2.2 REPRESENTATION OF LOGIC FUNCTION

The use of switching devices like transistors give rise to a special case of the Boolean algebra called as switching algebra.In switching algebra, all the variables assume one of the two values which are 0 and 1.

In Boolean algebra,0 is used to represent the'open'state or'false'state of logic gate. Similarly,1is used to represent the'closed'state or'true'state of logic gate.

A Boolean expression is an expression which consists of variables,constants (0-false and 1-true) and logical operators which results in true or false.

A Boolean function is an algebraic form of Boolean expression. A Boolean function of n-variables is represented by f(x1, x2, x3….xn). By using Boolean laws and theorems, we can simplify the Boolean functions of digital circuits. A brief note of different ways of representing a Boolean function is shown below.

- Sum-of-Products (SOP) Form
- Product-of-sums (POS) form
- Canonical forms

There are two types of canonical forms:

- Sum-of-minterms or Canonical SOP
- Product-of-max terms or Canonical POS

Boolean functions can be represented by using NAND gates and also by using K-map (Karnaugh map) method. We can standardize the Boolean expressions by using by two standard forms.

SOPform–Sum Of Products form

POSform–Product Of Sums form

Standardization of Booleane quations will make the implementation,evolution and simplification easier and more systematic.

SumofProduct(SOP)Form

Thesum-of-products(SOP)formisamethod(orform)ofsimplifyingtheBoolean expressionsoflogicgates.InthisSOPformofBooleanfunctionrepresentation,the

variablesareoperatedbyAND(product)toformaproducttermandalltheseproduct termsareORed(summedoradded)togethertogetthefinal function.

A sum-of-products form can be formed by adding (or summing) two or more producttermsusingaBooleanadditionoperation.Heretheproducttermsaredefinedbyusing theANDoperationandthesumtermisdefinedbyusingORoperation.

Thesum-of-productsformisalsocalledasDisjunctiveNormalFormastheproductterms areORedtogetherandDisjunctionoperationislogicalOR.Sum-of-productsformisalso called as Standard SOP.

SOPformrepresentationismostsuitabletousethemin FPGA(FieldProgrammableGate Arrays).

Examples

$AB + ABC + CDE$

$(\overline{AB}) + ABC + CD\overline{E}$

SOPformcanbeobtainedby

- WritinganANDtermforeachinputcombination,whichproducesHIGHoutput.
- Writingtheinputvariablesifthevalueis1,andwritethecomplementofthe variable if its value is 0.
- ORtheANDtermstoobtaintheoutputfunction.

Ex:Booleanexpressionformajorityfunction$F=A'BC+AB'C+ABC'+ABC$ Truth table:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Nowwritetheinputvariablescombinationwithhighoutput.$F=AB+BC+AC$. Checking

By Idempotence law, we know that

([ABC+ABC)]+ABC)=(ABC +ABC)=ABC

Now the function F=A'BC+AB'C+ABC'+ABC

=A'BC+AB'C+ABC'+([ABC+ABC)]+ABC)

=(ABC+ABC')+(ABC+AB'C)+(ABC+A'BC)

=AB(C+ C')+A(B+B')C+ (A+A')BC

=AB +BC +AC.

ProductofSums(POS)Form

The product of sums form is a method (or form) of simplifying the Boolean expressions of logic gates. In this POS form, all the variables are ORed, i.e. written as sums to form sum terms.

All these sum terms are ANDed (multiplied) together to get the product-of-sum form. This form is exactly opposite to the SOP form. So this can also be said as "Dual of SOP form".

Here the sum terms are defined by using the OR operation and the product term is defined by using ANDoperation. When twoormoresumterms are multiplied by aBoolean OR operation, the resultant output expression will be in the form of product-of-sums form or POS form.

The product-of-sums form is also called as Conjunctive Normal Form as the sum terms are ANDed together and Conjunction operation is logical AND. Product-of-sums form is also called as Standard POS.

Examples

(A+B)*(A+B+C)*(C+D) (A+B)¯*

(C + D + E¯)

POSformcanbeobtainedby

- WritinganORtermforeachinputcombination,whichproducesLOWoutput.
- Writingtheinputvariablesifthevalueis0,andwritethecomplementofthe variable if its value is 1.

- ANDtheORtermstoobtaintheoutput function.

Ex: Boolean expression for majority function F = (A + B + C) (A + B + C ') (A + B' + C) (A' + B + C)

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Nowwritetheinputvariablescombinationwithhighoutput.F=AB+BC+AC. Checking

ByIdempotencelaw,weknowthat

[(A+B +C) (A+B +C)](A+B +C) = [(A+B +C)](A+B+ C)= (A+B +C)

Nowthe function

F =(A+B) (B+ C)(A+C)

=(A+B+C) (A+B +C ')(A+B'+C) (A'+B +C)

=[(A+B +C)(A+B+C)](A+B +C) (A+B +C ') (A+B'+C)(A'+B +C)

=[(A+B+C) (A+B +C ')][(A+B +C) (A'+B +C)][(A+B+ C)(A+B'+C)]

=[(A+B) +(C *C')][(B+C)+(A*A')][(A+C)+(B* B')]

= [(A+B) +0][(B+ C)+ 0][(A+C) +0]=(A+B) (B+ C)(A+C)

CanonicalForm(StandardSOPandPOS Form)

AnyBooleanfunctionthatisexpressedasasum ofmintermsorasaproductofmax terms is said to be in its "canonical form".

