(d-d')$

To calculate safe udl of live load: follow same steps as in singly reinforced beams.

Problem 1 A doubly reinforced beam section is 250mm wide and 450mm deep to center of the tensile reinforcement. It is reinforced with 2 bars of 16mm diameter as compressive reinforcement at an effective cover 50mm and 4 bars of 25mm diameter as tensile steel. Using M15 concrete and Fe250 steel. Calculate the ultimate moment of resistance of the beam.

Given data:

$b=250\text{mm},$

$d=450\text{mm},$

$f_{ck}=15\text{ N/mm}^2,$

$f_y=250\text{ N/mm}^2,$

$d'=50\text{mm},$

$A_{sc}= 2- \#16, A_{st}=4- \#25$

Required: Mu Solution:

Step1: calculating Asc= no of bars x $\pi (\phi_c)^2/4$

$$=2 \pi(16)^2/4=402.12\text{mm}^2 \text{ Ast}$$

$$= \text{no of bars} \times \pi (\phi_t)^2/4=4 \times \pi(25)^2/4=1963.49\text{mm}^2$$

Step2: Xu,max=0.53d for Fe250

$$=0.53 \times 450=238.5\text{mm}$$

Step3: stress in compression (fsc)

$$f_{sc}=0.87.f_y=0.87 \times 250=217.5 \text{ N/mm}^2$$

step4: Ast2=(Asc. fsc)/(0.87. fy)

$$=(402.12 \times 217.5)/(0.87 \times 250)=402.12\text{mm}^2$$

step5: Ast=Ast1+Ast2, Ast1

$$=A_{st}-A_{st2}=1963.49-402.12=1561.37\text{mm}^2$$

step6: depth of neutral axis (Xu)

$$X_u=0.87.f_y.A_{st1}/(0.36.f_{ck}.b)$$

$$=0.87 \times 250 \times 1561.37/(0.36 \times 15 \times 250)$$

$$=251.5\text{mm}$$

Xu> Xu,max section is over reinforced.

Step7: Mu=0.149.fck.b.d²+fsc.Asc.

$$(d-d')=0.149 \times 15 \times 250 \times 450^2+217.5 \times 402.12 \times (450-50)=148.13 \times 10^6 \text{ N-mm}$$

$$Mu=148.13 \text{ kN-m.}$$

Problem 2 A doubly reinforced beam section is 250mm wide and 500mm deep to the center of the tensile reinforcement. It is reinforced with 2 bars of 18mm diameter as compression reinforcement at an effective cover of 40mm and 4 bars of 25mm diameter as tensile reinforcement using M15 concrete and Fe415 steel. Calculate MR of the section.

Given data:

$b=250\text{mm}$,

$d=500\text{mm}$,

$f_{ck}=15 \text{ N/mm}^2$,

$f_y=415 \text{ N/mm}^2$,

$d'=40\text{mm}$,

$A_{sc}= 2\text{-}\#18$,

$A_{st}=4\text{-}\#25$

Required: M_u

Solution:

Step1: calculating,

$$\begin{aligned} A_{st} &= \text{no's} \times \pi \cdot (\phi_t)^2 / 4 \\ &= 4 \times \pi \times (25)^2 / 4 = 1963.49 \text{mm}^2 \\ A_{sc} &= \text{no of bars} \times \pi (\phi_c)^2 / 4 = 2 \times \pi \times (18)^2 / 4 = 508.93 \text{mm}^2 \end{aligned}$$

Step2:

$$\begin{aligned} X_{u,\max} &= 0.48d \text{ for Fe415} \\ &= 0.48 \times 500 \\ &= 240 \text{mm} \end{aligned}$$

Step3: stress in compression $(f_{sc})d'/d$

$$\begin{aligned} &= 40/500 \\ &= 0.08 \end{aligned}$$

By referring table –F of sp16 d'/d f_{sc}

0.05 355 Y1

0.08 ? Y

0.1 353 Y2

$$\begin{aligned} Y &= Y1 + ((Y2-Y1)/(X2-X1))*(X-X1) \\ &= 355 + ((353-355)/(0.1-0.05))*(0.08-0.05) = 353.8 \text{ N/mm}^2 \\ F_{sc} &= 353.8 \text{ N/mm}^2 \end{aligned}$$

step4:

$$\begin{aligned} A_{st2} &= (A_{sc} \cdot f_{sc}) / (0.87 \cdot f_y) \\ &= (508.93 \times 353.8) / (0.87 \times 415) = 498.71 \text{ mm}^2 \end{aligned}$$

step5:

$$\begin{aligned} A_{st} &= A_{st1} + A_{st2}, \\ A_{st1} &= A_{st} - A_{st2} \\ &= 1963.49 - 498.71 \\ &= 1464.78 \text{ mm}^2 \text{ step6: depth of neutral axis } (X_u) \\ X_u &= (0.87 \cdot f_y \cdot A_{st} - f_{sc} \cdot A_{sc}) / (0.36 \cdot f_{ck} \cdot b) \\ &= (0.87 \times 415 \times 1963.49 - 353.8 \times 508.93) / (0.36 \times 15 \times 250) \\ &= 391.74 \text{ mm} \end{aligned}$$

$X_u > X_{u,max}$ section is over reinforced.

Step7:

$$\begin{aligned} M_u &= 0.138 \cdot f_{ck} \cdot b \cdot d^2 + f_{sc} \cdot A_{sc} \cdot (d - d') \\ &= 0.138 \times 15 \times 250 \times 500^2 + 353.8 \times 508.93 \times (500 - 40) \\ &= 212.2 \times 10^6 \text{ N-mm} \\ M_u &= 212.2 \text{ kN-m.} \end{aligned}$$