ItmainlyinvolvesintwoBooleanterms,"minterms"and"maxterms".

WhentheSOPformofaBooleanexpressionisincanonicalform,theneachofitsproduct termiscalled'minterm'.So,thecanonicalformofsumofproductsfunctionisalso

knownas"mintermcanonicalform"orSum-of-mintermsorstandardcanonicalSOP form.

Similarly,whenthePOSformofaBooleanexpressionisincanonicalform,theneachof its sum term is called 'maxterm'. So, the canonical form of product of sums function is also known as "maxterm canonical form or Product-of sum or standard canonical POS form".

Min terms

Amintermisdefinedasthe producttermofnvariables,inwhicheachofthenvariables will appear once either in its complemented orun-complemented form. The min termis denoted as mi where i is in the range of $0 \leq i < 2^n$.

A variable is in complemented form, if its value is assigned to 0, and the variable is un-complimented form, if its value is assigned to 1.

Fora2-variable(xandy)Booleanfunction,thepossiblemintermsare: x'y', x'y, xy' and xy.

Fora3-variable(x,yandz)Booleanfunction,thepossiblemintermsare: x'y'z', x'y'z, x'yz', x'yz, xy'z', xy'z, xyz' and xyz.

- 1–Minterms=mintermsforwhichthefunctionF=1.
- 0–Minterms=mintermsforwhichthefunctionF=0.

AnyBooleanfunctioncanbeexpressedasthesum(OR)ofits1-minterms.The representation of the equation will be

- F(listofvariables)=Σ(listof1-mintermindices) Ex:

F (x, y, z) = Σ (3, 5, 6, 7)

Theinverseofthefunctioncanbeexpressedasasum(OR)ofits0-minterms.The representation of the equation will be

- F(listofvariables)=Σ(listof0-mintermindices) Ex:

F' (x, y, z) = Σ (0,1, 2, 4)

Examplesofcanonicalformofsumofproductsexpressions(mintermcanonicalform):

i) Z=XY+XZ′

ii) F=XYZ′+ X′YZ+X′YZ′+XY′Z+ XYZ

InstandardSOPform,themaximumpossibleproducttermsfornnumberofvariables are givenby2^n.So,for2variableequations,theproductterms are 2^2 =4.Similarly,for 3 variable equations, the product terms are $2^3 = 8$.

Maxterms

A max term is defined as the product of n variables, within the range of $0 \leq i < 2^n$. The max term is denoted as Mi. In max term, each variable is complimented, if its value is assigned to 1, and each variable is un-complimented if its value is assigned to 0. Fora2-variable(xandy)Booleanfunction,thepossiblemaxtermsare: $x + y$, $x + y'$, $x' + y$ and $x' + y'$.

Fora3-variable(x,yandz)Booleanfunction,thepossiblemaxtermsare: $x+y+z, x+ y+z', x+y'+z, x+y'+z', x'+ y+z, x'+ y+z', x'+ y'+z$ andx'+y' $+ z'$.

- $1-$Maxterms=maxtermsforwhichthefunctionF=1.
- $0-$maxterms=maxtermsforwhichthefunctionF=0.

Any Boolean function can be expressed the product (AND) of its $0-$ max terms. Therepresentation of the equation will be

- F(listofvariables)=Π(listof0-maxtermindices) Ex:

F $(x, y, z) = \Pi (0, 1, 2, 4)$

The inverse of the function can be expressed as a product (AND) of its $1 -$ max terms. The representation of the equation will be

- F(listofvariables)=Π(listof1-maxtermindices) Ex:

F' $(x, y, z) = \Pi (3, 5, 6, 7)$

Examplesofcanonicalformofproductofsumsexpressions(maxtermcanonicalform):

i. Z=(X+Y) (X+Y′)

ii. F=(X′+Y+ Z′) (X′+Y+ Z) (X′+ Y′+ Z′)

In standard POS form, the maximum possible sum terms for n number of variables are given by 2^n. So, for 2 variable equations, the sum terms are $2^2 = 4$. Similarly, for 3 variable equations, the sum terms are $2^3 = 8$.

Tablefor2nmintermsand2nmaxterms

Thebelowtablewillmakeyouunderstandabouttherepresentationofthemeanterms andmaxtermsof3variables.

Variables			Min terms	Max terms
A	B	C	m_i	M_i
0	0	0	A' B' C' = m 0	A + B + C = M 0
0	0	1	A' B' C = m 1	A + B + C' = M 1
0	1	0	A' B C' = m 2	A + B' + C = M 2
0	1	1	A' B C = m 3	A + B' + C' = M 3
1	0	0	A B' C' = m 4	A' + B + C = M 4
1	0	1	A B' C = m 5	A' + B + C' = M 5
1	1	0	A B C' = m 6	A' + B' + C = M 6
1	1	1	A B C = m 7	A' + B' + C' = M 7

ConversionsofCanonical Forms

We can represent theone canonical formed equation inother canonical formi.e. we can represent the SOP form of equation in POS form and POS form equation in SOP form. Toconvertthecanonicalequations,weinterchangethe$\Sigma$and Πsymbolsafterlistingout the index numbers of the equations, which are excluded from the original form of equation.

Theimportantthingtorememberabout Booleanfunctionsisthat,theSOPandPOSforms are Duals to each other. There are 2 steps to follow to convert the canonical formof the equations. They are

Step1:Interchangingtheoperationalsymbols,ΣandΠintheequation.

Step 2: Use the De Morgan's principle of Duality to the index numbers of the Boolean function or writing the indexes of the terms that are not presented in the given form of equation.

ConversionofSOPformtoPOSform

ToconverttheSOPformintoPOSform,firstweshouldchangetheΣtoΠandthenwrite the numeric indexes of missing variables of the given Boolean function.

Example:

TheSOP function

$F=\sum A,B,C(0,2,3,5,7)=A'B'C'+AB'C'+AB'C+ABC'+ABC$ iswritten inPOSformby

Step1:changingtheoperationalsigntoΠ

Step 2: writing the missing indexes of the terms, 001, 100 and 110. Now write the sum form for these noted terms.

001 =(A+B +C) 100 =(A+B'+C') 110 =(A+B'+C')

WritingdownthenewequationintheformofPOSform,

$F=\Pi A,B,C (1, 4,6) = (A+B+ C)*(A+B'+C') * (A+B'+ C')$

ConversionofPOSformtoSOPform

ToconverthePOSformintoSOPform,firstweshouldchangetheΠtoΣandthenwrite the numeric indexes of missing variables of the given Boolean function.

Ex: The POS function $F = \Pi A, B, C (2, 3, 5) = A' C' + A B' C + ABC'$ is written in SOP form by

Step1:changingtheoperationalsigntoΣ

Step2:writingthemissingindexesoftheterms,000,001,100,110,and111.Nowwrite the product form for these noted terms.

000 =A'* B'*C'001=A'*B'* C 100=A* B* C'

110 =A* B* C'111=A*B * C

WritingdownthenewequationintheformofSOPform,

$F = \Sigma A, B, C (0, 1, 4, 6, 7) = (A' * B' * C') + (A' * B' * C) + (A * B' * C') + (A *B* C') + (A * B * C)$

ConversionofSOPformtostandardSOPformorCanonicalSOPform

We can include all the variables in each product term of the SOP form equation, which doesn't have all the variables by converting into standard SOP form. The normal SOP form function can be converted to standard SOP form by using the Boolean algebraic law, $(A + A' = 1)$ and by following the below steps.

Step1:

Bymultiplyingeachnon-standardproductermwith thesumofits missing variableand its complement, which results in 2 product terms

Step2:

Byrepeatingthestep1,untilallresultingproducttermscontainallvariables

By these two steps we can convert the SOP function into standard SOP function. In this process, for each missing variable in the function, the number of product terms will double.

Example:

ConvertthenonstandardSOPfunctionF=xy+xz+yz Sol:

F =xy+xz+yz

=xy(z+z') +x (y+y')z+(x+ x') yz

=x yz+ xyz'+xyz +xy'z+ xyz+ x'yz

=x yz+ xyz'+xy'z+ x'yz

Thestandard SOPformis F=xyz+xyz'+xy'z+x'yz

ConversionofPOSformtostandardPOSformor CanonicalPOSform

We can include all the variables in each product term of the POS form equation, which doesn't have all the variables by converting into standard POS form. The normal POS form function can be converted to standard POS form by using the Boolean algebraic law, (A * A' = 0) and by following the below steps.

Step1:

By adding each non-standard sum term to the product of its missing variable and its complement, which results in 2 sum terms

Step2:

ApplyingBooleanalgebraiclaw,A+BC=(A+B)*(A+C) Step 3:

Byrepeatingthestep1,untilallresultingsumtermscontainallvariables

BythesethreestepswecanconvertthePOSfunctionintostandardPOSfunction. Example:

F=(A'+B +C) * (B'+C +D') * (A+B'+C' +D)

Inthefirstterm,thevariableDorD'ismissing,soweaddD*D'=1toit.Then (A' + B + C + D*D') = (A' + B + C + D) * (A' + B + C + D')

Similarly, in thesecond term, the variableA or A'is missing,so we add A*A'= 1 to it. Then

(B'+C+D'+A*A') =(A+B'+C+D')* (A'+B' +C +D')

Thethirdtermisalreadyinthestandardform,asithasallthevariables.Nowthestandard POS

form equation of the function is

F=(A'+B+C+D)*(A'+B+C+D')*(A+B'+C+D')*(A'+B'+C+D')*

(A+B'+C'+D)